An Analysis of the Stability of Hinterland Container Transport Cooperation

Alberto Giudici, Tao Lu, Clemens Thielen, Rob Zuidwijk

Abstract. We study cooperation among hinterland container transport operators that may share transport capacity and demand in corridors between inland and sea ports. We model this transportation problem as a minimum cost flow problem and assume that operators share the total cost based on a bargaining outcome, which has been proven equivalent to the Shapley value. To examine the stability of such cooperation, we perform a sensitivity analysis of the membership of the Shapley value (the bargaining outcome) to the core (the set of stable outcomes) by leveraging a novel concept of parametric cooperative games. We obtain closed-form solutions for identical players that explicitly characterize the impact of overcapacity on the stability of cooperation. For more general cases, we develop a computational approach based on parametric optimization techniques. The numerical results indicate that our primary analytical result, that is, that overcapacity undermines stability, is generally valid, and that overcapacitated networks may permit stable cooperation in only a limited range of settings.

1. Introduction

Cooperation in transportation has the potential to reduce total costs, at the risk of exposing companies to the failure of the cooperation itself. Cost reductions have a direct positive impact on profits. Failure, in contrast, threatens the companies’ market position and generates additional costs and losses (Park and Ungson 2003). Therefore, prior to engaging in a cooperation, managers need to be able to evaluate the conditions under which the cooperation will endure. Although achieving cost savings motivates the formation of a cooperation, the division of these savings among participating companies may lead to failure. As highlighted by Basso et al. (2019), real-world cases of horizontal cooperation in ground transport are rare and disagreements regarding the division of benefits may prevent the formation of a cooperation. Design issues in the early phase of the collaboration are critical for real cooperations, such as the case of transport operators in the region of the port of Rotterdam (Ypsilantis and Zuidwijk 2019), or forestry transportation in Sweden (Frisk et al. 2010). Thus far, only limited guidelines exist to show how a transport setting affects the stability of a collaboration. This leads to the following question: Given the size of the cooperation as well as the transport network, costs, capacities, and orders, is it possible to predict whether a cooperation will be stable or not?

We focus on hinterland container transport, although we believe that our results are applicable to other transport domains as well.

The hinterland of a sea-port is the inland region of locations that can be served by transportation services from (import) or to (export) the sea terminals (Notteboom and Rodrigue 2007). On its way from the terminals at the port to a warehouse (or vice versa), a loaded container is moved through a sequence of transshipment and transport operations that might involve road, rail, and inland waterway transport. Terminal operators—both at the port and inland—and transport operators as well as other stakeholders are involved in the organization of container transport in the hinterland.

In this industry, very low margins and a strong pressure on the performance of transport chains drive
companies to cooperate. Hinterland costs account for 40%–80% of the overall door-to-door container transport costs (Notteboom 2004). Moreover, hinterland accessibility has become a mayor success factor for ports (de Langen 2004). Shippers, the cargo owners, increasingly require reliability of their hinterland transport chains (Port of Rotterdam 2018). Point-to-point connections between port and inland terminals, called corridors, are seen as an opportunity to alleviate the downsides of visiting congested port areas, and arise naturally in the development of ports (Notteboom and Rodrigue 2005). To make hinterland regions accessible, transport corridors between ports and inland terminals have to be formed by cooperating transport operators (Wilmsmeier, Monios, and Lambert 2011). A successful example in the Netherlands is that of Brabant Intermodal, where barge operators cooperated to consolidate their visits to port terminals (Veenstra, Zuidwijk, and Van Asperen 2012).

We study transport cooperation in which orders and transport capacity are shared. Companies strive to minimize the costs of their joint transport plans. Cost savings will then be divided following an agreed-upon mechanism, where the individual interests of each company ideally are advanced. Unfortunately, even if an agreement is reached, the cooperation is still exposed to the risk of failure. Indeed, self-interested negotiations do not take into account the interests of groups of companies, which might have incentives, as a subcoalition, to drop out (Park and Ungson 2003). The economic literature has established that the Shapley value (Shapley 1953) provides a reasonable prediction for the cost allocation when players engage in a noncooperative bargaining process (Gul 1989, Pérez-Castrillo and Wettstein 2001). For this reason, we focus on the Shapley value as the cost allocation concept in this paper. Thus, the question is under what conditions the Shapley value helps establish a stable coalition.

1.1. Illustrating Example

As an illustrating example, consider three operators $P_1$, $P_2$, and $P_3$ that offer transport between a port and its hinterland. Their networks partially overlap, which allows the formation of a corridor that serves demand from the port $s$ to a common inland terminal $t$ as shown in Figure 1. The unit transport cost faced by each operator for transport from origin to destination are given in Table 1. We assume that each operator chooses to share 15 units of orders and 30 units of capacity out of their total orders and capacity. The orders correspond to the demand from the port $s$ to the terminal $t$. Capacity is mapped to each operator’s arc in the graph $G_T$ in Figure 1. Either individually or as a cooperation, all orders are transported at minimum cost from node $s$ to node $t$ in $G_T$. Individually, operator $P_i$ would face a cost equal to $15c_i$ by using only her arc, which leads to a minimum total cost of 1125 without cooperation. When

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the three operators form a cooperation, the pooled orders are transported using pooled capacities and a minimum total cost of 975 can be achieved, which corresponds to about 10% savings. We can test whether the operators are able to form a stable cooperation if the cost allocation is based on a bargaining outcome. Assuming that the bargaining process follows Gul (1989), the Shapley value cost allocation $\Phi_i$ to each operator is reported in Table 1. The value $\Phi_i$ represents the share of the total cost of 975 that operator $P_i$ would agree to pay after the bargaining phase. Despite reaching such an agreement, it can be observed that operators $P_1$ and $P_3$ have an incentive to drop out of the cooperation: without operator $P_2$, they would face a cost of 600 for transporting their orders using only their arcs $P_1$ and $P_3$, which is lower than their allocated cost share of $\Phi_1 + \Phi_3 = 625$ in the three-way collaboration. Therefore, the cooperation would fail. Interestingly, if only a few orders more (18 rather than 15 units) were shared by each operator, or less capacity were pooled (25 rather than 30 units), no breakaway subcoalition would form and the cooperation would not suffer from failure (cf. Appendix A). Had the operators been guided by these insights ahead of bargaining, a more durable cooperation could have been designed.

1.2. Our Contribution

In the literature, it has been observed that sharing demand and capacity in a collaborative transport network leads to cost reductions, but may or may not result in a stable cooperation (cf. Frisk et al. 2010, Houghtalen, Ergun, and Sokol 2011). Most existing findings, however, are based on purely numerical

<table>
<thead>
<tr>
<th>Table 1. Parameter Setting for the Example: Per Unit Cost $c_i$ and Shapley Value Cost Allocation $\Phi_i$</th>
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<td>$c_i$</td>
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<td>$\Phi_i$</td>
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Figure 1. Example of a Collaborative Transport Network
observations. There is a lack of theoretical results and mathematical characterization for the structure of this problem. In this paper, we attempt to fill this gap.

To this end, we propose a novel methodology that combines cooperative game theory with parametric optimization to characterize the stability of cooperation in transport networks. Specifically, we first provide a mathematical characterization of the stability of the Shapley value as a function of a cost parameter, and then derive a closed-form characterization of whether the bargaining outcome (i.e., the Shapley value allocation) is stable for a special case with identical players. The obtained closed-form solutions are then extended in order to formally analyze the same setting on a richer network structure. For general cases, we develop a parametric optimization-based algorithm to efficiently evaluate the stability of cooperation. Furthermore, in the event that the Shapley value allocation is not stable, we introduce a measure of instability of cooperation, which is inspired by the concept of the $\epsilon$-Core in Shapley and Shubik (1966) and is similar to $\epsilon$-stability in Karsten, Slikker, and van Houtum (2015). We measure instability by computing the maximum gap between the Shapley value allocation to a coalition and the cost generated by this coalition: the larger the gap, the more willing that coalition is to drop out of the cooperation. Under mild conditions, we further prove that this measure is bounded whenever no subcoalition achieves higher cost savings than does the group of companies taken as a whole. To the best of our knowledge, we are the first to perform a sensitivity analysis of cooperative game solution concepts in transport networks that permits analytical results in conjunction with a general computational approach that exploits the structure of the game. Our results provide a systematic understanding about how demand, capacity, and operating costs impact the stability of cooperation.

The remainder of this paper is organized as follows: Section 2 reviews the literature on collaborative transport, and Section 3 provides an overview of basic concepts of cooperative game theory used in the paper. The mathematical model is formulated in Section 4. Our analytical results are presented in Section 5, and the numerical study is reported in Section 6. Section 7 concludes the paper.

2. Literature Review

Our work contributes to the literature on collaborative transport, which has been gaining traction over the past years. In this section, we review the studies most relevant to our work. For a broader overview, we refer interested readers to the comprehensive survey in Guajardo and Rönqvist (2016) and to an overview of practical challenges in collaborative transport in Basso et al. (2019). The main problem considered in this stream of research is to find a suitable mechanism (or solution concept) to allocate the total costs or profits generated between collaborating companies. In this paper, we take a different angle by asking whether a cooperation would be stable if the cost shares were determined by a reasonable bargaining outcome. As mentioned earlier, we adopt the Shapley value as the bargaining outcome as per the theoretical foundation by Gul (1989).

