Dynamic Multitasking and Managerial Investment Incentives*

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Abstract

We study non-contractible intangible investment in a dynamic agency model with multitasking. The manager’s short-term task determines current performance which deteriorates with investment in the firm’s future profitability, his long-term task. The optimal contract dynamically balances incentives for short- and long-term performance such that investment is distorted upwards (downwards) relative to first-best in firms with high (low) returns to investment. These distortions decrease as good performance relaxes endogenous financial constraints, implying negative (positive) investment-cash flow sensitivities. Our results shed light on how corporate investment policies, liquidity management and executive compensation structure differ across industries with different returns to intangible investment.

Keywords: Continuous time contracting, multiple tasks, delegated investment, managerial compensation, endogenous financing frictions, investment dynamics.


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1 Introduction

A manager responsible for a firm’s operations usually has some form of discretion in running the day-to-day business, which relies on his specific skills or private information. Due to the separation of ownership and control, this gives rise to an agency problem, which has been the focus of much of the recent dynamic financial contracting literature. Yet, in a dynamic world, firms also have to take strategic decisions and invest in order to maintain long-term profitability. Typically, this investment process also relies on information and skills of the same manager or is even delegated to him with discretion. In fact, many strategic investments, in particular those in intangibles such as investment in R&D, process innovation, product development, human capital, or distribution systems, share the following features: i) actual investment expenditures are not easily verifiable by firm owners as they are either not reported or reported expenses can be manipulated, ii) their outcome is uncertain, and iii) they have persistent effects on the firm’s profitability. The manager’s hidden actions, thus, affect both the firm’s current period payoffs as well as its long-term profitability, giving rise to a dynamic multitask problem.

To capture these ideas, we introduce delegated non-contractible investment in the firm’s future profitability – which we interpret as investment in intangible capital – into a continuous time cash flow diversion model. We analyze how the need to incentivize the manager to meet both short-term as well as long-term targets affects the optimal compensation scheme as well as the efficiency of the investment process. Under the optimal long-term contract, investment depends on the entire history of past performance and is distorted away from the first-best level. We show that both the sign of the investment distortion as well as the comparative statics of investment (with respect to realized cash flows, cor-

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1E.g., DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007), Biais et al. (2007), or Hoffmann and Pfeil (2010). See also Biais et al. (2013) for an excellent survey.

2Indeed, several forms of intangible or “soft” investment expenditures are – unlike hard investments in plants, property or other equipment – not reported explicitly in firms’ financial statements. For those intangible investments that are reported accounting rules often leave considerable discretion to managers “[...] to alter financial reports to either mislead some stakeholders about the underlying economic performance of the company, or to influence contractual outcomes that depend on reported accounting numbers” (Healy and Wahlen 1999), a practice referred to as earnings management. For instance, while R&D expenditures are reported in firms’ income statements, “[...] the notion of what outlays are considered R&D [...] can be difficult to assess, and often represents the manager’s discretionary choice” (Koh and Reeb 2015). Exploiting this discretion, e.g., by shifting core expenses to special items such as R&D, is referred to as classification shifting in the accounting literature (see, e.g., McVay 2006, Skaife et al. 2013, and Darrough et al. 2017, or, for related arguments also Bebchuk and Stole 1993, or Dutta and Reichelstein 2003).
porate governance or financial frictions) crucially depend on the technological returns to investment, i.e., the extent to which a firm’s long-term profitability depends on successful investment. Investment is distorted upwards relative to the first-best benchmark when returns to investment are high and it is distorted downwards when they are low. These distortions decrease in absolute terms with financial slack and, thus, past performance. In line with recent empirical evidence (see, e.g., Peters and Taylor 2017 or Hovakimian 2009), this implies a positive relation between intangible investment and realized cash flows in industries with low, and a negative relation in industries with high returns to such investment.3 Further, if the costs of raising external funds increase, so will both the distortions in investment as well as its sensitivity to past performance. Our multitask model with non-contractible investment, thus, provides a new perspective on investment-cash flow sensitivities as a measure of financial constraints accounting for investment in intangibles (cf., e.g., Brown and Petersen 2012 or Chen and Chen 2012).

The key agency frictions in our dynamic multitask theory of investment are due to i) unobservable cash flows and ii) non-contractibility of actual investment expenditures. An agency problem with respect to investment then arises endogenously as a result of compensation for short-term performance which is necessary to deter the manager from diverting cash flows for private consumption: Since his compensation is tied to current earnings figures, the manager has an incentive to cut profitable investment in order to boost the firm’s short-term profits at the expense of its long-term profitability.4 Our model, thus, shares with the literature on managerial short-sightedness the notion that short-term incentive schemes can induce managers to focus excessively on current period outcomes.5 However, in our model such unintended consequences of short-term incentive

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3 Much of the related empirical literature focusses on accounting measures of intangible investment input, such as R&D or SG&A expenditures, which, in light of our model, should be interpreted with caution given the considerable discretion in reporting these expenditures under current accounting standards (see, also footnote 2). Still, these findings are qualitatively in line with our model’s prediction which we formulate in terms of verifiable measures of investment output, such as, for the case of R&D investment, the number of patents granted or the market’s reaction to a new product launch.

4 These incentives should be particularly strong for intangible investment expenditures, which, in contrast to investments in physical capital, are mainly expensed rather than capitalized. Accordingly, financial constraints, which in our model arise endogenously from the agency model, should be particularly strong for these investments due to their low collateral value (see Almeida and Campello 2007).

5 This short-term bias has been attributed to career concerns (Narayanan 1985), takeover threats (Stein 1988), concerns about the firm’s stock price over a near-term horizon (Stein 1989), mispricing of long-term assets (Shleifer and Vishny 1990), reputational herding (Zwiebel 1995) and short-term financing (von Thadden 1995).
pay can be mitigated by tying compensation also to indicators of investment success such as project milestones or the number of patents granted as is common practice in particular in high-tech firms (see, e.g., Balkin et al. 2000).

Yet, as is well known from related (single-task) dynamic financial contracting models (see, e.g., DeMarzo and Sannikov 2006), exposing the manager to compensation risk in order to provide incentives is costly. In our model with bilateral risk neutrality this formally is a consequence of limited liability, which requires to terminate the contract if, after a series of bad outcomes, the manager is “too poor to be punished.” In this case he must be replaced, which is costly ex-post, but part of the ex-ante optimal long-term contract. Hence, the need to tie the manager’s pay to (imperfect) signals of investment success or failure, such as to incentivize investment, creates additional agency costs. The optimal investment schedule is then determined by trading-off the agency costs of investment with the potential efficiency gains as captured by the respective returns to investment.

We show that the agency costs of investment are non-monotonic in the implemented investment level. In particular, marginal agency costs are positive for low and negative for high levels of investment. As a consequence, investment is distorted downwards relative to the first-best (owner-manager) benchmark in firms with low returns to investment where first-best investment is low. By contrast, investment is distorted upwards relative to first-best when returns to investment and, thus, also first-best investment is high. To understand this result, consider a firm with high returns to investment and assume that the principal wants to implement the first-best investment level. Since the probability of an investment success is high and the probability of failure low given the high first-best investment level, punishment upon failure provides stronger incentives per unit of expected pay than a reward for success.6 Accordingly, the optimal compensation policy relies heavily on punishment such that the costs of providing incentives are concentrated on the failure event. Thus, if returns to investment are sufficiently high, agency costs can be reduced by distorting investment above the high first-best level, as this reduces the probability of having to bear the high incentive costs associated with punishing the manager for an investment failure. Similarly, if returns to investment are low, incentives

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6 Formally, the relative use of reward and punishment depends on the relative likelihood ratio of the two performance signals available to the principal (cf., Holmström 1979). If investment is sufficiently high, the likelihood ratio of an investment success is lower than that of an investment failure and vice versa if investment is sufficiently low.
are mainly provided by generous rewards. In this case, agency costs can be reduced by distorting investment downwards relative to the low first-best level, such as to reduce the probability of triggering a costly reward following investment success.

