Labour supply and saving decisions with uncertainty over sickness

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1. Introduction

Illness can severely constrain functioning. In the extreme, sickness reduces the market and non-market productivity of an individual's time to zero. Grossman (1972) defined such periods as 'sickness time'. This concept has been used by Grossman (op cit) and others (e.g., Wagstaff, 1986) in examining the demand for health and health care. To date, little attention has been given to the consequences of sickness time for households' time allocation and consumption decisions. This represents an important gap in the literature. Examination of the relationships between sickness time, on the one hand, and labour supply and saving, on the other, is essential to an understanding of the full economic consequences of ill-health. In addition to the obvious reduction in the well-being of individuals succumbing to illness, uncertainty over sickness is welfare depleting for all risk averse individuals. Such uncertainty can be expected to influence labour supply and saving behaviour. These behavioural responses will determine, in part, the economic impact of any shift in the distribution of ill-health. Further, correct specification of labour supply and consumption functions requires an understanding of how demographics, such as sickness, affect household allocation decisions. Such an understanding is also essential to the prediction of how individuals respond to welfare policies, such as the introduction, or extension, of social disability insurance.
This paper examines the impact of sickness time on labour supply and saving decisions. The certainty case is considered briefly in Section 2. Analysis of the more realistic case, with uncertainty over sickness time, is conducted first, in Section 3, within a single decision period. The following section introduces a dynamic context with two decision periods, allowing examination of the consequences of uncertainty over sickness time for saving, in addition to labour supply, decisions. This adds to a growing literature on precautionary saving. In Section 5 social insurance against sickness is introduced. The final section concludes.

2. Certainty case

In the presence of sickness time, the time constraint is: \( T - S = H + L \). \( T \) is the total time endowment (e.g., one year), \( (T - S) \) is the time available to divide between market work (\( H \)) and leisure (\( L \)), after the loss of time due to illness (\( S \)). Two important assumptions are made with respect to sickness time. First, unlike Grossman (1972), \( S \) is assumed exogenous. This assumption is made in order to focus on the consequences of exogenous shifts in the distribution of sickness time for the household allocation problem. Second, like work time, sickness time does not enter the utility function as a separate argument.

In this context, with utility defined over consumption and leisure, the impact of a change in sickness time is straightforward. As argued by Grossman and Benham (1974), an increase in sickness time reduces full income; \( y = w(T - S) + V \), where \( w \) - real wage, \( V \) - unearned income. There will be a decline in hours of work providing consumption is normal (see O'Donnell, 1994). Allowing unearned income to be a positive function of sickness time, through related transfers, would create a further negative effect on hours of work, provided leisure is normal.

3. Uncertainty within a single decision period

Now consider the problem faced by an individual who must decide how much of a period to work, without knowing how much of that period will be lost to sickness. A contract must be signed without knowledge of sickness time within the contract period, e.g. one year. If the contract is violated, it is assumed the individual loses the opportunity to obtain another for the next period. If the individual wishes to keep their job, any unforeseen sickness time must therefore be at the expense of leisure rather than work. The contract need not explicitly specify hours of work. In fact, the scenario perhaps best fits the type of contract held by a manager, with production or profit targets specified rather than hours. For such
workers, temporary interruptions to work due to illness must be compensated by
greater work at a later date within the contract period. ¹

Sickness time is assumed to be a random variable, with the individual holding a
subjective assessment of its distribution, \( F(S, \sigma) \); where \( \sigma \) is a riskiness parame-
ter (see Appendix A). Hours of work are chosen to maximise expected utility,
defined over consumption and leisure, given the distribution of the stochastic
variable and subject to the budget and time constraints. By substituting the
constraints into the utility function the optimisation problem can be written as

\[
\max_{H} \int_{S}^{T} U(wH + V, T - H - S) dF(S),
\]

s.t. \( T - S \geq H \geq O \geq -V/w. \) (1)