Collaborative operational planning problems have been considered by Houghtalen, Ergun, and Sokol (2011) and Frisk et al. (2010), who base their models on network flow problems (Ahuja, Magnanti, and Orlin 1993). Houghtalen, Ergun, and Sokol (2011) develop a capacity exchange pricing mechanism to drive self-interested behavior in a cooperative setting toward social optimum, and they observe numerically that overcapacity undermines the stability of cooperation. Frisk et al. (2010) perform a real-world case study on cooperative truck transport in the forestry industry in Sweden. The Shapley value is found to be difficult to accept by practitioners, despite being able to capture synergies between companies more effectively than other solution concepts. Along this line, Crijnsen, Dullaert, and Fleuren (2007) explain that simple rules to divide the gains in horizontal cooperation cannot precisely capture the contribution of each individual player. The network flow problem considered in our paper is similar to those studied in Houghtalen, Ergun, and Sokol (2011) and Frisk et al. (2010); nevertheless, we complement their work by systematically analyzing the impact of overcapacity on the stability of cooperation given that the cost allocation is determined by a bargaining outcome. Among other results, we theoretically validate the numerical finding of Houghtalen, Ergun, and Sokol (2011) under certain conditions.

Other papers have considered tactical decision problems in cooperative settings, focusing on a very different scale than in our work. Lozano et al. (2013) study horizontal cooperation between shippers that jointly determine weekly transit frequencies of transport connections to satisfy pooled demand. Agarwal and Ergun (2010) and Zheng et al. (2015) address a network design problem in a partially decentralized setting related to maritime container transport, where service frequency and cargo flow have to be defined.

In broader contexts, the Shapley value has been applied by Engevall, Göthe-Lundgren, and Värbrand (2004) and Özener, Ergun, and Savelbergh (2013) to estimate cost-to-serve customers in oil distribution networks and vendor-managed inventory settings, respectively. In Crijnsen et al. (2010), the Shapley value is used to decide the order in which a logistic service provider should approach new customers to provide increasing benefits to already confirmed ones.
Methodologically, our work is related to literature on parametric optimization. In particular, our algorithm is based on work from Eisner and Severance (1976), which solves a parametric linear problem without requiring an implementation of the simplex method for linear programs. The parametric setting of our model is further related to the work of Carstensen (1983), which constructively show that exponentially many points might be required to describe the solution of a linear parametric network flow problem. We contribute to this stream of literature by identifying an application of parametric optimization in cooperative game theory.

3. Basic Concepts

In this section, we recall basic concepts of cooperative game theory that are used throughout the paper.

In contrast to its noncooperative counterpart, cooperative game theory studies the bargaining process in a cooperative setting under complete information where contracts might be enforced (Nash 1953). Cooperative games can be represented as transferable utility games (TU games) in characteristic form. Given a set \( N = \{1, 2, \ldots, n\} \) of companies (or players), the cost structure of the cooperation is described by a cost vector \( c = (c_S)_{S \subseteq N} \in \mathbb{R}^2^N \), where the component \( c_S \) represents the cost generated by coalition \( S \subseteq N \) (Serrano 2004). Games are characterized in terms of their properties. A game is called subadditive if \( c_S + c_T \geq c_{S \cup T} \) for all \( S, T \subseteq N \) with \( S \cap T = \emptyset \), and convex if \( c_{S \cup T} + c_{S \cap T} \leq c_S + c_T \) for all \( S, T \subseteq N \). Subadditivity implies that joining forces does not increase costs. Convexity, on the other hand, represents an advantageous situation where the incentive to join the cooperation grows with the cooperation size (Shapley 1971).

An allocation vector \( (x_i)_{i \in N} \in \mathbb{R}^n \) describes the cost \( x_i \) allocated to each player \( i \in N \), corresponding to the agreed outcome of the bargaining.

A solution concept \( \Psi \) represents a cost sharing mechanism and maps TU games to allocation vectors, that is, \( \Psi : T U^n \rightarrow \mathbb{R}^n \) with \( \Psi(c) = (x_i)_{i \in N} \), where \( T U^n \) is the set of \( n \)-person TU games and \( c = (c_S)_{S \subseteq N} \in T U^n \). Certain properties are sought: efficiency requires the total generated cost \( c_N \) to be completely allocated to the players \( (\sum_{i \in N} x_i = c_N) \), while individual rationality mandates that individual players do no worse under cooperation \( (x_i \leq c_i \text{ for each } i \in N) \). We focus on the Shapley value \( \Phi \in \mathbb{R}^n \) (Shapley 1953), with the component \( \Phi_i \) for a player \( i \in N \) defined as follows:

\[
\Phi_i = \frac{1}{n} \sum_{S \subseteq N \setminus \{i\}} \left( \frac{n-1}{|S|} \right)^{-1} (c_{S \cup \{i\}} - c_S).
\]

The allocation \( \Phi_i \) is a weighted average of the marginal cost of player \( i \) to any coalitions she can join. Results from the so-called Nash-program proved that the Shapley value is the outcome of a non-cooperative game that models bargaining (Serrano 2004). Thus, (1) provides an explicit form for the bargained cost division. Pérez-Castrillo and Wettstein (2001) show that the Shapley value coincides with the perfect subgame equilibrium outcomes of a non-cooperative game. Engevall, Göthe-Lundgren, and Värbrand (2004) show that, whenever the game needs to be constructed, the computation time for the Shapley value is negligible.

The concept of the core describes the set of allocation vectors that do not give any subcoalition an incentive to leave the cooperation (Gillies 1959). Formally, the core is the set \( \mathcal{C} \subseteq \mathbb{R}^n \) defined as

\[
\mathcal{C} := \{ (x_i)_{i \in N} \in \mathbb{R}^n : \sum_{i \in N} x_i = c_N, \sum_{i \in S} x_i \leq c_S \text{ for all } S \subseteq N \}. \tag{2}
\]

The inequalities in (2) capture the so-called coalitional rationality of an allocation vector. It requires a given allocation \( (x_i)_{i \in N} \) not to assign a total cost \( \sum_{i \in S} x_i \) to a coalition \( S \) that is higher than the cost \( c_S \) that coalition would generate by itself. If such a case occurs, the players in \( S \) would form a subcoalition and abandon the larger game. Shapley (1971) shows that, if a game is convex, the Shapley value is within the core.

4. Model Definition

The current approach in the literature to test the properties of a collaboration is to sample the parameter space of a transport setting, generate one game for each sample, and then test the desired properties. We instead propose an evaluation of the stability of the Shapley value that uses parametric solutions of the problem and results in a parametric sensitivity analysis.

We start by defining the transport problem and the resulting cooperative game in a classical sense, later expanding them to the parametric case.

Let \( N = \{1, \ldots, n\} \) be the set of companies that jointly operate the transport network given by the directed graph \( G = (V, R) \), with node set \( V \) and arc set \( R \). The set of potentially shared services need not constitute the whole network operated by each company. All transport demand originates at the source node \( s \in V \), for example a sea port, and must be transported to a single inland destination node \( t \in V \), where \( s \neq t \) (uniqueness of the destination is without loss of generality, see Online Appendix A). The single-source problem describes a scenario where demand originates either from a single terminal or from multiple, tightly-grouped terminals. It is assumed that all transport demand can be transported on time with pooled services, which allows us to exclude the time element from explicit consideration and represents a realistic assumption when considering time-insensitive cargo. For each company \( i \in N \), let \( R_i \subseteq R \) and \( k_i \in \mathbb{N}_{\geq 0} \) be the set of services owned and the amount of demand shared by \( i \), respectively. We assume that no arc is owned by multiple companies, that is, \( R' \cap R'' = \emptyset \) for
and that all arcs are assigned to companies, that is, \( \bigcup_{i \in N} R^i = R \). Transport of containers in the hinterland of a port is done either by truck, train, or barge, which are distinguished here in terms of capacity and unit transport cost. For each service \( r \in R \), let \( u_r \in \mathbb{N}_{\geq 0} \) be its shared capacity and \( c_r \in \mathbb{N}_{\geq 0} \) the unit transport cost on service \( r \). We assume that companies share only part of their total vehicle or barge capacity per service. Capacities of transport means vary from a few TEUs (Twenty-feet Equivalent Units, a standard in container size) for trucks to hundreds for barges and trains, which justifies discreteness of flow.\(^1\)

Given a group of operators \( S \subseteq N \), let \( R^S := \bigcup_{i \in S} R^i \) and \( k^S := \sum_{r \in S} k^r \) be the set of services controlled and the amount of demand pooled by \( S \), respectively. We concentrate only on total transport costs, requiring each coalition \( S \) to find a feasible flow allocation \((f_r)_{r \in R^S}\) transporting \( k^S \) units of flow from \( s \) to \( t \) on the graph \( G^S := (V, R^S) \) such that \( f_r \leq u_r \) for all \( r \in R^S \), that minimizes the total transport cost \( \sum_{r \in R^S} c_r f_r \). If we denote by \( \delta^+(v) \) and \( \delta^-(v) \) the set of outgoing and incoming arcs of node \( v \in V \), respectively, we can define the following integer programming formulation \( P^S \) for this problem:

\[
\begin{align*}
c_S := \min & \quad \sum_{r \in R^S} c_r f_r \tag{3a} \\
\text{s.t.} \quad & f_r \leq u_r \quad \forall r \in R^S \tag{3b} \\
& \sum_{r \in \delta^+(v)} f_r - \sum_{r \in \delta^-(v)} f_r = 0 \quad \forall v \in V \setminus \{s,t\} \tag{3c} \\
& \sum_{r \in \delta^+(t)} f_r = k^S \tag{3d} \\
& \sum_{r \in \delta^-(t)} f_r = k^S \tag{3e} \\
& f_r \in \mathbb{N}_{\geq 0} \quad \forall r \in R^S \tag{3f}
\end{align*}
\]