While the sign of investment distortions is in our model determined by the profitability of the investment technology, their size depends on the firm’s past performance, creating novel implications for investment dynamics. In particular, when the manager’s track record – the history of cash flows and investment outcomes under his tenure – improves, his stake in the firm under the optimal contract increases, mitigating the agency problem and, thus, reducing investment distortions. Accordingly, investment expenditures are negatively related to cash flows in overinvesting firms (i.e., in firms with high returns to investment), while in underinvesting firms (i.e., those with low returns to investment), the relation is positive.

We show that this dependence of investment on past performance is stronger the more severe the firm’s financial constraints. Concretely, we consider a standard implementation of the optimal contract in which the manager’s track record is captured by a measure of the firm’s financial slack in the form of cash holdings (cf., e.g., Biais et al. 2007). When cash holdings are depleted, the firm is no longer able to honor its payments to investors, triggering a costly restructuring of the firm. This involves replacing the incumbent manager and raising new external funds. We show that if the costs of raising external funds, i.e., financial constraints, increase, investment distortions become more severe and investment becomes more sensitive to cash flows for any given level of financial slack. Hence, investment increases and its sensitivity to financial slack becomes more negative in firms with high returns to investment, while investment decreases and its sensitivity to financial slack becomes more positive in firms with low returns to investment.

Our model, thus, contributes to the debate about the relationship between investment-cash flow sensitivities and financial constraints, which here arise endogenously from the agency problem. By focussing on investment in future profitability per unit of (physical) capital instead of standard capital investment, our model can be viewed as a first step towards accounting for the increasing importance of intangible investment, e.g., in R&D or knowledge or organizational capital (see, for instance, Brown and Petersen 2009). We

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7Since cash is required to cover investment expenditures, potential operating losses, and payments to investors, while it increases with cash flows, the level of cash holdings is a summary measure of the entire performance history.
show that investment-cash flow sensitivities as an empirical measure of financial constraints need to be interpreted with caution in order to account for the distinct features of intangible investment. There are two main reasons: First, financially constrained firms may feature both positive as well as negative investment-cash flow sensitivities depending on firm- or industry-specific returns to intangible investment. Concretely, our model predicts investment-cash flow sensitivities to be negative in industries where intangible investment is particularly important (see also recent empirical evidence in Hovakimian 2009 and Peters and Taylor 2017) and positive else. Hence, our model offers an explanation for the puzzling findings in Chen and Chen (2012) that on average (i.e., not controlling for intangible investment intensity) investment-cash flow sensitivities have disappeared despite financial constraints still being a first-order concern for firms. The second empirical challenge is that, in light of our model of non-contractible (intangible) investment, accounting measures of intangible investment input, such as R&D or SG&A expenditures, which are predominantly used in the empirical literature, might be unreliable given the considerable discretion in reporting these expenditures under current accounting standards (cf., footnote 2). Taking this feature of intangible investment seriously, we suggest that empirical tests should focus instead on verifiable measures of investment output – such as, for the case of R&D investment, the number of patents granted or the market’s reaction to new product launches. Indeed, considering output measure of intangible investment next to the more commonly used input measures might allow to establish more robust empirical findings also beyond the concrete predictions of our model.\footnote{The use of such output measures is already quite common in studies measuring firms’ innovation efficiency, employing, in particular, the NBER patent database (see for instance Hall et al. 2005 or Kogan et al. 2017).}

The multitask nature of our dynamic optimal contracting model further allows to derive new testable implications regarding the relation between pay for short-term performance and long-term investment. If, in our model, the cash flow diversion problem becomes more severe, e.g., due to worse corporate governance, then i) the optimal contract has to be more sensitive with respect to short-term performance. This exacerbates the agency problem with respect to long-term investment, such that, ii) investment distortions increase in absolute terms. That is, steeper short-term incentives are associated with higher investment expenditures in firms with high returns to investment and with lower investment expenditures in firms with low returns to investment.
In addition, our model provides clear predictions regarding the dependence of executive pay on the performance of intangible investment projects (see e.g. Balkin et al. (2000) for empirical evidence). Notably, our model predicts an asymmetric response of compensation to investment success and failure, with pay being more sensitive to investment success in firms with low returns to investment, where incentives are mainly provided via reward. The opposite is true in firms with high returns to investment, where incentives are mainly provided via punishment.

Ultimately, the need to incentivize intangible investment creates additional volatility in the firm’s cash position which varies endogenously with the returns to intangible investment via the implemented investment profile and the pay for performance sensitivities used to incentivize it. We show that the average volatility of firms’ cash holdings is not monotonically related to the returns to investment, but hump-shaped as we move from industries in which successful intangible investment is less important (such as in oil, mining, or textile industries) to those in which it is highly important (such as in the pharma or the high-tech sector). This has direct implications for corporate liquidity management. In particular, our model predicts firms with intermediate returns to investment to build particularly high cash buffers before stipulating payouts, e.g., in the form of dividends. The reason is that cash holdings are particularly volatile for these firms, which exacerbates financing frictions, thus, making precautionary cash holdings highly valuable.

**Related Literature.** Optimal investment with dynamic agency is also analyzed in the two-period model of Gertler (1992) as well as the multiperiod discrete time models in Quadrini (2004), Clementi and Hopenhayn (2006), and DeMarzo and Fishman (2007). Further, DeMarzo et al. (2012) analyze dynamic agency in a neoclassical investment setting using continuous time methods. All of these contributions consider capital investment which can be verified by firm owners. In our model, by contrast, the manager privately controls investment and, thus, has to be incentivized to invest in the firm owners’ interest. Further, (intangible) investment in our model does not scale cash flows by increasing the capital stock as in neoclassical investment settings, but rather affects mean cash flows per unit of (constant) firm size.

From a methodological perspective, our model is most closely related to Hoffmann and Pfeil (2010) who introduce exogenous (Poisson) shocks to firm profitability in a dynamic
cash flow diversion model à la DeMarzo and Sannikov (2006). While the firm’s long-term profitability in their model is purely determined by exogenous “lucky” shocks, it can be controlled by the manager in the present analysis via investment expenditures, giving rise to a fully dynamic multitask problem (see Holmström and Milgrom 1991). In contemporaneous work, Gryglewicz et al. (2018) and Hackbarth et al. (2018) also analyze dynamic multitask problems in which the manager controls both current cash flows via a short-term action as well as the firm’s future earnings via a long-term action. However, while the manager’s long term action in their models controls firm size in the spirit of standard neoclassical capital investment, in our model of intangible investment it persistently affects future profitability per unit of constant firm size. Accordingly, also the nature of observable performance signals, as a basis for incentivizing long-term investment, differ in the two settings: In our model (intangible) investment success and failure are rare events – corresponding, e.g., to the approval or rejection of a patent application – and the main mechanism driving our results is based on their (relative) informativeness which depends on the implemented investment level. This mechanism is absent in Gryglewicz et al. (2018) and Hackbarth et al. (2018) where capital investment controls the drift of the capital stock such that performance signals arrive continuously over time according to a diffusion process.9 Their analysis of the agency conflicts in capital investment, thus, complements ours which captures delegated intangible investment, e.g., in knowledge or organizational capital.