The constraint requires that leisure, hours of work and consumption are all
non-negative. ² This formulation is consistent with the general problem, consid-
ered by Dardanoni (1988), of maximising a two argument utility function, with
one random argument \((T - H - S)\) and one fixed \((wH + V)\), through the choice of
a control variable \((H)\). The impact of a rightward shift in the distribution \( F(S, \sigma) \)
is qualitatively the same as an increase in \( S \) in the certainty case. In the previous
section, the latter was argued to have a negative effect on work hours under the
mild assumption of normality of consumption. This assumption, together with
concavity of utility and (absolute) risk aversion non-decreasing in \( H \), is sufficient
for hours of work to be decreasing in a mean preserving increase in the dispersion
of the distribution of sickness (see Appendix A). That is, the model predicts
increased uncertainty over sickness time results in less time being committed to
the labour market. As uncertainty increases, hours of work are reduced in order to
raise the expected value of leisure. This decreases the probability of being left with
very little leisure, even if a bad draw of sickness time is made.

The assumptions of concavity of utility and normality of consumption are
weak. The assumption of non-decreasing risk aversion in \( H \) is conventional
(Dardanoni, op cit) and is based on the premise that an individual is unlikely to
display less aversion to risk when the expected value of the random determinant of
utility (leisure) declines and the value of the certain component (consumption)
increases.

The result is dependent upon a decision framework which assumes a pre-com-
mitment to work rather than leisure. Under the alternative assumption, stochastic
sickness time results in randomness in consumption rather than leisure. It can be
shown, using the same analysis and assumptions as given in Appendix A, ³ that

¹ This model assumes there is no opportunity to take out insurance against sickness absence. The
availability of such insurance is considered in Section 5, in the context of a two period model.
² \( V \) and \( w \) are assumed non-negative and positive respectively.
³ In this case, risk aversion is assumed non-decreasing in \( L \).
increased uncertainty over sickness now results in the opposite impact on labour supply. Faced with greater randomness in consumption, the individual will increase expected labour supply (reduce pre-committed leisure) in order to limit the probability of experiencing a very low level of consumption. For primary earners employed, for example, as managers with contracts specifying production or profit targets, a pre-commitment to market work, rather than leisure, is more realistic. It seems plausible that such individuals will react to increased uncertainty over time lost to sickness by negotiating reduced production targets and, hence, hours of work. This model explains why it may be optimal for individuals to reduce their labour supply, perhaps opting for part-time work, as they age and, hence, face greater uncertainty over their health within each contract period. However, for secondary earners with many non-market claims on their time from children or elderly relatives, a pre-commitment to non-market time is, arguably, more realistic.

4. Uncertainty across two decision periods

Now let there be two periods, the present and the future. At the beginning of each period, it is assumed the individual knows the value sickness time will take in that period. However, at the beginning of the first period (i.e. period 0), the individual does not know how much of their time endowment in the second period (i.e. period 1) will be lost to sickness. The individual is assumed to hold a subjective assessment of the distribution of second period sickness, $F(S_1)$. In order to concentrate on the impact of uncertainty over sickness time, it is assumed second period wages and prices are known in the first. Utility is again assumed a function of consumption and leisure and the function is assumed to be additively separable over time. Finally, there is assumed to be no bequest motive. This general framework is consistent with that employed by Killingsworth (1983, pp. 239–261) in examining the consequences of wage rate uncertainty. The solution method adopted here follows Killingsworth (op cit).

The first period expected utility maximisation problem is,

$$\max EU = \max_{H_0, z_t} \left\{ U^0 \left[ (1 + r)Z_0 + w_0H_0 - Z_1, T_0 - H_0 - S_0 \right] + (1 + \rho)^{-1} E[U^{*1}] \right\}$$

$$E[U^{*1}] = \int_{S}^{S_1} U^{*1} dF(S_1)$$

$$= \int_{S}^{S_1} \max_{H_1} U^1 \left[ (1 + r)Z_1 + w_1H_1, T_1 - H_1 - S_1 \right] dF(S_1),$$

(2)
where $Z_t$, $t = 0, 1$ are assets held over from period $t - 1$ and $\rho$ is the marginal rate of time preference.\footnote{A superscript $t = 0, 1$ indicates first and second period utility respectively. For other terms, a subscript $t = 0, 1$ indicates the period for which it is evaluated. Subscripts on utilities indicates derivatives.}