Here, (3b) ensures that transportation orders per service do not exceed the available capacity, (3c) ensures that incoming and outgoing flow at each node \( v \neq s,t \) are equal, (3d) and (3e) require that flow demand at source and sink nodes is met, and (3f) forces integrality of the flow \((f_r)_{r \in R^S}\). It is well-known that constraint (3f) can be relaxed whenever capacities and demands are integer. An integer optimal solution can then be found by solving the LP relaxation of \( P^S \) whenever a feasible solution exists (cf. Ahuja, Magnanti, and Orin 1993). Assuming that problem \( P^S \) is feasible for each \( S \subseteq N \), the cooperative game \( c = (c_S)_{S \subseteq N} \) is obtained by solving the problems \( \{P^S\}_{S \subseteq N} \).

In order to obtain insight into the dependence of the stability of the cooperation on costs, we perturb one of the players’ arc costs by an additive parametric term \( \lambda \in \Lambda \subseteq \mathbb{R}_+ \). Let \( i_p \in N \) be the company owning service \( r_1 \in R^{i_p} \) having parametric unit transport cost \( c_{r_1}(\lambda) = c_{r_1} + \lambda \), where \( c_{r_1} \) is the original unit transport cost on arc \( r_1 \). We assume that \( \Lambda \) is such that \( c_{r_1}(\lambda) \geq 0 \) for all \( \lambda \in \Lambda \). The choice of company \( i_p \in N \) is arbitrary. The introduction of the parameter \( \lambda \) requires an update to the objective function (3a) in problem \( P^S \), which now takes the form \( \sum_{r \in R^S} c_r f_r + \lambda f_{r_1} \). Clearly, this change affects only the cost \( c_S \) of the coalitions \( S \subseteq N \) for which \( i_p \in S \) and leads to the parametric version \( P^S(\lambda) \) of problem \( P^S \).

Thus, rather than the optimal value \( c_S \), the cost curve \( c_S(\lambda) \) will be computed as a function of \( \lambda \). Given \( \lambda \in \Lambda \), the cost \( c_S(\lambda) \) is the optimal objective value of \( P^S(\lambda) \). In our case, the cost curve is a piecewise linear, non-decreasing, concave function (Gal 2010). A parameter value \( \lambda \) at which the slope of the cost curve \( c_S(\lambda) \) changes is called a breakpoint. We denote the set of breakpoints of \( c_S(\lambda) \) by \( \mathcal{B}_S \). The number of breakpoints is a natural measure of problem complexity, as the set of optimal solutions changes exactly at the breakpoints. As shown by Carstensen (1983), the number of breakpoints can be exponential in the instance size. Our parametrization also changes the definition of the cooperative game. Indeed, unlike a classical cooperative game in characteristic function form, we let the cost functions \( c_S(\lambda) \) be the cost curves for the parametric problems \( P^S(\lambda) \). We denote the cost functions by \( c(\lambda) := (c_S(\lambda))_{S \subseteq N} \) and the parametric minimum cost flow cooperative game by \((N, c(\lambda))\). However, in order to simplify notation, we usually identify the game \((N, c(\lambda))\) with \( c(\lambda) \) when the set of players is clear from context.

Solution concepts themselves are now parametrized. For company \( i \in N \), the Shapley value allocation \( \Phi_i \) changes from (1) to the following expression:

\[
\Phi_i(\lambda) = \frac{1}{n} \sum_{S \subseteq N \setminus \{i\} } \left( \frac{n-1}{|S|} \right) (c_S(\lambda) - c_S(\lambda - \lambda)). \tag{4}
\]

The core \( \mathcal{C}(\lambda) \subseteq \mathbb{R}^n \) of the game \( c(\lambda) \) is now defined as

\[
\mathcal{C}(\lambda) = \left\{ x \in \mathbb{R}^n : \sum_{i \in N} x_i = c_S(\lambda) \text{ and } \sum_{i \in S} x_i \leq c_S(\lambda) \quad \forall S \subseteq N \right\} \tag{5}
\]

Overall, we set up a parametric description of the transport problem and apply this to the overlaying cooperative game and solution concepts. Given an interval of interest \( \Lambda = [\Lambda, \Lambda] \) with \( \Lambda < \Lambda \), our next step is to tackle the problem of describing the set of values \( \lambda \in \Lambda \) for which \( \Phi(\lambda) \in \mathcal{C}(\lambda) \). It is clear that this will depend on the problem instance, so capacity, demands, and costs will appear in the description of this set of values, which will describe the transport settings leading to stability of the cooperation.

Two basic properties need to be tested: subadditivity of the game and nonemptiness of the core.

**Proposition 1.** The parametric minimum cost flow game \( c(\lambda) \) is subadditive and has a nonempty core for all \( \lambda \in \Lambda \).

The proof is given in Online Appendix B.
Subadditivity implies that formation of the grand-coalition is optimal for the cooperation and that the Shapley value is individually rational, whereas non-emptiness of the core implies that testing the membership $\Phi(\lambda) \in \mathcal{C}(\lambda)$ is a nontrivial problem for all $\lambda \in \Lambda$. Moreover, the Shapley value is not guaranteed to belong to the core in general as shown by the example provided in Section 1.

5. Results
In this section, we lay out the mathematical properties supporting our proposed sensitivity analysis (Section 5.1). These properties are exploited to characterize stability of cooperation in a stylized corridor setting in Section 5.2, which is extended to a more involved network structure in Section 6.3. We define the $\varepsilon$-distance to measure instability of cooperation in Section 5.3.

5.1. Sensitivity Analysis
For any given value $\lambda \in \Lambda$, we have $\Phi(\lambda) \in \mathcal{C}(\lambda)$ if and only if

$$\sum_{i \in N} \Phi_i(\lambda) = c_N(\lambda)$$  \hspace{1cm} (6)

and

$$\Phi_S(\lambda) \leq c_S(\lambda) \quad \forall S \subseteq N,$$  \hspace{1cm} (7)

where $\Phi_S(\lambda) := \sum_{i \in S} \Phi_i(\lambda)$ is the total marginal cost of the players in coalition $S$. Although (6) is always satisfied as the Shapley value is an efficient solution concept (Shapley 1953), (7) is not guaranteed to hold. For a given coalition $S$, both sides of the inequality in (7) are piecewise linear functions of $\lambda$. Indeed, the Shapley value is obtained as a linear combination of piecewise linear functions. Let $\mathcal{B}$ be the set of all breakdowns of the cost functions, that is, $\mathcal{B} := \bigcup_{S \subseteq N} \mathcal{B}_S$. Each set $\mathcal{B}_S$ is finite because $|\mathcal{B}_S| \leq u_i$ for all $S \subseteq N$ and nonempty as we add the points $\check{\lambda}$ and $\bar{\lambda}$. We further assume that the breakpoints in $\mathcal{B}$ are sorted in increasing order, so $\mathcal{B} = \{\lambda_0 = \check{\lambda}, \lambda_1, \lambda_2, \ldots, \lambda_i, \lambda_{i+1} = \bar{\lambda}\}$ with $\lambda_i < \lambda_{i+1}$, $i = 1, \ldots, l$. It follows that, for $i \in \{0, \ldots, l\}$ and $\lambda \in [\lambda_i, \lambda_{i+1})$, the functions on both sides of (7) are linear.

Linearity implies that for each $S \subseteq N$, the inequality $\sum_{i \in S} \Phi_i(\lambda) \leq c_S(\lambda)$ is either valid for all $\lambda \in [\lambda_i, \lambda_{i+1})$, there exists no $\lambda \in [\lambda_i, \lambda_{i+1})$ for which it holds, or there exists $\lambda^S \in [\lambda_i, \lambda_{i+1})$ for which the inequality is valid on exactly one of the subintervals $[\lambda_i, \lambda^S]_\lambda$ and $[\lambda^S, \lambda_{i+1})_\lambda$ (see Figure 2).

It follows that the Shapley value transitions from inside the core to outside, or vice-versa, at most once between breakpoints, and that we can find those points by checking intersections between lines. Furthermore, the observations above result in the following pointwise stability tests of the cooperation: instability at one sample value extends to a neighborhood that can be computed explicitly.