Our dynamic multitask problem is also related to Zhu (2018) who considers a model of persistent moral hazard in which the agent can choose between a short-term or a long-term action in every period. He characterizes the optimal contract that always induces the long-term action. Our focus is instead on determining the optimal interior investment level at every possible history while abstracting from problems of persistent private information. Relatedly, Varas (2017) studies a multitasking model of project completion in which the agent can complete a project faster by reducing its quality which is a one-time action. In

9Accordingly also the predictions regarding investment distortions differ fundamentally. In particular, while in our model overinvestment arises when investment in intangible capital is very profitable, Gryglewicz et al. (2018) find that overinvestment is more likely when the technology governing capital investment is inefficient. Then, to counteract the dilution in the manager’s stake due to unexpected firm growth, the optimal contract in their model may feature excessive capital investment. By contrast, in Hackbarth et al. (2018) it is never optimal to distort the manager’s long-term action above the first-best level since firm owners do not fully incorporate the benefits of long-term firm growth due to a classic debt overhang problem.
contrast, we consider a fully dynamic repeated problem in which the agent can divert cash flow for private consumption as well as move funds within the firm in order to meet either short- or long-term targets. In this sense, our model is related to the capital budgeting literature starting with Harris and Raviv (1996) and extended to a dynamic setting in Malenko (2018).

The remainder of the paper is organized as follows. We introduce the model in Section 2. In Section 3 we derive the optimal dynamic contract and provide a detailed discussion of optimal compensation and investment. Section 4 provides comparative statics and derives empirical implications. Section 5 concludes. All proofs as well as some additional technical material are contained in the Appendix.

2 The Model

We consider an infinite horizon continuous time principal-agent model of a firm whose owners (the principal) hire a manager (the agent) to operate the business, which includes investment in the firm’s future profitability. We now, first, present the firm’s cash flow process and investment technology. Second, we introduce the agency problem between firm owners and the manager, and formulate the optimal contracting problem.

2.1 Cash Flow Process and Investment Technology

The firm’s cash flows, \( Y = \{Y_t : t \geq 0\} \), net of investment expenditures, \( I = \{I_t \in [0, \bar{I}] : t \geq 0\} \), evolve according to

\[
dY_t = (\mu_t - I_t) \, dt + \sigma \, dZ_t, \tag{1}
\]

where \( \mu_t \) denotes the prevailing drift rate of cash flows, which we refer to as the firm’s “profitability,” \( \sigma \) the instantaneous volatility, and \( Z \) is a standard Brownian motion on a complete probability space. For notational simplicity we restrict attention in the main text to two possible profitability levels \( \mu_t \in \{\mu^l, \mu^h\} \), where \( 0 < \mu^l < \mu^h \). It is straightforward to extend the analysis allowing for an arbitrary number of profitability levels (see Appendix B.2, where we show that our main results continue to hold).

Investment affects the firm’s future profitability by determining its ability to adopt and commercialize new technologies as they become available.\(^{10}\) As illustrated in Figure

\(^{10}\)Concretely, investment expenditures can be interpreted as a firm’s choice of *absorptive capacity* in the
the industry is subject to (rare) exogenous technology shocks governed by a Poisson process \( N \) with intensity \( \nu \), indicating the availability of a new technology. If there is a technology shock \( (dN_t = 1) \), the firm is able to adopt the new technology with probability \( p(I_t) \) and its future profitability will be high \( (\mu_{t+} := \lim_{s \to t} \mu_s = \mu_h) \). We refer to this event as an investment success, which can be interpreted as moving to or staying at the research frontier. With probability \( 1 - p(I_t) \) the firm is, however, unable to adopt the new technology sufficiently quickly to stay competitive, and its future profitability will, thus, be low \( (\mu_{t+} = \mu_l) \). We refer to this as an investment failure, which can be interpreted as falling or staying behind the research frontier. For future reference, note that the occurrence of an investment success is, thus, governed by a Poisson process \( N^g \) with arrival rate \( \nu p(I_t) \) and the occurrence of an investment failure by a Poisson process \( N^b \) with arrival rate \( \nu (1 - p(I_t)) \). In between two technology shocks, profitability remains unchanged such that investment has persistent effects. Finally, we stipulate that the success probability \( p(\cdot) \in [0, 1] \) is an increasing and strictly concave function of the investment amount \( I \), satisfying \( p(0) = 0 \) and \( p(\bar{I}) = 1 \).

### 2.2 Agency Problem

Firm owners have to hire a manager to (profitably) run the business which relies on the manager’s specific skills or private information. Both parties are risk neutral and the manager is protected by limited liability and has no initial wealth. Hence, owners have to bear the costs of setting up the firm (normalized to zero) and cover operating losses. In order to capture liquidity needs on behalf of the manager, we assume, as is standard in dynamic contracting models (see, e.g., DeMarzo and Duffie 1999 or DeMarzo and Sannikov 2006), that he is relatively impatient, i.e., the discount rates of the manager \( (\gamma) \) and firm owners \( (r) \) satisfy \( \gamma > r \).

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sense of Cohen and Levinthal (1990), describing its capability of “assimilating new, external information and apply it to commercial ends.” While we frame the investment technology within the context of technology adoption it is, hence, more widely applicable. E.g., one could similarly think of the industry being subject to unpredictable demand shocks, to which the firm can react quickly enough only if it has invested sufficient resources, e.g., into building capacity in market research or product design.

Investment in our model, thus, depreciates instantly, reflecting the need to keep up with fast technological progress. Indeed, evidence suggests that, e.g., R&D investment depreciates much faster than physical capital (see, Bernstein and Mamuneas 2006). Nevertheless, the main drivers of our results do not depend on this assumption and our insights should extend also to the case where investment depreciates gradually over time, which however significantly complicates the analysis.
Running the firm requires some form of discretion over the firm’s cash flows, which we model as follows: Firm owners do not observe cash flows \( Y \) directly but only the manager’s contractible reports \( \hat{Y} \). The difference between actual and reported cash flows is determined by the manager’s hidden action, which is the source of the agency problem. In particular, we assume that the manager can divert cash flows for private consumption. To capture possible costs of concealing and taking funds out of the firm we stipulate that the manager can consume fraction \( \lambda \in (0, 1] \) of each unit diverted.

As investment similarly requires a considerable understanding of the firm’s internal processes as well as the market it is operating in, it is also delegated to the manager with discretion. To model this in a concise way we assume that, in addition to cash flows, also actual investment expenditures \( I \) are not contractible. Hence, the manager can freely use cash flows for investment purposes or, respectively, inflate net cash flows \( Y \) by deviating from the prescribed investment schedule (see (1)). While firm owners may observe (contractible) reports of \( I \), e.g., as documented in the firm’s income statement, these reports do not have to coincide with actual investment expenditures as the manager can divert funds from his investment budget for private benefit, or misrepresent running expenses as investments.\(^{12}\) Hence, independently of whether reports of investment expenditures are

\(^{12}\)Indeed, several relevant components of intangible investment have to be reported (such as R&D), however, accounting rules leave considerable discretion to managers over which expenses to classify in the respective categories. This allows managers, e.g., to shift core expenses to other expense categories (such
available or not, to provide incentives the contract has to condition on the (contractible) investment outcome, as governed by the Poisson processes $N^g$ and $N^b$.