Assuming interior solutions, the first order conditions are,

$$w_0 U_C^0 - U_L^0 = 0.$$  \hspace{1cm} (3)

$$-U_C^0 + \frac{(1 + r)}{(1 + \rho)}E[U_C^*] = 0.$$ \hspace{1cm} (4)

In order to derive the comparative dynamic effects of changes in the distribution of $S_1$, let $S_1' = \nu S_1 + m$ and replace $S_1$ with $S_1'$ in the optimisation problem. Examining the impact of a shift in the distribution of $S_1$, holding the variance constant, involves considering $dm \neq 0$ and $d\nu = 0$. On the other hand, a mean preserving change in the dispersion of the distribution occurs where $dm = -\mu_S d\nu$; where $\mu_S$ is the mean of $S_1$.

The second period problem can be solved to yield the following expressions (see O'Donnell, 1994), which are useful in examining the impact of uncertainty faced in the first period:

$$\frac{d^2 U'^*}{dZ_1^2} = (1 + r)^2 \left( \frac{\gamma_1}{\Theta_1} \right),$$ \hspace{1cm} (5)

$$\frac{d^2 U'^*}{dZ_1 dS_1} = -(1 + r) w_1 \left( \frac{\gamma_1}{\Theta_1} \right),$$ \hspace{1cm} (6)

$$\frac{d^2 U'^*}{dS_1^2} = w_1^2 \left( \frac{\gamma_1}{\Theta_1} \right),$$ \hspace{1cm} (7)

where: $\gamma_1 = U_{CC}^1 U_{LL}^1 - (U_{CL}^1)^2 > 0$, by concavity, and $\Theta_1 = w_1^2 - 2 w_1 U_{CL}^1 + U_{LL}^1 < 0$ from the s.o.c.

Using (5) and (6), implicit differentiation of the first order conditions (3)–(4) gives,

$$dH_0 = -\frac{\alpha_0}{\theta_0} dZ_1,$$ \hspace{1cm} (8)

$$\left[ U_{CC}^0 + \frac{(1 + r)^2}{1 + \rho}E\left( \frac{\gamma_1}{\Theta_1} \right) \right]dZ_1 + \alpha_0 dH_0 = \frac{1 + r}{1 + \rho} w_1 E\left( S_1 \frac{\gamma_1}{\Theta_1} \right) d\nu$$

$$+ \frac{1 + r}{1 + \rho} w_1 E\left( \frac{\gamma_1}{\Theta_1} \right) dm,$$ \hspace{1cm} (9)
where: \( \alpha_0 = U_{CL}^0 - w_0 U_{CC}^0 \) and \( \Theta_0 \) is defined analogously to \( \Theta_1 \). Substituting for \( dH_0 \) in (9) using (8) and setting \( dv = 0 \) yields, after simplification,

\[
\frac{dZ_1}{dm} = \frac{(1 + r)}{(1 + \rho)} \frac{w_1 E}{\gamma_1} \left( \frac{\gamma_1}{\Theta_1} \right) \left( \frac{1 + r}{\Theta_0} + \frac{(1 + r)^2}{(1 + \rho)} E \left( \frac{\gamma_1}{\Theta_1} \right) \right)
\]

(10)

Also,

\[
\frac{dH_0}{dm} = - \left( \frac{\alpha_0}{\Theta_0} \right) \frac{dZ_1}{dm} \cdot \frac{1}{\gamma_0}
\]

(11)

With a concave utility function, \( \gamma_1 > 0 \) and \( \Theta_1 < 0, t = 0, 1 \). Therefore, from (10), an increase in expected sickness time increases savings under a relatively weak restriction on preferences. Providing leisure is normal, \( \alpha_0 > 0 \) (see O'Donnell, 1994), and current period labour supply also increases in response to an increase in anticipated sickness time.

A couple of intuitive interpretations of these results can be given. An increase in sickness time in the second period represents a fall in full income in that period. If both consumption and leisure are normal, the demand for both will fall, increasing the marginal utility of both arguments in the second period. Since the optimal life-cycle consumption and leisure paths involve equating discounted marginal utilities across periods, the quantities of consumption and leisure demanded in the current period must also decline in order to restore equilibrium consumption/leisure paths. Further, an increase in expected sickness time in the second period represents a fall in anticipated lifetime full income. A decline in demand for all normal commodities would be predicted.