**Figure 2.** Evaluating the Inequality $\Phi_S(\lambda) \leq c_S(\lambda)$ to Analyze the Stability of the Shapley Value

**Proposition 2.** If there exists $\hat{\lambda} \in \Lambda$ and $\hat{S} \subseteq N$ such that $\Phi_S(\lambda) > c_S(\hat{\lambda})$, then

$$\forall \varepsilon \in \left[0, \frac{\Phi_S(\hat{\lambda}) - c_S(\hat{\lambda})}{2K}\right]: \Phi(\lambda) \in \mathcal{C}(\lambda) \quad \forall \lambda \in \mathcal{B}_S(\hat{\lambda}),$$  \hspace{1cm} (8)

where $\mathcal{B}_S(\hat{\lambda}) := (\hat{\lambda} - \varepsilon, \hat{\lambda} + \varepsilon)$, $K \geq 0$ is a Lipschitz constant of the functions $\{c_S(\lambda)\}_{S \subseteq N}$ and $\{\Phi_S(\lambda)\}_{S \subseteq N}$, that is, $\forall \lambda', \lambda'' \in \Lambda, \quad |c_S(\lambda') - c_S(\lambda'')| \leq K \cdot |\lambda' - \lambda''|$ and $|\Phi_S(\lambda') - \Phi_S(\lambda'')| \leq K \cdot |\lambda' - \lambda''|$ for all $S \subseteq N$.²

**Proof.** The proof is given in Online Appendix C.

5.2. Corridor with Identical Players
We start by analyzing stability of the collaboration in the case of cooperation on a corridor with identical companies. The closed-form solutions we obtain here align with insights for complex problems in the literature (Agarwal and Ergun 2010, Houghtalen, Ergun, and Sokol 2011).

In a corridor, each company $i \in \{1, \ldots, n\}$ offers a transport connection $r_i$ between the port $s$ to a single inland terminal $t$ (see Figure 3). For company $i$, we denote the amount of pooled demand by $k_i$. For service $r_i$, we denote its capacity by $u_i$ and its unit cost by $c_i$.

The amount of demand $k_i$, the capacity $u_i$, and the per unit cost $c_i$ of arc $r_i$ are assumed to be independent of $i$, that is, $k_i = k$, $u_i = u$, and $c_i = c_0$ for all $i \in \{1, \ldots, n\} \setminus \{i_p\}$. Company $i_p$, however, is assigned a parametric arc cost $c_{i_p} = c_0 + \lambda$.

To characterize stability of the cooperation, we need to solve the set of inequalities (7) for the problem just defined. We provide a constructive proof where the cost game and the Shapley value $\Phi(\lambda)$ are explicitly computed and the inequalities $\Phi_S(\lambda) \leq c_S(\lambda)$ are
solved for all $S \subseteq N$. We note that the cases $k/u = 0$ or $k/u = 1$ are trivial, meaning that the Shapley value is in the core for those cases.

**Theorem 1.** In the case of cooperation on a corridor with identical players, whether the Shapley value is in the core or not depends on the value of the ratio $k/u$ compared with the size $n = |N| \geq 2$ of the grand coalition as follows:

$$
\Phi(0) \in \mathcal{C}(0) \quad \text{for all} \quad 0 \leq \frac{k}{u} \leq 1, \quad (9a)
$$

$$
\Phi(\lambda) \in \mathcal{C}(\lambda) \quad \forall \lambda > 0 \quad \text{if and only if} \quad \frac{(2n - 1)(n - 2)}{2n^2 - 3n} \leq \frac{k}{u} \leq 1, \quad (9b)
$$

$$
\Phi(\lambda) \in \mathcal{C}(\lambda) \quad \forall -c_0 \leq \lambda < 0 \quad \text{if and only if} \quad 0 \leq \frac{k}{u} \leq \frac{2n - 2}{2n^2 - 3n}. \quad (9c)
$$

**Proof.** We provide an intuitive explanation of the proof, which is given in Online Appendix D. Here, we focus on the case $\lambda > 0$, the other cases are treated similarly.

The proof is based on a decomposition of the minimum cost flow game $c(\lambda)$ as a linear combination of two simpler, nonparametric games $c^0$ and $c^+$, and on linearity of the Shapley value on the vector space of $N$-person games (Shapley 1953). This decomposition means that $c_S(\lambda) = c_S^0 + \lambda c_S^+$ for all $S \subseteq N$, so linearity of the Shapley value implies that $\Phi(c(\lambda)) = \Phi(c^0 + \lambda c^+) = \Phi(c^0) + \lambda \Phi(c^+)$. This implies that each inequality $\sum_{i \in S} \Phi_i(\lambda) \leq c_S(\lambda)$ for $S \subseteq N$ can be rewritten as follows:

$$
\sum_{i \in S} \Phi_i(\lambda) \leq c_S(\lambda) \quad \Leftrightarrow \quad \sum_{i \in S} (\Phi_i(c^0) + \lambda \Phi_i(c^+)) \leq c_S^0 + \lambda c_S^+ \quad (\Leftrightarrow \sum_{i \in S} \Phi_i(c^0) \leq c_S^0) \quad (\Leftrightarrow \sum_{i \in S} \Phi_i(c^+) \leq c_S^+ \quad (as \lambda > 0))
$$

This greatly simplifies the problem at hand by removing the dependency on $\lambda$ and allowing for a direct calculation of the solutions of the last inequality. \[ \square \]

In the following, we denote the terms $(2n - 1)(n - 2)/(2n^2 - 3n)$ in (9b) and $(2n - 2)/(2n^2 - 3n)$ in (9c) by $f^+(n)$ and $f^-(n)$, respectively. We note that these expressions are asymptotic, for large values of $n$, to the simpler expressions $(n - 1)/n$ and $1/n$, respectively. Moreover, as can be seen in Figure 4, these values and the respective asymptotes are close for $n \geq 3$ as well. Indeed, the shaded areas are obtained by the actual bounds and the dotted lines represent the simpler expressions.

In this case, stability is insensitive to the absolute value of the cost parameter $\lambda$, but depends on its sign, the network saturation, and the number of players as detailed in conditions (9). Mathematically, this independence of the absolute value of $\lambda$ is explained by linearity of the Shapley value, as can be seen from the proof provided above. Dependency on the sign of $\lambda$ follows, instead, from the impact a sign change has on the flow allocation between players. Indeed, for $\lambda < 0$, the arc of player $i_p$ becomes the cheapest and will be used first, while it is the most expensive arc (and, thus, will be used last) for $\lambda > 0$. This dependence of the stability on the cost parameter $\lambda$ holds only for this case, as shown by the numerical experiments of Section 6.

Our result shows explicitly that overcapacity hinders the stability of a cooperation when $\lambda > 0$, that is, when a single player’s cost exceeds that of the others: for low values of the demand-over-capacity ratio $k/u$, the cooperation is unstable. Moreover, our result shows that the overcapacity threshold is a function of the number of companies only. The threshold value $k/u = (2n - 1)(n - 2)/(2n^2 - 3n) = f^+(n)$ can be interpreted by looking at its asymptote $(n - 1)/n$, which corresponds to the amount of orders leading to a saturation of $n - 1$ companies’ transport capacity. As $f^+(n) < (n - 1)/n$, cooperation is achieved just before reaching saturation of $n - 1$ companies. For $\lambda > 0$, we face the situation where company $i_p$ has a transport cost that is higher than that of any other company. For $k/u \geq (n - 1)/n$, all companies with the lowest cost have their capacity fully utilized. Despite company $i_p$’s shared capacity being used last, the cost reduction this company achieves is spread among the other companies, reducing their total cost.

A symmetric observation holds when $\lambda < 0$, that is, when a single player operates transport at a lower unit cost than that of all others. Indeed, for $\lambda < 0$, the cooperation is stable in the overcapacitated regime of low values of the demand-over-capacity ratio $k/u$. The threshold $f^-(n)$ is just greater than $1/n$, which corresponds to the amount of orders that can be transported by a single player, $i_p$, in this case. This means that the cooperation is stable when all the orders of the cooperation can be executed by a single company.
In summary, by considering stylized corridors, we obtain closed-form solutions that characterize the transport setting leading to stability. While being in line with Agarwal and Ergun (2010) and Houghtalen, Ergun, and Sokol (2011), who show how overcapacity is related to instability, we extend their work by providing an exact threshold that quantifies the overcapacity level leading to instability.

In the same setting of Theorem 1, we further support practitioners in deciding an acceptable number of partners to seek to obtain a stable cooperation:

**Corollary 1.** Given demand k and capacity u > 0 such that k ≤ u, the maximum size of a stable cooperation on a corridor with identical players when λ > 0 is \( \hat{n}^+ := \lfloor (5u - 3k - \sqrt{9u^2 - 14uk + 9k^2})/(4(u - k)) \rfloor \). In case λ < 0, the maximum size of a stable cooperation on a corridor with identical players is \( \hat{n}^- := \lfloor (2u + 3k + \sqrt{9k^2 - 4ku + 4u^2})/(4k) \rfloor \).