The principal can commit to a long-term compensation contract $(U, \tau)$ specifying – based on reported cash flows and investment outcomes – wage payments to the agent, $dU_t$, as well as the (random) time $\tau$ when the agent is fired and replaced, which incurs the principal fixed costs $k > 0$. Still, because limited liability requires the cumulative wage process $U = \{U_t : 0 \leq t \leq \tau\}$ to be non-decreasing, firing and replacing the agent may be necessary for incentive provision and will indeed be part of the ex-ante optimal contract. We further assume that the agent cannot save privately, implying that $dY_t - d\hat{Y}_t \geq 0$, i.e., he can only underreport cash flows.$^{13}$ Hence, the agent’s consumption flow at time $t$ equals the sum of wage payments and the utility from consuming diverted funds (if any):

$$dC_t = dU_t + \lambda \left[ dY_t - d\hat{Y}_t dt \right].$$

When the agent is fired he receives his outside option, which is normalized to zero. The incumbent agent’s total expected wealth at the start of his employment, hence, is

$$w_0 = \mathbb{E}^S_0 \left[ \int_0^\tau e^{-\gamma t} dC_t \right],$$

(2)

where the expectation is conditional on time zero information and depends on the agent’s reporting and investment strategy $S = \{\hat{Y}_t, I_t : 0 \leq t \leq \tau\}.$ $^{14}$ The actual value of $w_0 > 0$ in (2) is determined by the two parties’ relative bargaining power. For concreteness, in what follows, we will assume that the principal enjoys all bargaining power and the agent accepts any contract with $w_0 \geq 0.$ $^{15}$

If the agent is replaced, the principal’s value is given by $L_\tau$, which denotes the (endogenous) expected profit from the relationship with a new agent, net of replacement costs $k$. We assume that $k$ is sufficiently small such that continuing the firm is optimal. Hence, at $t = 0$, the principal’s total expected profit, delivering the agent an expected payoff of $w_0$ as R&D) to improve core earnings, a form of earnings management referred to as classification shifting (see footnote 2).

$^{13}$Given that the agent is risk-neutral and relatively impatient, this is without loss of generality.

$^{14}$Formally, $\mathbb{E}^S$ denotes the expectation under the probability measure $Q^S$ induced by $S$. If obvious, we will not state the measure associated with the expectation operator in the following.

$^{15}$This is not crucial for our results. We just require relative bargaining power to be constant over time.
and given initial profitability $\mu_0$, is

$$f_0 = \mathbb{E}_0 \left[ \int_0^\tau e^{-rt} (d\tilde{Y}_t - dU_t) + e^{-r\tau} L_\tau \right]. \quad (3)$$

Given a compensation contract $(U, \tau)$, the agent chooses a feasible strategy $S$ to maximize his initial expected payoff $w_0$ from (2). A strategy $S$ is called incentive compatible if it solves the agent’s maximization problem. Hence, an incentive compatible contract can be described by the triple $(S^*, U, \tau)$, where $S^*$ is the (incentive compatible) strategy that the principal wants to induce. The associated (global) incentive constraint is given by

$$\mathbb{E}_0^{S^*} \left[ \int_0^\tau e^{-\gamma t} dC_t \right] \geq \mathbb{E}_0^{\tilde{S}} \left[ \int_0^\tau e^{-\gamma t} dC_t \right], \quad \text{for any } \tilde{S} \neq S^*. \quad (4)$$

We can simplify the analysis considerably by relying on a version of the revelation principle, which allows us to restrict attention to truth-telling contracts, implementing $\hat{Y} = Y$.

The contracting problem then is to find an incentive compatible truth-telling contract maximizing the principal’s expected profit $f_0$ for given initial profitability $\mu_0$, delivering expected payoff $w_0$ to the agent and satisfying limited liability. We refer to the solution of this constrained maximization problem, which includes the optimal investment profile, as the optimal contract.

A heuristic outline of the timing of events taking place in any infinitesimal time interval $[t, t + dt]$ prior to replacement of the incumbent manager, is illustrated in 1. Note, in particular, that the agent chooses investment prior to observing whether a technology shock occurred or not, which is the main important “sequentiality” in our continuous time model, while compensation and possible replacement of the manager occur thereafter.

## 3 Model Solution

In this section we solve for the optimal contract using a dynamic programming approach. As we will show, the optimal contract can be written in terms of the agent’s continuation payoff as the single state variable. We derive the dynamics of this key state variable along

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16For a formal argument compare Lemma 1 and Proposition 2 in DeMarzo and Sannikov (2006).

17Formally, $I$ is predictable with respect to $\mathcal{F} = \{\mathcal{F}_t, t \geq 0\}$, the filtration generated by $(Z, N^g, N^b)$, while $U$ is adapted to $\tilde{\mathcal{F}} = \{\tilde{\mathcal{F}}_t, t \geq 0\}$, the filtration generated by $(\hat{Y}, N^g, N^b)$ and $\tau$ is an $\tilde{\mathcal{F}}$-measurable stopping time.
3.1 Continuation Payoff and Local Incentive Compatibility

For any truth-telling contract, define the agent’s continuation payoff, \( w_t \), as his future expected discounted payoff at time \( t \), given he will follow strategy \( S^* \) from \( t \) onwards, i.e.,

\[
w_t = E_t^{S^*} \left[ \int_t^\tau e^{-\gamma(s-t)} dC_s \right].
\]  

(5)

While \( w_t \) is the continuation payoff after observing whether a technology shock occurs in \( t \) or not, investment expenditures can only condition on the respective value before this uncertainty is resolved, which we denote by \( w_{t-} := \lim_{s \uparrow t} w_s \). This variable will serve, together with profitability \( \mu_t \), as the state variable in the recursive formulation of the optimal contracting problem. Applying standard martingale techniques, the evolution of \( w_{t-} \) can be characterized as follows:

**Lemma 1.** For any contract, there exist some predictable processes \( \alpha = \{\alpha_t : 0 \leq t \leq \tau\} \) and \( \beta^j = \{\beta^j_t : 0 \leq t \leq \tau\}, j \in \{g, b\} \), such that, if the manager follows the recommended strategy \( S^* = \{Y_t, I^*_t : 0 \leq t \leq \tau\} \), his continuation payoff at any moment of time evolves according to:

\[
dw_{t-} = \gamma w_{t-} dt - dU_t + \alpha_t (d\hat{Y}_t - (\mu_t - I^*_t) dt)
\]

\[
+ \beta^g_t (dN^g_t - \nu p(I^*_t)) dt + \beta^b_t (dN^b_t - \nu (1 - p(I^*_t))) dt,
\]

where \( d\hat{Y}_t - (\mu_t - I^*_t) dt \) is the increment of a Brownian motion and \( dN^g_t, dN^b_t \) are increments of standard Poisson processes with arrival rates \( \nu p(I_t) \) and \( \nu (1 - p(I_t)) \) respectively.

To build intuition, let us discuss in more detail the evolution of \( w_{t-} \) in (6). Due to promise keeping, the agent’s promised wealth, \( w_{t-} \), has to grow at his discount rate, \( \gamma \), while it must decrease with direct wage payments, \( dU_t \). It also depends on his reporting strategy via sensitivity \( \alpha_t \) and on the investment outcome via sensitivities \( \beta^g_t \) and \( \beta^b_t \). If the agent underreports cash flows, such that he can immediately consume \( \lambda(dY_t - d\hat{Y}_t) \), his
continuation payoff is reduced by $\alpha_t(dY_t - \hat{Y}_t)$. Incentive compatibility for truth-telling, thus, requires $\alpha_t \geq \lambda$. Next, consider a deviation from the recommended investment $I^*_t$. When the agent reduces investment $I_t$ marginally below $I^*_t$, from (1), net current cash flows $dY_t$ improve at the same rate, and the agent’s continuation payoff $w_{t-}$ grows with sensitivity $\alpha_t$. However, cutting investment implies that the probability of an investment success — triggering “reward” $\beta^g_t$ — decreases at rate $\nu p'(I^*_t)$, while the probability of a failure — triggering “punishment” $\beta^b_t$ — increases by $\nu p'(I^*_t)$. Hence, the agent has no incentives to invest less than the recommended level $I^*_t \in (0, T]$ if and only if $\nu p'(I^*_t) (\beta^g_t - \beta^b_t) \geq \alpha_t$.