The results of Section 2 suggest an increase in current period sickness reduces current labour supply, whilst in a dynamic setting hours of work increase with the expectation of future sickness. It is likely that sickness will be positively serially correlated, such that an increase in first period sickness raises the expectation of second period sickness. In this case, an increase in current sickness has two contradictory effects on current hours of work. Cross-section evidence typically reveals a negative effect of ill-health on labour supply (e.g., Grossman and Benham, 1974; Lambrinos, 1981). This suggests the direct effect of current sickness, generally, outweighs its effect through the expectation of future sickness.

Setting \( dm = -\mu dv \) in (8)–(9) and solving yields the comparative dynamic

<sup>5</sup> I thank a referee for drawing my attention to this.
From concavity, the denominator of (12) is negative and so the impact of changes in uncertainty over sickness time on the savings decision is dependent on the sign of the covariance term in the numerator. This covariance can be signed by making an assumption about how attitudes toward risk vary with sickness time. Following Killingsworth (1983, p. 259), a risk premium, $\pi$, can be derived (see O'Donnell, 1994),

$$
\pi = -\frac{\sigma^2_z}{2} \left( \frac{w_1}{U_{L}^{*+1}} \gamma_1 \right),
$$

where $\sigma^2_z$ is the variance of $S_1$ and the risk premium is defined with all terms in parentheses evaluated at maximised utility, given sickness time set at its expected value ($\mu_z$). Concavity and a positive marginal utility of leisure ensures the risk premium is positive.

Assuming aversion to sickness time risk is greater at a higher expected value of $S_1$, the risk premium is increasing in $\mu_z$. The basis for this assumption is that, as the expected value of sickness time increases, an individual anticipates having less time available for work and leisure and is therefore more averse to a given volatility in sickness time, since a bad draw from the distribution will leave them with very little time to divide between these activities. From $dU_{L}^{*+1}/dS_1 = -U_{L}^{*+1}$ and (7), under concavity, the marginal utility of leisure is increasing with $\mu_z$. Given this, the risk premium can only be increasing in $\mu_z$ if this is true of $(-\gamma_1/\Theta_1)$. Providing the derivative never changes sign, the latter implies the function $(-\gamma_1/\Theta_1)$ is increasing in the realisation of $S_1$ and so, $\text{cov}[\gamma_1/\Theta_1, S_1] < 0$. 

$$
\text{cov}[\gamma_1/\Theta_1, S_1] < 0.
$$
From (12), under the assumptions of concavity, aversion to risk increasing in the expected value of sickness time and no change in the sign of $\partial(\gamma_1/\Theta_1)/\partial S_1$, greater uncertainty over sickness time is predicted to lead to higher savings. That is, uncertainty over sickness time provides a precautionary motive for saving (Kimball, 1990). From (13), under the further assumption of normality of leisure, labour supply in the current period is also predicted to rise in the face of greater sickness time uncertainty. Although there is a plausible conceptual basis for three of the four assumptions necessary for these results, this is not true for the requirement that there be no change in the sign on the derivative of $\gamma_1/\Theta_1$ with respect to $S_1$.

The intuition behind these results is not difficult. Greater uncertainty over sickness time means facing a larger probability of realising a very high value of sickness time and consequently having little time available for work and leisure. An individual displaying increasing aversion to sickness time risk wishes to avoid very low values of consumption and leisure, and hence utility, and hedges against this by working harder and saving more in the first period. By doing so, they assemble a stock of assets which are insurance against being forced to experience very low values of both consumption and leisure in the second period, even if they draw a large value of sickness time.

In order to concentrate on the impact of sickness time, the results above were generated under the assumption that second period wage rates are known in the first. Of course, increased uncertainty over health may be expected to induce uncertainty over both future sickness time and wages. Killingsworth (1983, pp. 239-261) used the methodology applied above to examine the impact of increased wage uncertainty on labour supply and savings. Under similar assumptions with respect to concavity, normality and risk aversion, and with the additional assumption of a continuously forward sloping labour supply curve, Killingsworth (op cit) predicts both hours and savings are increasing with wage uncertainty. Of course, it does not necessarily follow, even under the assumptions stated, that a simultaneous increase in wage and sickness time uncertainty would increase saving and labour market activity.