**Proof.** By solving the inequality \( k/u \geq (2n - 1)(n - 2)/(2n^2 - 3n) \) in (9b) for \( n \), one obtains \( n \leq (5u - 3k - \sqrt{9u^2 - 14uk + 9k^2})/(4(u - k)) \). Rounding down is necessary as \( n \) is integer. Similarly, by solving \( k/u \leq (2n - 2)/(2n^2 - 3n) \) in (9c), one obtains the result after rounding down. \( \square \)

### 5.3. The \( \varepsilon \)-Distance

Stability of the Shapley value is determined by testing the inequalities \( \sum_{i \in S} \Phi_i(\lambda) \leq c_S(\lambda) \). If one is invalid, subcoalitions will form. Should a coalition \( S \subseteq N \) be allocated a total share \( \sum_{i \in S} \Phi_i(\lambda) \) greater than the cost \( c_S(\lambda) \) it generates, it may drop out of the grand-coalition \( N \) to take advantage of the lower cost \( c_S(\lambda) \) instead of \( \sum_{i \in S} \Phi_i(\lambda) \). Clearly, the magnitude of the gap \( \sum_{i \in S} \Phi_i(\lambda) - c_S(\lambda) \) is ignored from this perspective. To address this shortcoming, we define a measure of instability based on the concept of the \( \varepsilon \)-Core (Shapley and Shubik 1966). The \( \varepsilon \)-Core is the set of efficient pay-off allocations where coalitional rationality is relaxed by a given threshold that can be interpreted as a cost for dropping out of the grand-coalition, or an incentive to stay (Shapley and Shubik 1966). For the parametric game \( c(\lambda) \) and \( \varepsilon \in \mathbb{R} \), the \( \varepsilon \)-Core \( \mathcal{C}_\varepsilon(\lambda) \) is defined as follows:

\[
\mathcal{C}_\varepsilon(\lambda) := \left\{ x \in \mathbb{R}^n : \sum_{i \in N} x_i = c_N(\lambda) \text{ and } \sum_{i \in S} x_i \leq c_S(\lambda) + \varepsilon \quad \forall \emptyset \neq S \subseteq N \right\}.
\]  

(10)

We measure the instability of the Shapley value as its maximum deviation from coalitional rationality, and define the following distance.

**Definition 1.** Given a parametric cooperative game \( c(\lambda) \), the \( \varepsilon \)-distance \( \varepsilon_\Phi(\lambda) \) of the Shapley value \( \Phi(\lambda) \) is given by

\[
\varepsilon_\Phi(\lambda) := \max_{S \subseteq N} \left\{ \sum_{i \in S} \Phi_i(\lambda) - c_S(\lambda) \right\}.
\]  

(11)

Efficiency of the Shapley value implies that \( \varepsilon_\Phi(\lambda) \geq 0 \), and, if \( \Phi(\lambda) \in \mathcal{C}(\lambda) \) then \( \varepsilon_\Phi(\lambda) = 0 \). If \( \Phi(\lambda) \notin \mathcal{C}(\lambda) \), it follows that \( \varepsilon_\Phi(\lambda) > 0 \) is the smallest \( \varepsilon \)-value for which the Shapley value belongs to the \( \varepsilon \)-Core. Moreover, if \( \Phi(\lambda) \notin \mathcal{C}(\lambda) \), then \( \Phi(\lambda) \in \mathcal{C}_\varepsilon(\lambda) \) showing that stability has been violated by an amount \( \varepsilon_\Phi(\lambda) \).

Our definition of \( \varepsilon \)-distance is comparable to that of \( \varepsilon \)-stability introduced by Karsten, Slikker, and van
Houtum (2015). In their case, for a vector $\mathbf{e} \in \mathbb{R}^n$, an allocation $x$ for a game $(N, c)$ is $\varepsilon$-stable if $\sum_{i \in S} x_i \leq c_S + \varepsilon(S) \sum_{i \in S} e_i$ for all $\emptyset \neq S \subseteq N$. Note that if $x$ is $\varepsilon$-stable, then $x \in C_{\sum_{i \in N} e_i}$ and the $\varepsilon$-distance $\varepsilon_x$ for the allocation $x$ would satisfy $\varepsilon_x \leq \sum_{i \in N} e_i$.

If we define the synergy $s_S(\lambda)$ for coalition $S \subseteq N$ as the cost reduction generated by the cooperation between companies in $S$, then,

$$s_S(\lambda) := \sum_{i \in S} c_i(\lambda) - c_S(\lambda),$$

it follows that the $\varepsilon$-distance and the synergy are related by the following result.

**Theorem 2.** Given a subadditive parametric cost game $c(\lambda)$ and an individually rational and efficient solution concept $\Psi(\lambda)$, the following holds:

$$s_S(\lambda) \leq s_N(\lambda) \quad \forall S \subseteq N \Rightarrow \Psi(\lambda) \in C_{s_N(\lambda)} \quad \forall \lambda \in \Lambda,$$

(13)

where $C_{s_N(\lambda)}$ is the $\varepsilon$-Core for $\varepsilon = s_N(\lambda)$.

**Proof.** The proof is given in Online Appendix E. ⊓⊔

This general result shows that if the synergy $s_S(\lambda)$ of each coalition $S \subseteq N$ is lower than the grand coalition’s synergy $s_N(\lambda)$, then the gain from dropping out for any coalition is at most $s_N(\lambda)$. Given a fixed value of the parameter $\lambda$, high values of synergy $s_S(\lambda)$ stand for high reductions of transport costs generated by the cooperation. Indeed, the total cost without cooperation $\sum_{i \in S} c_i(\lambda)$ is lowered by the cost $c_S(\lambda)$ generated under cooperation. Subadditivity of the game ensures that $s_S(\lambda) \geq 0$, whereas additivity would lead to $s_S(\lambda) = 0$ for all $S \subseteq N$.

Note that $s_S(\lambda)$ is defined independently of any solution concept. A similar notion of synergy of a coalition is given in Lozano et al. (2013), who define it as: Synergy($S$) := $(\sum_{i \in S} c_i) - c_S$. In contrast, we do not rescale by the total cost generated by the coalition.

**Corollary 2.** Given a parametric minimum cost flow game $c(\lambda)$, it follows that

$$\Phi(\lambda) \in C_{s_N(\lambda)} \quad \forall \lambda \in \Lambda.$$ (14)

**Proof.** The proof is given in Online Appendix F. ⊓⊔

Interpreting the value of the $\varepsilon$-distance might be difficult because incentives are reported in absolute terms. To overcome this problem, we define the relative $\varepsilon$-distance $\varepsilon(\lambda)$ as the $\varepsilon$-distance relative to the total cost generated by the subcoalition:

$$\varepsilon(\lambda) := \max_{S \subseteq N} \left\{ \frac{\sum_{i \in S} \Phi_i(\lambda) - c_S(\lambda)}{c_S(\lambda)} \right\}.$$ (15)

Efficiency of the Shapley value implies that $\varepsilon(\lambda) \geq 0$. Moreover, $\varepsilon(\lambda) > 0$ if and only if $\Phi(\lambda) / C(\lambda)$, in case $\varepsilon(\lambda) > 0$, each coalition $S$ maximizing (15) is unstable, that is, $\sum_{i \in S} (\Phi_i(\lambda) - c_S(\lambda)) > 0$. Such a measure quantifies the magnitude of the incentive to form subcoalitions relative to the total cost generated by each sub-coalition itself. Clearly, from the value $\varepsilon(\lambda)$ it cannot be concluded if $\Phi(\lambda) \in C_{\varepsilon(\lambda)}(\lambda)$.

The computation of the relative $\varepsilon$-distance $\varepsilon(\lambda)$ requires deriving the solution concept, that is, the Shapley value $\Phi(\lambda)$. In case this operation is complex or expensive, we provide the following upper bound that is defined by the coalitional costs $c_S(\lambda)$ only.

**Corollary 3.** Given a parametric minimum cost flow game $c(\lambda)$, it follows that

$$\varepsilon(\lambda) \leq \frac{s_N(\lambda)}{\min_{S \subseteq N} |S|^2 c_S(\lambda)} \quad \forall \lambda \in \Lambda.$$ (16)

**Proof.** From Corollary 2, we obtain that $\sum_{i \in S} (\Phi_i(\lambda) - c_S(\lambda))$ for all coalitions $S \subseteq N$. Division by the smallest cost of a coalition that can violate coalitional irrationality yields the claimed upper bound on (15). ⊓⊔

The previous result shows that the maximum relative deviation from stability is bounded by the cooperation synergy relative to the cost generated by the cheapest coalition.

6. Generalizations

Obtaining a closed-form expression (in terms of the parameters of the game $c(\lambda)$) for the solution set of $\Phi(\lambda) / C(\lambda)$ is a challenging task. This holds especially because the costs $c_S(\lambda)$ ($S \subseteq N$) are optimal solutions to an optimization problem. Therefore, as soon as we generalize the transport setting, we opt for a numerical approach that exploits the Eisner-Severance method (Eisner and Severance 1976) for the construction of the cost curves (see Appendix B for a detailed description). Once the parametric game $c(\lambda)$ has been constructed, the Shapley value is obtained numerically by working directly with the piecewise linear cost curves $(c_S(\lambda))_{S \subseteq N}$. Thanks to the observation from Section 5.1, solving $\Phi(\lambda) / C(\lambda)$ for $\lambda \in \Lambda$ translates into the problem of solving linear inequalities.