Likewise, marginally increasing investment above $I^*_t$ increases the probability of receiving reward $\beta^g_t$ by $\nu p'(I^*_t)$ and decreases the probability of punishment $\beta^b_t$ by $\nu p'(I^*_t)$. However, as additional investment has to be financed out of current cash flows, the continuation payoff $w_{t-}$ is reduced by sensitivity $\alpha_t$. Hence, the agent has no incentives to invest more than the recommended level $I^*_t \in [0, T]$ if and only if $\nu p'(I^*_t) (\beta^g_t - \beta^b_t) \leq \alpha_t$. These observations lead to the local incentive compatibility conditions in Lemma 2 below.

**Lemma 2.** The truth-telling contract $\{S^*, U, \tau\}$ with recommended investment $I^*_t \in (0, T]$ is incentive compatible if and only if

$$\alpha_t \geq \lambda$$

(7)

and

$$\nu p'(I^*_t) (\beta^g_t - \beta^b_t) = \alpha_t$$

(8)

holds for all $t \in [0, \tau)$, almost surely.\(^{18}\) Further, the limited liability constraint implies for all $t \in [0, \tau)$ that

$$\beta^j_t \geq -w_{t-}, j \in \{g, b\}.$$  

(9)

Incentive constraint (8) reflects the interaction between the problem of non-contractible investment and the cash flow diversion problem: As the agent’s continuation value must be tied to reported cash flows ($\alpha_t > 0$), it also has to increase following an investment success and decrease following a failure to provide incentives for investment. More precisely, the sensitivity of the agent’s continuation payoff with respect to the investment outcome, $\beta^g_t - \beta^b_t$, has to increase with the sensitivity to reported cash flows, $\alpha_t$, for any given

\(^{18}\)If $I^*_t = 0$ (8) is replaced by $\nu p'(I_t) (\beta^g_t - \beta^b_t) \leq \alpha_t$, in case $I^*_t = T$ we instead require $\nu p'(I_t) (\beta^g_t - \beta^b_t) \geq \alpha_t$. 

level of investment. Thus, a more severe cash flow diversion problem, reflected by greater diversion benefits $\lambda$, would require to impose more risk on the agent’s income in two respects: First, from (7), the sensitivity to current cash flows, $\alpha_t$, has to increase, because with higher diversion benefits the agent’s income has to be linked more closely to reported cash flows in order to induce truthful reporting. Second, with his income being more sensitive to current cash flows, the agent has an incentive to inflate those by deviating from the recommended investment level. Hence, according to (8), his income has to be more responsive to the investment outcome, i.e., to future profitability as well.

3.2 Optimal Contract

The optimal contract can now be derived using the dynamic programming approach. Denote by $f^i(w)$, $i \in \{l, h\}$ the principal’s value function, that is, the highest profit the principal can attain under any incentive compatible contract delivering expected payoff $w$ to the agent and given the prevailing drift rate of cash flows, $\mu^i$, $i \in \{l, h\}$, where we drop time subscripts for notational convenience.

The contracting problem can be greatly simplified by noting that, following any history, the optimal (continuation) contract is independent of the current profitability state $\mu^i$. Intuitively this result follows since, for a given level of investment, the probability of an investment success or failure does not depend on current profitability and neither do the basic cash flow diversion problem nor the agency problem with respect to investment. As the optimal compensation and investment policies are, thus, independent of the prevailing profitability level, the principal’s value in the two profitability states only differs by an additive constant, which captures the direct benefit from being at the research frontier, $\mu^h - \mu^l$, properly accounting for the Markov-switching structure.

Lemma 3. The optimal contract $(S^*, U, \tau)$ is independent of current profitability $\mu^i$ and the principal’s value functions in the high and in the low profitability state satisfy

$$f(w) := f^l(w) = f^h(w) - \Delta,$$

with

$$\Delta := \frac{1}{r + \nu} (\mu^h - \mu^l).$$

From Lemma 3 we can now characterize the optimal contract based on the agent’s con-
tinuation payoff \( w \) as the single state variable. Consider, first, the optimal compensation policy. Clearly, the principal can always compensate the agent directly by paying him a lump-sum of \( dU > 0 \) (at marginal costs of \(-1\)) and then move to the optimal contract with reduced continuation payoff \((w - dU)\). However, deferring compensation may be valuable: A higher promised wealth, \( w \), relaxes future limited liability constraints as the agent can be punished for poor results by “clawing-back” previously deferred payments instead of having to inefficiently fire and replace him at cost \( k \). In contrast to the costs of deferring compensation, which are due to the wedge in discount rates \((\gamma > r)\), this benefit declines, however, as the agent’s continuation payoff, \( w \), increases and with it the probability of inefficient firing. This is reflected in the concavity of \( f(w) \), which we show formally in the proof of Proposition 1 below. As a consequence, the principal is effectively risk averse with respect to variation in the agent’s continuation value. One implication is that compensation is optimally deferred until a threshold \( \bar{w} \) is reached, where \( f'(\bar{w}) = -1 \) and the agent is paid cash.

Next, since the principal discounts at rate \( r \), his expected flow of value at time \( t \) must be \( rf^i(w)dt \). This has to be equal to the expected instantaneous net cash flow \((\mu - I)dt\) plus the expected change in his value function, which can be computed using Itô’s lemma and the change of variables formula for jump processes. Hence, on the interval without cash compensation, \( w \in [0, \bar{w}] \), the principal’s value function in the low profitability state must satisfy the following HJB equation \((f^h(w) \text{ then follows from (10)})\):

\[
(r + \nu) f(w) = \max_{\alpha, \beta, I} \left\{ \mu^i - I + \nu p(I) \Delta + \frac{1}{2} \sigma^2 \alpha^2 f''(w) \right. \\
+ \left[ \gamma w - \nu \left[ \beta^g p(I) + \beta^b (1 - p(I)) \right] \right] f'(w) \\
+ \nu p(I) f(w + \beta^g) + \nu (1 - p(I)) f(w + \beta^b) \right\} \\
\text{s.t. (7), (8), (9).}
\]

Since cash transfers cause \( w \) to reflect at \( \bar{w} \), the value function extends linearly to the right of the compensation threshold, i.e., \( f(w) = f(\bar{w}) - (w - \bar{w}) \) for \( w \geq \bar{w} \). To solve for the optimal contract we have to pin down a solution to (12) and the compensation threshold \( \bar{w} \). For this we require three boundary conditions: The first (“value matching”) condition

\[
f(0) = f(w^*) - k,
\]

(13)
with \( w^* \in \arg \max_w \{ f(w) \} \), reflects that upon firing the incumbent agent at \( w = 0 \), the principal receives \( f(w^*) \) from the relation with a new agent and bears replacement costs \( k \). The second is a standard “smooth pasting” condition at the compensation threshold

\[
f'(\bar{w}) = -1, \tag{14}
\]

while the third boundary condition (“super contact”) guarantees the optimal choice of \( \bar{w} \)

\[
f''(\bar{w}) = 0. \tag{15}
\]

The following Proposition characterizes the optimal contract, i.e., the solution to the boundary value problem in (12)-(15).