5. Social disability insurance

The above results suggest an individual displaying increasing aversion to sickness time risk will react to greater uncertainty by working more and increasing their holding of assets; a form of self insurance. The literature on precautionary savings demonstrates the sensitivity of such results to the availability of alternative forms of insurance (e.g., Kotlikoff, 1986). Whilst a risk averse individual will purchase actuarially fair sickness/disability insurance, this opportunity may not exist due to the problems of adverse selection and moral hazard preventing the existence of a complete set of insurance markets (Rothschild and Stiglitz, 1976).
Observation confirms the lack of such market opportunities. In most developed countries this void in markets is filled by social disability insurance. In this section the comparative dynamic effects of changes in the distribution of sickness time on labour supply and saving decisions are re-considered in the presence of sickness related income transfers available through social insurance. The finance of the transfers is not considered since the aim is not to establish the net effect of social disability insurance on labour supply and saving behaviour.

The optimisation problem differs from that examined in the previous section by the presence of a transfer \( V(S) \), which offers financial compensation for sickness \( (V_s > 0) \). As is clear from the solutions given in Appendix B, provided \( w_1 > V_{S_1} \), \( \forall S_1 \), a change in the expected value of second period sickness shifts first period saving and labour supply in the same direction as it did in the absence of sickness related transfers. The condition on the relative magnitude of the wage and marginal compensation ensures a rise in anticipated sickness time corresponds to a fall in expected full income at the margin and so this result makes intuitive sense.

It seems realistic to assume income transfers will be set such that, \( w_1 > V_{S_1} \), in order to contain moral hazard. Further, providing \( V_{S_1} > 0 \), the introduction of sickness related transfers reduces the marginal (positive) impact of anticipated sickness time on saving, and labour supply. This result is entirely as expected. The provision of compensation limits the expected reduction in full income in the second period and reduces the need for inter-temporal substitution.

In order to obtain unambiguous results with respect to the impact of a change in uncertainty over sickness time, compensation must be assumed proportional to sickness time and the degree of proportionality assumed known. Under these assumptions, and those of the previous section, an increase in uncertainty over sickness time still results in an increase in savings and labour supply (if \( w_1 > V_{S_1} \)). But the marginal impact of uncertainty over sickness is less than in the absence of compensation (if \( V_{S_1} > 0 \)). Again this makes intuitive sense, with some form of social insurance, there is less need for self insurance against uncertain prospects.

6. Discussion

This paper has considered the consequences of time lost to sickness for labour supply and saving decisions. A certain increase in sickness time reduces full income and results in less time being devoted to both work and leisure. An

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6. In a generalised model which allowed compensation to be a \( \textit{continuous} \) function of both current and past earnings, in addition to sickness time, unambiguous signs for the comparative dynamics could not be obtained.

7. The assumption of proportionality is inconsistent with the rules of some social disability insurance systems. For example, in the UK an individual is paid Sickness Benefit for the first 28 weeks of absence from work before moving onto Invalidity Benefit paid at a different rate.
individual deciding how much of a period to commit to the labour market, without knowledge of how much will be lost to sickness, is predicted to commit less, the greater is uncertainty over sickness time. Current labour supply and saving decisions depend upon the distribution of future sickness time. An increase in expected sickness is predicted to raise both current labour supply and savings. An increase in uncertainty over sickness has the same effect. When financial compensation for sickness is available through social insurance, these results hold but the magnitude of the marginal impact of sickness time on labour supply and savings declines.

The results have a number of implications for policy and research. Usually, ill-health is anticipated to affect labour market activity through a direct effect on the productivity of individuals experiencing illness. This analysis emphasises the wider economic consequences of sickness. The prospect of ill-health affects the economic behaviour not only of individuals who are or will become sick, but also individuals who are potentially, but never actually, unwell. The introduction of an effective treatment is often claimed to benefit the economy through raising labour inputs of recipients of care. This analysis suggests there is also a negative effect on labour supply, as a consequence of reducing the expected value and degree of uncertainty over sickness. Indeed, since potential victims of an illness typically outnumber those who will actually succumb, the net impact of an effective treatment on labour market activity may be negative.