In what follows, we conduct several tests in which we drop several assumptions made in the case treated in Theorem 1. In Section 6.1, we consider the case where players’ costs are allowed to take two different values. This is extended in Section 6.2, where two different demand and capacity levels are considered as well. Finally, the network structure is generalized in Section 6.3, where both a mathematical and a numerical analysis are carried out.
6.1. Corridor with High and Low Costs
We again consider collaboration on a corridor as in Section 5.2, and test whether our insights obtained from Theorem 1 still hold when unit costs are no longer identical. As opposed to Theorem 1, we assume different costs: \( n_L < n \) companies have low cost \( c_L \), company \( i_p \) has a cost of \( c_p = c_L + \lambda \), for \( \lambda \in \Lambda \), and the remaining \( n_H := n - n_L - 1 \) companies have cost \( c_H > c_L \).

We repeatedly generate the parametric game for increasing demand levels from \( k = k_0 \) to \( k = u \) for each value \( n_L = 0, 1, \ldots, n - 1 \), keeping all other parameters fixed.\(^3\) For each instance, the optimal objective value \( c_S(\lambda) \) for the parametric problem \( P^S(\lambda) \) is computed for each coalition \( S \subseteq N \) by using Algorithm B.1 described in Appendix B. We find the intervals in \( \Lambda \) for which \( \Phi(\lambda) \in C(\lambda) \). Parameter regions of stability are shown shaded in Figure 5 for \( n_L = 0, 1, 2, 3, 4 \) and \( n = 5 \). The case \( n_L = 4 \), which coincides with the situation studied in Theorem 1, has been inserted for comparison.

We find that stability is sensitive to changes in \( \lambda \) as soon as players’ costs are heterogeneous. Regions (I) and (II) are inherited from the scenario with identical costs studied in Theorem 1. Indeed, region (I) appears for values of \( k/u \) higher than the threshold \((2n - 1)(n - 2)/(2n^2 - 3n)\) and positive values of \( \lambda \), while region (II) is located at values of \( k/u \) lower than the threshold \((2n - 1)/(2n^2 - 3n)\) and negative values of \( \lambda \). Unlike for Theorem 1, the extension of those regions now also depends on the absolute value of \( \lambda \) and not only on its sign. Notably, the stability regions inherited from the identical cost scenario appear consistently throughout the experiments, even when the size \( n \) of the cooperation increases.

From Figure 5, it can be seen that region (I) is formed beginning at \( k^+ = 24 \). Let \( \delta^+(n, n_L) := k^+/u \) be the numerical threshold obtained, where \( k^+ \) is the lowest demand value \( k \) contained in region (I). Similarly, we denote the value below which region (II) is formed by \( k^- \) and the corresponding numerical threshold by \( \delta^-(n, n_L) := k^-/u \). In the case of Figure 5, we have \( k^- = 6 \). Table 2 reports average values for the absolute relative deviation of the numerically obtained thresholds \( \delta^+(n, n_L) \) and \( \delta^-(n, n_L) \) from the theoretical ones obtained in Theorem 1. Given a test value of the capacity \( u \), the values \( \delta^+(n, n_L) \) have been computed for each \( n = 4, \ldots, \delta \) and \( n_L = 1, \ldots, n - 1 \), and the absolute relative error \( |\delta^+(n, n_L) - f^+(n)|/f^+(n) \) has been computed. The same procedure has been performed for

**Figure 5.** Regions of Stability (Shaded) of the Five-Players Cooperation

Notes. The cases \( n_L = 4, n_H = 0 \) and \( n_L = 0, n_H = 4 \) coincide with the situation studied in Theorem 1. * Indicates the case where a complete transfer of order is possible between companies with low and high cost, as explained further in the text.
Table 2. Relative Deviation of the Numerically Obtained Thresholds $\delta^+(n, n_L)$ and $\delta^-(n, n_L)$ from the Theoretical Ones $f^+(n) = \frac{(2n - 1)u - 2n}{2n - 3u}$ and $f^-(n) = \frac{2n - 2}{2n - 3u}$. Respectively

<table>
<thead>
<tr>
<th>Case $\lambda &gt; 0$</th>
<th>$f^+(n)$</th>
<th>Mean relative error (%)</th>
</tr>
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<tr>
<td>$n$</td>
<td>$u = 30$</td>
<td>$u = 40$</td>
</tr>
<tr>
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<td>0.70</td>
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</tr>
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<table>
<thead>
<tr>
<th>Case $\lambda &lt; 0$</th>
<th>$f^-(n)$</th>
<th>Mean relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>$u = 30$</td>
<td>$u = 40$</td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
<td>8</td>
<td>0.13</td>
<td>11.56</td>
</tr>
</tbody>
</table>

Note. The reported values are the average over the cases $n_L \in \{1, \ldots, n - 1\}$ of the absolute relative error $|\delta^+(n, n_L) - f^+(n)|/f^+(n)$.

This result sheds light on corridor formation, as this structure can be stable even in the case of an overcapacitated network. This could potentially benefit areas of the hinterland where container flows are small and cooperation is not seen as an option because economies of scale cannot be achieved. Our results show that, even in the absence of this cost reduction, cooperation can nonetheless be stable and beneficial.

6.2. Corridor Cases with Varying Demand and Capacity

In the previous section, the network saturation $k/u$ could be computed from the demand $k$ and capacity $u$. This is not the case when demand and capacity are not the same for all players. We observe that the total demand $K := \sum_{i \in N} k_i$ over total capacity $U := \sum_{i \in N} u_i$ ratio $K/U$ allows for a comparison with the previous case.

Let $k_L$ and $k_H$ be the demand for players with low or high cost, respectively, and let $\Delta_k := |k_H - k_L|$ be the

Figure 6. The Relative $\varepsilon$-Distance $\bar{\varepsilon}_\Phi(\lambda)$ for a Five-Players Cooperation with $n_L = 1$ (Values in Percent) (cf. Figure 5)
demand gap between the players. We assume: $n_L < n$ companies with low cost $c_L$ have demand $d_L$, company $i_p$ with cost of $c_{i_p} = c_L + \lambda_i$ for $\lambda \in \Lambda$ has demand $d_L$, and the remaining $n_H := n - n_L - 1$ companies with high cost $c_H$ have demand $d_H$.

Then, we test the stability of cooperation for increasing values of $k = k_0, \ldots, u - \Delta_k$ in two cases where either $k_L = k + \Delta_k$ ($k_H = k_L$), or $k_H = k + \Delta_k$). Note that it is not necessary that $k_L < k_H$. We generate plots as in Figure 5 and compute the numerical thresholds $\delta^+(n, n_L)$ and $\delta^-(n, n_L)$ using the lowest total demand $K^*$ at which the saturated region of stability exists, and the highest total demand $K^*$ below which the region of stability for negative values of $\lambda$ exists, respectively. In Table 3, we report the mean relative error $\frac{\delta^+(n, n_L) - f^+(n)}{f^+(n)}$ over $n_L = 1, \ldots, n - 2$ for two sample values of $\Delta_k$. We observe that the numerical threshold $\delta^+(n, n_L)$ is close to the value $f^+(n)$ obtained from Theorem 1, but a higher gap is present between $\delta^-(n, n_L)$ and $f^-(n)$ for $\lambda < 0$. The demand gap $\Delta_k$ can explain this difference as low values of saturation of the network cannot be reached. Indeed, even if one company is only transporting $k = 1$ order, others would have $k = 1 + \Delta_k$ orders, so the network is still far from the target values of low saturation.

Similar results are obtained if we assume that players’ capacities are different. Again we test for the case where either $u_L = u + \Delta_u$ and $u_H = u$, the capacity for players with low or high costs, respectively, or $u_L = u$ and $u_H = u + \Delta_u$. The player $i_p$ with parametric cost is counted as a player with low cost. Table 4 reports values of the mean relative error that again shows results close to the theoretical threshold obtained from Theorem 1 for the case of $\lambda > 0$ only.

Overall, these numerical tests show that the total demand to capacity ratio, compared with the threshold $f^+(n)$, is a main discriminant for the stability of a cooperation. We can also conclude that the region of stability for low values of the demand-over-capacity ratio for $\lambda < 0$ is more susceptible to variations in the amount of orders and of capacity in the network.

### 6.3. Vertical Cooperation Opportunity

In this section we discuss the case where the companies also have the opportunity of collaborating in a vertical transport setting. Although being simple in its formulation, this case generalizes the network of the cases considered previously. The analysis performed here shows that our theoretical results can be extended to a slightly richer network and our algorithmic approach is not bound to specific networks.

Consider the case where each company in a cooperation executes a segment of a joint transport route and, at the same time, is able to execute direct transport from origin to destination. The sequence of segments constitutes a path that can be used only when all companies cooperate. One example of such a configuration can be found in the case of intermodal container transport where each company operates a transport service in a vertically integrated sequence of transport operations, but also has the opportunity of self-arranging direct transport.