**Proposition 1.** The optimal truth-telling contract with non-contractible investment takes the following form: Optimal investment \( I(w) \) as well as the sensitivities \( \alpha(w) = \lambda \) and \( \beta_j(w), j \in \{g, b\}, \) are independent of \( \mu_i, i \in \{l, h\}, \) and chosen as maximizers in (12). The incumbent agent’s continuation payoff evolves according to (6) with \( \alpha_t = \lambda, \beta^g_t = \beta^g(w_{t-}), \) \( j \in \{g, b\} \) and \( I_t = I(w_{t-}), \forall t. \) Optimal compensation satisfies \( dU_t = \max\{w_{t-} - \bar{w}, 0\}. \) The incumbent agent is replaced when \( w_{t-} = 0. \) The principal’s expected payoff at any point in time is given by \( f_i(w_t), i \in \{h, l\}, \) which satisfies \( f(w) := f_l(w) = f_h(w) - \Delta, \) where \( f(w) \) is concave, strictly so for \( 0 \leq w < \bar{w} \) and solves, for \( w \in [0, \bar{w}] \), the HJB equation in (12) subject to the boundary conditions (13) to (15).

As is standard in dynamic cash flow diversion models (see, e.g., DeMarzo and Sannikov 2006), the agent’s incentive constraint with respect to cash flow diversion (7) binds under the optimal contract. Formally, this result follows from the concavity of \( f(w) \) and the observation that increasing \( \alpha \) would increase the instantaneous volatility of \( w. \) Given that the basic cash flow diversion problem is by now well now understood, in the following Section 3.3, we focus on the investment task and discuss in detail the optimal investment schedule, as well as the optimal choice of reward and punishment used to incentivize investment under the optimal contract of Proposition 1.

### 3.3 Optimal Investment

Incentive compatibility requires that the optimal contract contains a certain degree of
reward following an investment success, $\beta^g(w) > 0$, and/or punishment after a failure, $\beta^b(w) < 0$. However, providing these incentives for investment is costly such that, in determining the optimal level of investment, the principal trades off the potential for higher profitability with the additional agency costs of providing incentives for investment. To see this formally, note that from (12) we can write the principal’s problem of finding the optimal investment level as follows:

$$I(w) \in \arg \max_{I \in [0, I]} \{ \nu p(I) \Delta - I - \Phi(w, I) \},$$

(16)

with $\Phi(w, I) = \min_{\beta^g, \beta^b s.t.(8),(9)} \left\{ \nu p(I)[f(w) + \beta^g f'(w + \beta^g)] + \nu(1 - p(I))[f(w) + \beta^b f'(w + \beta^b)] \right\}$. (17)

From (16), optimal investment is chosen to maximize the (expected) technological returns to investment, $\nu p(I) \Delta$, net of investment expenditures, $I$, and the agency costs of providing incentives for investment denoted by $\Phi$. Since firm owners are effectively risk-averse with respect to variation in the manager’s compensation, as reflected in the concavity of $f(w)$, these agency costs are strictly positive whenever incentive compatible rewards for investment success, $\beta^g$, and punishments for investment failure, $\beta^b$, induce additional variation in the agent’s continuation payoff $w$. Accordingly, the optimal combination of reward and punishment to implement a given level of investment are determined from (17), minimizing the agency costs of providing incentives for investment, $\Phi$.

In the following, we now first study optimal compensation and investment distortions for a given value of the agent’s continuation utility $w$. Then, we analyze the implied compensation and investment dynamics when $w$ evolves as described in Proposition 1.

**Optimal Incentives for Investment.** Minimizing the agency costs of investment $\Phi$ in (17), with Lagrange multiplier $\eta(w)$ attached to the incentive compatibility constraint (8), implies that interior optimal values $\beta^g(w)$ and $\beta^b(w)$ have to satisfy

$$\left[ f'(w) - f'(w + \beta^i(w)) \right] / LR^i(I(w)) = \eta(w),$$

(18)

19If, for $i = b$, (18) has no solution $\beta^b(w) \geq -w$, the limited liability constraint (9) binds such that $\beta^b(w) = -w$ and the respective value of $\beta^g(w)$ follows directly from incentive compatibility (8). For a given level of investment, $I$, a sufficient condition for the limited liability constraint to be slack is: $w \geq \lambda / (\nu p(I))$. 

Electronic copy available at: https://ssrn.com/abstract=3164471
for \(i \in \{g, b\}\). Here \(LR^i(I) := \frac{d}{dI} \log (\Pr (\mu = \mu^i | I))\) denotes the \textit{likelihood ratio} of the respective investment outcome, i.e., \(LR^g(I) = p'(I)/p(I)\) for an investment success and \(LR^b(I) = -p'(I)/(1 - p(I))\) for an investment failure. As is well known \(|LR^i(I)|\) measures the strength of incentive provision per unit of expected reward or punishment and, in that sense, how \textit{informative} a performance signal – here, an investment outcome – is for providing incentives (see, e.g., Holmström 1979). Intuitively, according to \((18)\), reward for investment success and punishment for failure are, hence, optimally chosen such as to equalize the respective expected cost of providing incentives (arising from the principal’s “risk aversion”) per unit of incentive provision.

To build intuition, consider, first, the benchmark case with contractible investment (henceforth, \(CI\) – see Appendix B.1 for a complete characterization). Then, as the contract does not have to provide incentives for investment, optimal risk-sharing is attained by setting \(\beta^g_{CI}(w) = \beta^b_{CI}(w) = 0\). I.e., the principal, who is effectively risk-averse, bears all risk from the investment shock and perfectly insures the risk-neutral agent. This seemingly counterintuitive result follows from the fact that the principal is risk-averse with respect to variation not in his own, but in the agent’s income, such that optimal risk sharing (corresponding to \(\eta(w) = 0\) in \((18)\)) prevails whenever the agent’s compensation is not contingent on investment outcomes that occur with positive probability.

Instead, with non-contractible investment, the constrained optimal solution, subject to incentive compatibility, will usually have to deviate from optimal risk-sharing. According to \((18)\), this is optimally done by shifting most of the required incentive costs to the more informative investment outcome, i.e., by setting\(^{20}\)

\[
\frac{f'(w + \beta^b(w)) - f'(w)}{f'(w) - f'(w + \beta^g(w))} = \left| \frac{LR^b(I(w))}{LR^g(I(w))} \right|.
\]  \hspace{1cm} \text{(19)}

From \(|LR^b(I)/LR^g(I)| = p(I)/(1 - p(I))\) incentive costs are, thus, mainly incurred following the investment outcome that is less likely to occur.\(^{21}\) Now, as the probability of successful investment increases in the implemented investment level, we obtain the fol-

\(^{20}\)In that sense, optimal incentive pay conditions more heavily on the more informative investment outcome. Note, however, that \((19)\) does not necessarily imply that \(|\beta'(w)| > |\beta'(w)|\) whenever \(|LR^i(I(w))| > |LR^j(I(w))|\) as the curvature of \(f\) changes with \(w\).

\(^{21}\)Note that this simple description of the relative likelihood ratio is due to the fact that, with binary investment outcomes, the incentive effect of a payment conditioning on either investment outcome is given by \(|p'(I)|\). In Appendix B.2 we consider a more general setup with an arbitrary number of investment outcomes for which \((18)\) still holds and show how our main insights extend.
lowing result comparing the composition of incentive costs for different sets of parameters
\( \psi \in \{ \mu^h, \mu^l, \nu, \lambda, r, \gamma, \sigma, k \} \) which map into different values of \( I(w) \).\(^{22}\)

**Lemma 4.** Fix \( w \in (0, \overline{w}) \) and consider two sets of parameter values \( \psi' \) and \( \psi'' \), such that \( I(w)|_{\psi'} < I(w)|_{\psi''} \) under the optimal contract of Proposition 1. Then, as long as the limited liability constraint does not bind, rewards for investment success, \( \beta^g(w) \), and punishments for investment failure, \( \beta^b(w) \), are chosen such that the ratio of the costs of providing incentives through punishment relative to the costs of providing incentives through reward is strictly increasing in \( I(w)|_{\psi} \), i.e.,

\[
\frac{f'(w + \beta^b(w)) - f'(w)}{f'(w) - f'(w + \beta^b(w))}|_{\psi'} < \frac{f'(w + \beta^b(w)) - f'(w)}{f'(w) - f'(w + \beta^b(w))}|_{\psi''}.
\]

Further, \( \beta^g(w) \to 0 \) as \( I(w)|_{\psi} \to \overline{T} \), while for \( I(w)|_{\psi} \to 0 \), as the agency problem vanishes, \( -\beta^b(w)/\beta^g(w) \to 0 \), i.e., the first unit of investment is incentivized with rewards only. For all \( I(w)|_{\psi} \notin \{0, \overline{T}\} \), both reward and punishment are used, i.e., \( \beta^g(w)|_{\psi} > 0 > \beta^b(w)|_{\psi} \).