The analysis also points to an effect of ill-health on macroeconomic activity through savings behaviour. Since sickness time is likely to increase in both expected value and dispersion with age, as the population ages, assuming no change in the age-specific distribution of sickness, the dynamic model predicts the population savings rate will increase. However, due to improvements in medicine and other factors, age-specific distributions of health are shifting to the right and becoming more compressed. According to the model, this will reduce age-specific savings rates. Such predictions suggest consideration should be given to the appropriate inclusion of health variables in the specification of consumption functions.

The theoretical models could be developed in a number of ways. Since health has been assumed exogenous, the models do not incorporate medical care. Selden (1993) shows that preventive medicine, which changes the distribution of second period sickness, can substitute for saving in a two period model. If curative medicine was also introduced to the dynamic model of the present analysis, one might expect that greater uncertainty over sickness would lead to greater saving, as a precaution against the need to finance large medical expenditures. Of course, once medical care is introduced, health insurance must also be considered.

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8 From a welfare economics perspective, both the increased productivity of recipients of care and the reduced uncertainty over sickness are welfare enhancing.
Conventional theory predicts the demand for health insurance will increase with uncertainty over sickness. This would be expected to reduce the impact of the latter on savings. Further work is required to determine whether, and under what circumstances, these results would be generated by a dynamic model, like that of Section 4, extended to incorporate endogenous sickness time (with a stochastic component), medical care and health insurance.

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Appendix A

Uncertainty within a single decision period – Comparative static effect of increased risk

Consider the impact on hours of work (H) of an increase in the riskiness parameter, \( \sigma \), associated with the distribution of sickness time, \( F(S, \sigma) \). An increase in \( \sigma \) represents a Rothschild–Stiglitz (R–S) (mean preserving) increase in risk iff:

\[
\int_S^S F_{\sigma}(S, \sigma) dS = 0, \quad \text{(A.1)}
\]

and

\[
\int_S^S F_{\sigma}(S, \sigma) dS \geq 0 \quad \forall \Phi \in [S, S], \quad \text{(A.2)}
\]

where

\[
F_{\sigma} = \frac{\partial F}{\partial \sigma}
\]

(Dardanoni, 1988; Diamond and Stiglitz, 1974).

Assuming an interior solution, the first and second order conditions for the solution to the optimisation problem (1) are,

\[
wE_{UC} - E_{UL} = 0, \quad \text{(A.3)}
\]

\[
D = w^2E_{CC} - 2wE_{CL} + E_{LL} < 0, \quad \text{(A.4)}
\]

where \( E \) is an expectation operator.
Implicit differentiation of the f.o.c., assuming the limits of the distribution are independent of \( \sigma \), gives,

\[
\frac{\partial H}{\partial \sigma} = \frac{\int_S^s (U_L - wU_C) dF_{\sigma} }{D}.
\]  
(A.5)

Following Dardanoni (op cit), integrating the numerator by parts twice, using the assumed independence of the tails of the distribution from \( \sigma \) and (A.1) gives,

\[
\int_S^s (U_L - wU_C) dF_{\sigma} = \int_S^s (U_{LLL} - wU_{LLL}) \left[ \int_t^s F_{\sigma}(t, \sigma) dt \right] dS.
\]  
(A.6)

This, (A.2) and \( D < 0 \), implies,

\[
\text{sgn} \left( \frac{\partial H}{\partial \sigma} \right) = \text{sgn} (wU_{LLL} - U_{LLL}).
\]  
(A.7)

Using the logic of Dardanoni (op cit), a two good index of absolute risk aversion, \( R_A = -(U_L/U_L) \), is assumed to be non-decreasing in \( H \) such that,

\[
\frac{\partial R_A}{\partial H} = \left[ \frac{(-wU_{LLL} + U_{LLL})U_L + U_{LLL}(wU_{LC} - U_{LL})}{U_L^2} \right] \geq 0.
\]  
(A.8)

Providing consumption is a normal good, \((wU_{LC} - U_{LL}) > 0\), and by concavity, \( U_{LL} < 0 \). So \( \partial R_A/\partial H > 0 \) requires, \((-wU_{LLL} + U_{LLL}) > 0\). From (A.7), this implies, \((\partial H/\partial \sigma) < 0\).