Formally, we consider the network given in Figure 7. Each company $i \in N = \{1, \ldots, n\}$ executes transport on arc $\bar{r}_i$, representing the direct transport option, and arc $\bar{r}_i$, representing the segment in the vertical cooperation path joining origin $s$ and destination $t$. All companies but company $i_p$ have identical unit transport costs $c_i = c$ and $c_{i_p} = \bar{c}$. Company $i_p$ has a parametric arc cost either on arc $\bar{r}_i$, that is, $c_{i_p} = \bar{c} + \lambda$, or on $\bar{r}_i$, that is, $c_{i_p} = c + \lambda$, that is used to inspect the sensitivity of the stability of the Shapley value. Each of these two cases will be treated separately later. The direct transport capacity $u_i = u$ on arc $\bar{r}_i$, and the vertical cooperation transport capacity $u_i = \bar{u}$ on arc $\bar{r}_i$ are identical for all companies $i \in N$, as well as the amount of orders $k^i = k$ to be transported from origin node $s$ to destination node $t$. We let $c_S(\lambda)$ denote total cost of the minimum cost flow generated by coalition $S \subset N$ when the set of arcs $\mathcal{R}_S = \bigcup_{i \in S} \{\bar{r}_i, \bar{r}_i\}$ is used and the amount of orders $k^S = \sum_{i \in S} k^i$ is pooled. We then obtain a parametric minimum cost flow game as in Section 4.
We denote the resulting parametric minimum cost flow game on the graph given in Figure 7 by \(c^i(\lambda)\) and, for simplicity, refer to it as the vertical cooperation game. In Section 6.3.1, we discuss the theoretical properties of the cooperation, whereas Section 6.3.2 presents the numerical experiments that complement the theoretical findings.

6.3.1. Theoretical Analysis. We denote the Shapley value and the core of the vertical cooperation game \(c^i(\lambda)\) by \(\Phi^i(\lambda)\) and \(C^i(\lambda)\), respectively.

We first consider the case where \(c_{r_\nu} = \tilde{c} + \lambda\) and assume that \(\lambda \geq -c\) in order to ensure that \(c_{r_\nu} \geq 0\). Here, we obtain the following theorem:

**Theorem 3.** Consider the vertical cooperation game \(c^i(\lambda)\), where \(c_{r_\nu} = \tilde{c} + \lambda\). For all values of direct unit transport cost \(c\) and capacity \(u\), vertical unit transport cost \(\tilde{c}\) and capacity \(\tilde{u}\), amount of orders \(k\) and number of players \(n\), we have

\[
\Phi^i(\lambda) \in C^i(\lambda) \quad \text{for all } \lambda \in [-c, +\infty). \tag{17}
\]

In other words, when the parametric cost is on one of the arcs in the vertical cooperation path, the Shapley value \(\Phi^i(\lambda)\) is stable for all values of \(\lambda\).

**Proof.** The proof is given in Online Appendix G. □

Figure 7. Transport Network for Corridor Cooperation with Opportunity of Vertical Cooperation

![Transport Network](image)

Note. Arc \(r_i\) represents a direct connection, whereas path \((\overline{r}_1, \ldots, \overline{r}_n)\) represents the joint vertical service.

**Theorem 3** shows that, given identical costs for direct transport, adding a vertical cooperation opportunity can only benefit the stability of the Shapley value. Moreover, it becomes clear that this case does not require any further computational investigation as stability holds for every parameter setting.

We now consider the case where the parameter \(\lambda\) is on the direct arc \(r_\nu\) of player \(i_\nu\), that is, \(c_{r_\nu} = c + \lambda\). We assume that \(\lambda \geq -c\) to ensure nonnegativity of the unit transport cost, and that the total unit cost \(n\tilde{c}\) for vertical cooperation transport is at most that of direct transport, that is, \(n\tilde{c} \leq c\). Otherwise the vertical cooperation game would reduce to the horizontal cooperation case studied in Theorem 1. Indeed, if \(n\tilde{c} > c\), then the vertical cooperation path is never used.

Given the parameter being on arc \(r_\nu\), if we consider only cooperation on the direct transport arcs \(\{r_i : i \in N\}\), we obtain the case of cooperation on a corridor with identical players treated in Theorem 1. We denote the corresponding cooperative game by \(c^i(\lambda)\) and show the following relation between \(c^i(\lambda)\) and \(c^i(\lambda)\):

**Theorem 4.** In the vertical cooperation game \(c^i(\lambda)\) where \(c_{r_\nu} = c + \lambda\), we have that, for all values of direct unit transport cost \(c\) and capacity \(u\), vertical unit transport cost \(\tilde{c}\) such that \(n\tilde{c} \leq c\) and capacity \(\tilde{u}\), amount of orders \(k\) and number of players \(n\), the following holds: For each value of \(\lambda \in [-c, +\infty)\), stability of the Shapley value \(\Phi^i(\lambda)\) in the horizontal cooperation game \(c^i(\lambda)\) implies stability of the Shapley value \(\Phi^i(\lambda)\) in the vertical cooperation game \(c^i(\lambda)\). More formally:

\[
\Phi^i(\lambda) \in C^i(\lambda) \implies \Phi^i(\lambda) \in C^i(\lambda). \tag{18}
\]

The converse does, in general, not hold true.

**Proof.** We provide an intuitive explanation of the proof, which is given in Online Appendix G. Here, we focus on the case \(\lambda > 0\).
Figure 8. Regions of Stability for the Vertical Cooperation Game

Note. All subfigures share the same vertical axis shown on the left.

Like the proof of Theorem 1, this proof is based on a decomposition of the game $c^\star(\lambda)$ as a linear combination of the horizontal cooperation game $c^\lambda(\lambda)$ and a new game $\bar{c}(\lambda)$ that is obtained as the algebraic difference of $c^\lambda(\lambda)$ and $c^\lambda(\lambda)$. Since, in this case, the arc $r_i$ with parametric cost is a direct transport arc, we obtain that the horizontal cooperation game $c^\lambda(\lambda)$ can be decomposed as in the proof of Theorem 1, that is, $c^\lambda(\lambda) = c^\lambda + \lambda c^\star$.

We denote the Shapley values for the games $c^\star$ and $c^\lambda$ by $\Phi^\star = \Phi(c^\star)$ and $\Phi^\lambda = \Phi(c^\lambda)$, respectively. Combining the decomposition of $c^\lambda(\lambda)$ with that of $c^\lambda(\lambda)$, we obtain that $c^\lambda(\lambda) = c^\lambda + \lambda c^\star + \bar{c}(\lambda)$ and the Shapley value $\Phi^\lambda(\lambda)$ can be obtained by using linearity because it can be computed explicitly for each of the games $c^\lambda, c^\star$, and $\bar{c}(\lambda)$.

Testing coalitional rationality for $S \subseteq N$ means testing whether $\sum_{i \in S} \Phi_i(\lambda) \leq c^\star_S(\lambda)$, which can be rewritten as follows:

$$\sum_{i \in S} \Phi_i(\lambda) \leq c^\star_S(\lambda)$$

$$\Leftrightarrow \sum_{i \in S} \Phi_i + \lambda \sum_{i \in S} \Phi_i^\star + |S| \frac{\gamma_N(\lambda)}{n} \leq c^\lambda + \lambda c^\star_S$$

$$\Leftrightarrow \lambda \sum_{i \in S} \Phi_i + |S| \frac{\gamma_N(\lambda)}{n} \leq \lambda c^\star_S$$

This greatly simplifies the problem and provides a simple way to complete the proof. Indeed, it now suffices to study the term $|S| \frac{\gamma_N(\lambda)}{n}$ because a closed-form solution to the set of inequalities $\sum_{i \in S} \Phi_i(\lambda) \leq c^\star_S(S \subseteq N)$ has already been obtained in the proof of Theorem 1.

From Theorem 4, we can conclude that, independently of the unit cost $\bar{c}$ and capacity $\bar{u}$ on the vertical cooperation path, the vertical cooperation opportunity is never disadvantageous for stability. However, it is not clear whether and—if so—by how much the region of stability is enlarged. We test this in the numerical experiments of the following section.

6.3.2. Numerical Study. To test whether the region of stability of the vertical cooperation game $c^\star(\lambda)$ extends beyond that of the horizontal cooperation game $c^\lambda(\lambda)$, we consider the following numerical experiments.

We assume $n = 4$, direct unit transport cost $c = 100$ and vertical unit transport cost $\bar{c} \in \{25, 24, 23, 20\}$, direct transport capacity $u = 60$ and vertical transport capacity $\bar{u} = 120$. Knowing from Theorem 1 that the stability of the Shapley value in the game $c^\lambda(\lambda)$ depends on the ratio $k/u'$ we test stability for a varying amount $k$ of orders between $k = 0$ and $k = u = 60$. The parameter $\lambda$ varies in the interval $\Lambda = [-100, 100]$. Note that we set vertical transport capacity equal to the direct transport capacity of two players, and generate a plot for each value of $\bar{c}$.

Our results are shown in Figure 8, in which the shaded areas correspond to the regions of stability of the vertical cooperation game. We observe that, in case $n\bar{c} = 100 = c$, we obtain the result described in Theorem 1 for $n = 4$ players. As soon as vertical cooperation becomes beneficial for $\bar{c} = 24$, meaning that $n\bar{c} = 96 < 100 = c$, the region of stability expands for all values of the demand-over-capacity ratio $k/u$. This behavior is consistent across the remaining two cases of $\bar{c} = 23$ and $\bar{c} = 20$.