This dependence of the relative costs of providing incentives through punishment rather than through reward on the implemented level of investment will be key in understanding the distortions in investment which we will analyze next.

**Investment Distortions.** In the first-best benchmark without agency frictions, a profit-maximizing risk-neutral owner-manager with discount rate \( r \) would choose the following investment level for all \( t \geq 0 \):\(^{23}\)

\[
I_{FB} \in \arg \max_{t \in [0, T]} \left\{ \nu p(t) \Delta - I \right\}.
\]  

This first-best investment schedule is constant and satisfies, if interior, the first-order condition \( \nu p'(I) \Delta = 1 \), which trades off the potential for higher profitability against the marginal investment expenditures.

\(^{22}\)As will be shown below, optimal investment \( I(w) \) varies also with \( w \) such that the following comparative statics similarly hold fixing \( \psi \) and considering different levels of \( w \).

\(^{23}\)As we show in Appendix B.1, the same investment level would be implemented in the case where only the cash flow diversion problem is present and investment is contractible. This is due to the fact that both the returns to investment, captured by \( \Delta \), as well as its costs, \( I \), are independent of the agency problem. Notably, this is different in the neoclassical investment model considered by DeMarzo et al. (2012), where the returns to investment (Tobin’s Q) are reduced by a cash flow diversion problem. Thus, even though investment is contractible in their model, it is always distorted below first-best.
In order to determine whether, with non-contractible investment expenditures, the firm invests too much or too little relative to this (first-best) benchmark, it is crucial to understand how the agency costs of investment in (17) change in $I$, i.e., to sign the marginal agency costs of investment $\partial \Phi(w, I(w))/\partial I$. If these are positive, investment is distorted downwards relative to first-best, while it is distorted upwards if marginal agency costs are negative. We will now show that agency costs of delegated investment are non-monotonic in the incentivized investment level $I$ such that indeed both under- as well as overinvestment can arise in equilibrium.

To see this, it is instructive to look, first, at the boundary cases where the problem in (20) has a corner solution. When the investment technology is very unprofitable, such that, in the extreme case, $I_{FB} = 0$, the principal can trivially implement the first-best investment level without incurring any agency costs as there is no need to provide incentives – optimal risk-sharing can be attained. However, also when the investment technology is highly profitable, such that $I_{FB} = \bar{I}$, first-best investment can be incentivized without inducing (additional) volatility in $w$, i.e., maintaining optimal risk-sharing. Now, the shadow value on the incentive constraint in (18) is zero as an investment failure becomes extremely informative, in the sense of the respective likelihood ratio going to infinity. Intuitively, when the probability of success is $p(\bar{I}) = 1$, from Lemma 4, all incentives are optimally provided “off-equilibrium” by relying on punishment only, as long as this does not violate limited liability. Since agency costs, as given in (17), are zero for $\beta^g(w) = 0$ and $p(\bar{I}) = 1$, optimal risk-sharing prevails and optimal investment equals the first-best level.

**Lemma 5.** Fix $w \in [0, \bar{w}]$. Then, as long as the limited liability constraint does not bind, investment under the optimal contract of Proposition 1 is equal to first-best whenever first-best investment attains a corner solution, i.e., $I(w) = I_{FB}$ for $I_{FB} \in \{0, \bar{I}\}$.

The discussion above makes clear that first-best investment can be implemented without incurring any agency costs if $I_{FB} = 0$, or $I_{FB} = \bar{I}$. However, this is no longer true for interior values of $I_{FB}$ which, from Lemma 4, are optimally implemented using both reward, $\beta^g > 0$, as well as punishment, $\beta^b < 0$, thus, implying a deviation from optimal risk-sharing. Since agency costs to incentivize first-best investment are, hence, strictly positive if $I_{FB}$ is interior, and zero when $I_{FB}$ attains a corner solution, it is easy to see that marginal agency costs are positive if first-best investment is sufficiently low and they
are negative if first-best investment is sufficiently high. As a result, investment under the optimal contract will be distorted downwards in the former and upwards in the latter case.

**Proposition 2.** Fix \( w \in (0, \bar{w}) \). Then, as long as the limited liability constraint does not bind, investment is distorted downwards, \( I(w) < I_{FB} \), for sufficiently low values of \( I_{FB} > 0 \), while it is distorted upwards, \( I(w) > I_{FB} \), for sufficiently high values of \( I_{FB} < \bar{I} \).

To build intuition, note that increasing investment has two basic effects on agency costs: First, higher investment requires stronger incentives (see (8)), and, second, it affects the outcome distribution, thus, increasing the probability of having to reward the agent for a favorable investment outcome and decreasing the probability of having to punish him for an investment failure. While the first effect unambiguously tends to increase agency costs pushing towards underinvestment, the second effect can have either sign depending on the relative costs of providing incentives through reward rather than punishment. As Proposition 2 shows, overinvestment may, hence, arise if incentives are predominantly given through punishment for bad outcomes, which from Lemma 4 is the case if the investment technology is sufficiently profitable such that an (unlikely) investment failure constitutes a very informative performance signal as measured by a high relative likelihood ratio. Then, in order to reduce the probability of having to bear the high agency costs associated with an investment failure, it is optimal to increase \( I(w) \) above the first-best value.\(^{24}\)

The opposite case applies if the investment technology is rather unprofitable implying low first-best investment, which would be optimally incentivized with high rewards following an informative investment success. Then, to avoid the associated high incentive costs, investment \( I(w) \) is optimally distorted downwards relative to first-best.

It is now instructive to look at a numerical example quantifying how large \( I_{FB} \) needs to be for overinvestment to occur under the optimal contract. To highlight that \( I_{FB} \) captures the profitability of the investment technology independent of the agency problem, we do so by determining a cut-off value for the “returns to investment” \( \Delta \mu := \mu^h - \mu^l \). Note that \( \Delta \mu \) is a purely technological parameter in that it does not affect the agency problem for a given investment schedule.\(^{25}\)

\(^{24}\)Note that this intuition is robust also beyond our simple binary state setting. In Appendix B.2 we allow for an arbitrary number of investment outcomes and derive, analogous to Proposition 2, conditions for when over- and underinvestment occurs. In particular, we show that, under standard assumptions, there is overinvestment whenever low investment outcomes (akin to an investment failure in the binary model) are sufficiently informative for providing incentives, as reflected in a high relative likelihood ratio.

\(^{25}\)I.e., if one had to implement a fixed (exogenously given) investment schedule, then the optimal
Example 1. Consider $p(I) = \phi \sqrt{I}$, and fix parameters $\nu = 1.2$, $\sigma = 10$, $\lambda = 0.5$, $k = 15$, $\gamma = 0.15$, $r = 0.1$, $\phi = 0.95$. Then the manager overinvests under the optimal contract (for all $w \in (0, \bar{w})$ for which the limited liability constraint is slack), if and only if “returns to investment” $\Delta \mu$ are larger than 1.77 which, given the other parameter values, is equivalent to first-best investment $I_{FB}$ above 0.6 or a success probability $p(I_{FB})$ above 0.74.