Appendix B

Social Disability Insurance – Derivation of comparative dynamics

The first period constrained optimisation problem with a sickness transfer \( V(S) \) is,

\[
\max_{H_0,Z_1} \mathbb{E}[U] = \max \left\{ U \left[ (1 + r) Z_0 + w_0 H_0 + V(S_0) - Z_1, T_0 - H_0 - S_0 \right] + (1 + \rho)^{-1} \mathbb{E}[U^1] \right\},
\]

\[
\mathbb{E}[U^1] = \int_{S_1}^{S} \max_{H_1} U \left[ (1 + r) Z_1 + w_1 H_1 + V(S_1), T_1 - H_1 - S_1 \right] f(S_1) dS_1.
\]  
(B.1)
The comparative dynamics are generated in the same way as for the simpler model. The second period problem yields (5) plus the following expressions:

\[
\frac{d^2U_{t+1}}{dZ_t dS_t} = (1 + r)(V_{s_t} - w_t) \left( \frac{\gamma_1}{\Theta_1} \right) .
\]

(B.2)

\[
\frac{d^2U_{t+1}}{dS_t^2} = (w_t - V_{s_t})^2 \left( \frac{\gamma_1}{\Theta_1} \right) + U_{C_t} V_{s_t}. \]

(B.3)

The first order conditions for the first period problem are exactly as before. Implicitly differentiating these, using the above results, and substituting gives,

\[
\frac{dZ_1}{dm} = \frac{(1 + r)}{(1 + \rho) E \left( (w_t - V_{s_t}) \frac{\gamma_1}{\Theta_1} \right)} \frac{\gamma_0}{\Theta_0} + \frac{(1 + r)^2}{(1 + \rho)} E \left( \frac{\gamma_1}{\Theta_1} \right)
\]

and (11).

The impact of a change in the spread of the distribution on saving behaviour is now given by,

\[
\left( \frac{dZ_1}{dv} \right)_{dm=\mu, dv} = \frac{(1 + r)}{(1 + \rho) E \left( (w_t - V_{s_t}) \frac{\gamma_1}{\Theta_1} (S_t - \mu_s) \right)} \frac{\gamma_0}{\Theta_0} + \frac{(1 + r)^2}{(1 + \rho)} E \left( \frac{\gamma_1}{\Theta_1} \right).
\]

(B.5)

Assuming compensation is proportional to sickness time and the degree of proportionality is known, (B.5) can be written as,

\[
\left( \frac{dZ_1}{dv} \right)_{dm=-\mu, dv} = \frac{(1 + r)}{(1 + \rho) (w_t - V_{s_t}) \text{cov} \left[ \frac{\gamma_1}{\Theta_1}, S_t \right]} \frac{\gamma_0}{\Theta_0} + \frac{(1 + r)^2}{(1 + \rho)} E \left( \frac{\gamma_1}{\Theta_1} \right)
\]

(B.6)

A risk premium can be derived as before, to give,

\[
\pi' = \frac{\sigma_s^2}{2} \left( \frac{(w_t - V_{s_t})^2}{V_{s_t} U_{C_t} \Theta_1^* - U_{L_t} \Theta_1^*} \frac{\gamma_1}{\Theta_1} \right).
\]

(B.7)

where the assumption of proportionality \((V_{s_t,s_t} = 0)\) has been used.

Again all expressions inside the parentheses are evaluated at maximised utility, with sickness time at its expected value. From \(dU_{t+1} / dS_t = U_{C_t} V_{s_t} - U_{L_t} \) and (B.3), the denominator of the first expression inside the brackets is decreasing in.
$s_1$ (given $V_s s_1 = 0$) and so it is decreasing in $\mu_z$. Therefore, for the risk premium to be increasing in $\mu_z$, $\gamma_1 / \Theta_1$ must be decreasing in this value, and so, assuming the derivative never changes sign, in the variable $s_1$. Then from (B.6), provided $\omega_1 > V_s$, an increase in uncertainty over sickness time still results in an increase in savings. The impact on labour supply is again obtained from (13).

References


Kimball, M.S., 1990, Precautionary saving in the small and in the large, Econometrica 58, 53–73.