From the experiments, we can conclude that, as soon as vertical transport is advantageous (i.e., its unit transport cost is lower than that of direct transport), the stability of the vertical cooperation game becomes less dependent on the demand-over-capacity ratio than in the horizontal cooperation on a corridor case (compare Figure 8 with Figures 5 and 6). This decreased dependency on the amount of spare capacity depends on the value of the unit cost of vertical transport. This result adds to the current understanding that horizontal cooperation in transportation is stable only for coalitions of small size (Agarwal and Ergün 2010, Basso et al. 2019). Indeed, we observe that, when a service requiring the joint effort of all companies is advantageous for all, then the cooperation gains in stability.
7. Conclusion
Cooperation in the hinterland container transport sector can improve the performance of hinterland connections. However, although reducing costs and improving the competitive position of ports, cooperation exposes members to the risk of its failure. For this reason, we study the relationship between transport setting and stability of cooperation from a cost sharing perspective. We propose a sensitivity analysis method to test the stability of bargained cost shares. This approach combines results from linear parametric optimization with key concepts from cooperative game theory. By using methods from linear parametric optimization, we generate parametric cooperative games for more complex instances. Our approach computes parameter intervals leading to stability, thus extending the sampling-based analysis available in the literature on collaborative transport. Furthermore, we introduce a measure of instability that quantifies the deviation from stability based on the ϵ-Core (Shapley and Shubik 1966). Overall, we prove that the demand-over-capacity ratio—when compared with a function of the size of the cooperation—is the main discriminant for stability of horizontal cooperation in transportation for network flow-like cost structures. Moreover, we show that, even for overcapacitated networks, a stable, or limited unstable, cooperation is possible. Given the complexity of the formal analysis, our analytical results are limited to the case of identical companies cooperating on two different networks. In our numerical experiments, instead, we study the effect that heterogeneity of companies has on our theoretical results.

There are several directions for future research. First, we assume a simple transport model. Including a time dimension in the model could lead to a parametric analysis of the dependency of cooperation stability on time-related parameters, such as speed and frequency of connections. Second, extending and generalizing the type of networks studied could further reveal the role played by the network structure itself. Third, other concepts from cooperative game theory can be parameterized using the proposed definition of parametric cooperative games. Finally, a stochastic optimization model could be considered to improve the representation of the actual decision-making process, at the cost of finding a suitable representation of the bargaining process.

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Appendix A. Introductory Example Analysis
If we denote the amount of orders and capacity of each company by k and u, respectively, then the problem to be solved for each coalition S ⊆ N := \{P_1, P_2, P_3\} is a minimum cost flow problem on the graph in Figure 1. Node s is the source and node t the sink (see the model definition given in Section 4). Table A.1 shows the cost c_5 generated by each subset S of companies as well as the Shapley value allocation to each player P_i (i = 1, 2, 3) and each coalition. The Shapley value has been computed using expression (1) in Section 3. Note that, for coalition S ⊆ N, \( \Phi_S := \sum_{\lambda \in \Lambda} c_S(\lambda) \). Table A.1 presents three cases: the case k = 15 is the one described in Section 1, the cases k = 18 and u = 25 refer to the mentioned situations where demand is increased to 18 units or capacity is lowered to 25 units.

The bold value in case k = 15 highlights instability of coalition \{P_1, P_3\} due to the cost generated by this coalition being lower than the Shapley value allocation. In all other cases, the Shapley value allocation \( \Phi_S \) is at most as large as the cost c_5 proving stability of the Shapley value and of the cooperation.

Appendix B. Algorithmic Approach
The parametric minimum cost flow game c(\lambda) can be efficiently constructed by computing the cost curves c_S(\lambda) for each coalition S ⊆ N following the Eisner-Severance method (Eisner and Severance 1976), which we recall here. We first explain the method with the help of Figure B.1, then provide a formal definition in Algorithm B.1. We use the same notation as in Section 4.

The construction of the cost curve c_S(\lambda) over an interval \( \Lambda := [\lambda_L, \lambda_R] \) is performed by updating a piecewise linear curve \( f(\lambda) \) until it converges to \( c_S(\lambda) \). During the execution of the algorithm, optimal solutions to the minimum cost flow problem at query values of \( \lambda \in \Lambda \) will be computed. For an optimal solution \( f' \) found at a query value \( \lambda' \), let \( l_{\lambda'}(\lambda) := \sum_{r \in \Lambda} c_{S_i}(\lambda') + \lambda r_{S_i} \) be the parametric objective value for the optimal solution \( f' \). Note that \( l_{\lambda'}(\lambda') = c_S(\lambda') \), that is, \( l_{\lambda'}(\lambda) \) is optimal at \( \lambda' \), and \( l_{\lambda'}(\lambda) \geq c_S(\lambda) \) for \( \lambda \neq \lambda' \), as \( f' \) is not granted to be an optimal solution to the minimum cost flow problem for \( \lambda \neq \lambda' \).

Algorithm B.1 (Construction of Cost Curve c_S(\lambda) for \( \lambda \in \Lambda \))

1: \( A \leftarrow l_{\lambda_L} \cap l_1 \)
2: if \( l_{\lambda_L} = l_1 \) then
3: \( f(\lambda) \leftarrow l_{\lambda_L} \)
4: else
5: \( f \leftarrow l_{\lambda_L} \cap l_1 \)
6: while \( \lambda \neq \lambda' \) do
7: \( \lambda' \leftarrow \text{pop an element from } A \)
8: if \( |f \cap l_{\lambda_L}| \leq 2 \) then
9: \( A \leftarrow A \cup (f \cap l_{\lambda_L}) \)
10: \( f \leftarrow f \cup l_{\lambda_L} \)
11: \( f(\lambda) \) is the optimal cost curve c_S(\lambda) for \( \lambda \in \Lambda \).
Figure B.1. Sequential Construction of the Parametric Cost Curve $c_S(\lambda)$ with the Eisner-Severance Method

Assuming that $c_S(\lambda)$ is not linear (i.e., it has at least one breakpoint) implies that $I'(\lambda)$ is piecewise linear and that $I'(\lambda)$ has at least one breakpoint since the start of the algorithm. The starting point, indeed, is the computation of the lines $l_1(\lambda) := l_{11}(\lambda)$ and $l_2(\lambda) := l_{12}(\lambda)$ obtained from the optimal solutions at query values given by the extremes of $\Lambda$. The curve $I'(\lambda)$ is defined at first by the point-wise minimum of the lines $l_1(\lambda)$ and $l_2(\lambda): I'(\lambda) = \min(l_1(\lambda), l_2(\lambda))$ (see Figure B.1(a)). During each update, a new line $l_{1(\lambda)} := l_{1c}(\lambda)$ is computed for a query value of $\lambda'$ corresponding to one of the breakpoints of $I'(\lambda)$. This operation can have two outcomes: either the point $(\lambda', l_{1c}(\lambda'))$ lies on the curve $I'(\lambda)$ and this breakpoint does not need to be tested further, or $(\lambda', l_{1c}(\lambda'))$ lies below $I'(\lambda')$. In the second case, the optimal solution found by solving the minimum cost flow problem generates the line $l_{1c}(\lambda)$ which intersects $I'(\lambda)$ (see $l_3$ in Figure B.1(b)). In this case, $I'(\lambda)$ can be updated to the point-wise minimum between $I'(\lambda)$ and $l_{1c}(\lambda)$, leading to new breakpoints to test in the following updates (see Figure B.1(c)). The method terminates when no further breakpoints need to be tested.

From a computational perspective, these updates performed within the algorithm require only $2PS - 1$ queries to a solver for the minimum cost flow problem, where $PS$ is the number of breakpoints of $c_S(\lambda)$ in the interval $\Lambda$ (Jenkins 1990).

Intuitively, the result is correct because the performed operations are equivalent to updating an upper and a lower bound until the two converge to the cost curve itself. The upper bound is obtained from optimal solutions, while the lower bound is a piecewise linear and concave approximation of the cost curve. A formal proof is provided in Eisner and Severance (1976).

We now provide a formal description in Algorithm B.1, where we use the following notation: given two lines $l_1(\lambda)$ and $l_2(\lambda)$ in the plane, we denote by $l_1 \land l_2$ the piecewise linear function obtained by the point-wise minimum of the two lines, and by $l_1 \land l_2$ the set of the intersection points between the two lines. For ease of notation, we write $l_{1c}$ for the line $l_{1c}(\lambda)$ obtained from an optimal solution at $\lambda = \lambda'$.

Note that the condition at Line 2 treats the case where the cost curve $c_S(\lambda)$ has no breakpoints, meaning that it is a linear function. In this case, the fact that the two lines $l_{1L}$ and $l_{1L}$ coincide implies that each of them is optimal for the other extreme point as well. Lines 5–11 formalize the explanation given above. The set $A$ contains the query points to evaluate.

Endnotes

1 By assuming that capacity and demand are integral, we might consider continuous flows. This is in contrast with Agarwal and Ergun (2010), who assume continuity of flow variables due to homogeneity and bigger transport capacity.

2 In this case, the constant $K$ can be chosen as the highest slope of all functions.

3 Parameters are set as follows: $k_0 = 0$, $u = 30$, $c_L = 20$, $c_H = 30$; company $i$ has parametric cost $c_i = c_L + \lambda$, $\lambda \in \Lambda = [-20, 40]$; for $i \in \{2, \ldots, n_L + 1\}$, $c_i = c_L$, whereas for $i \in \{n_L + 2, \ldots, n\}$, $c_i = c_H$ ($c_L < c_H$).

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