Compensation and Investment Dynamics. So far, we studied how investment distortions and the incentive scheme used to implement investment depend on the profitability of the investment technology as captured by $I_{FB}$. For this analysis we kept the agent’s continuation value fixed. In the following we will now analyze the resulting dynamics of investment and compensation when $w$ evolves as described in Proposition 1. In order to do so, it is useful to look, first, at extreme values of the state space, i.e., $w = \bar{w}$ and $w = 0$.

As the agent’s continuation value reaches the compensation boundary ($w = \bar{w}$), he has accumulated so much wealth inside the firm that the agency problem is relaxed sufficiently for firm owners’ effective risk-aversion to disappear. In fact, rewards are taken out as cash payments and do not induce costly variation in $w$ (see Proposition 1). Hence, it is optimal not to punish the agent and to rely exclusively on rewards to incentivize investment. As a result, agency costs of investment are zero and investment equals first-best.\footnote{To see this formally, substitute $\bar{w}$ in (16) and note that $f(w)$ extends linearly to the right of $\bar{w}$.}

For $w < \bar{w}$ the principal’s value function is strictly concave such that both rewards and punishments are used according to the optimal compensation policy in (19). However, for sufficiently low values of the agent’s continuation payoff, punishment is restricted by the limited liability constraint (9). In fact, only rewards can be used to incentivize any $I > 0$ in the limiting case as we approach the termination boundary ($w = 0$) as the agent is “too poor to be punished.” Now, whenever incentives are provided mainly through rewards, an investment success is the investment outcome that is particularly costly in terms of incentive provision. As a result, investment will be optimally distorted downwards for all $I_{FB} \in (0, \bar{T})$. The following Lemma summarizes these observations.

Lemma 6. Fix $I_{FB} \in (0, \bar{T})$. Then under the optimal contract of Proposition 1

i) For $w \geq 0$ sufficiently small, the limited liability constraint for $\beta^b(w)$ binds such that

The contract would be the same for all values of $\Delta \mu$. In contrast, the other parameters determining $I_{FB}$ in (20) affect the agency problem directly, i.e., not just via the optimal investment schedule.
the agent is instantly fired following a failure. Investment is distorted downwards, \( I(w) < I_{FB} \).

ii) For \( w < \bar{w} \) sufficiently large, incentives are provided through punishment and reward, which are optimally chosen according to (19). Investment may be distorted upwards or downwards as shown in Proposition 2.

iii) For \( w \geq \bar{w} \), incentives are provided through rewards only and investment is equal to first-best, \( I(w) = I_{FB} \).

Figure 2 illustrates the compensation and investment dynamics implied by Lemma 6 in an equilibrium with overinvestment (high \( I_{FB} \) in left panels), and in one with underinvestment (low \( I_{FB} \) in right panels). As in Example 1, the two scenarios are generated by varying the “returns to investment” \( \Delta \mu := \mu^h - \mu^l \), while all other parameters and functional forms are kept constant. As shown in Lemma 6, there is no punishment \( (\beta^b(w) = 0) \) if the agent’s track record is very poor \( (w \to 0) \) or if it is very good \( (w \to \bar{w}) \), while \( \beta^b(w) \) is strictly negative in between. The upper panels of Figure 2 illustrate cases where the punishment policy \( \beta^b(w) \leq 0 \) is in fact U-shaped in the agent’s continuation value \( w \). The lower the degree of punishment, keeping all else constant, the more generous rewards are needed to satisfy incentive compatibility. Notably, this is true independently of whether the lack of punishment is imposed by the binding limited liability constraint (for low \( w \)), or if it is in fact optimal not to punish (for high \( w \)). As a result, also the reward policy \( \beta^g(w) \geq 0 \) in our numerical example is U-shaped in \( w \).

Turning to the optimal investment policy \( I(w) \) as depicted in the lower panels of Figure 2, note first that investment is distorted downwards relative to first-best for low returns to investment \( \Delta \mu \) (right panel) and upwards for high \( \Delta \mu \) (left panel), as long as \( w \) is sufficiently large, which corresponds to a slack limited liability constraint. In both cases investment \( I(w) \) approaches the respective first-best value at the compensation boundary, i.e., for \( w \to \bar{w} \). Hence, in the case with high \( \Delta \mu \) (left panel), investment is distorted upwards and decreases with the agent’s stake in the firm \( w \), while, for low \( \Delta \mu \) (right panel), it is distorted downwards and increases with \( w \). In line with empirical evidence, our model, thus, predicts positive investment-cash flow sensitivities in firms with low returns to investment and (excluding financially distressed firms) negative investment-cash flow sensitivities in firms with high returns to investment.
Figure 2: The upper panels illustrate the punishment and reward dynamics and the lower panels the investment dynamics under the optimal contract for low $\Delta \mu = 1.1$ and for high $\Delta \mu = 2.2$. All remaining parameters are set as in Example 1.

4 Empirical Implications

In this section, we use our dynamic agency model with non-contractible investment to derive empirical implications for the level and dynamics of investment in intangibles, the structure of executive compensation, liquidity management and payout policies. For this we have to map key model variables and parameters into observables. To do so, we first discuss the relevance of our key agency friction, the non-observability of actual investment expenditures, for different forms of investment and its implications for empirical tests based on reported investment expenditures. Second, we show how to interpret key variables of the optimal contract within an implementation.

Input- vs. Output-Based Measures of Investment. In order to test our model’s predictions regarding investment one has to find reliable empirical measures of the non-contractible investment expenditures we seek to model. Recall that by focussing on non-contractible investment in future profitability per unit of (physical) capital instead of standard capital investment, our model seeks to explain intangible investment such as in-
vestment in knowledge or organizational capital. Common input proxies for these types of intangible investment include R&D expenditures or parts of selling, general and administrative (SG&A) spending (see, e.g., Peters and Taylor 2017). However, within these applications, actual and reported investment expenditures need not coincide perfectly. For instance, while R&D investments have to be expensed and reported when material (FASB 1974), accounting rules such as U.S. Generally Accepted Accounting Principles (GAAP) do leave considerable discretion to managers regarding which costs to classify as R&D. 27 Exploiting this discretion – e.g., to artificially improve short-term “core” earnings at the cost of long-term profitability – is referred to as “classification shifting” in the accounting literature. This is similarly relevant for investment in organizational capital (including human capital, brands, customer relationships and distribution system) for which only relatively coarse measures, e.g., in the form of SG&A, are available.

These examples illustrate the relevance of our key agency friction, the non-observability of actual investment expenditures, for different types of intangible investment. Our theoretical analysis focusses on the optimal dynamic investment incentives in such settings in which reported investment expenditures do not need to coincide with actual investment. Under the optimal contract, managers invest in the firm owners’ interest, which uniquely pins down actual investment. Reported investment expenditures on the other hand affect neither the manager’s nor investors’ value and, hence, are not uniquely determined under the optimal contract. In fact, the manager and investors could agree on any “reporting convention” not necessarily just truthtelling. 28 The possibility of classification shifting – i.e., misreporting – in equilibrium poses a challenge for any empirical test taking reported investment expenditures as a measure of actual intangible investment. In light of our model, empirical tests of firms’ actual investment in intangibles such as R&D should,

27 The Financial Accounting Standards Board (1974) acknowledges that “the differences [in research and development] among enterprises and industries are so great that a detailed prescription of the activities and related costs includable in research and development, either for all companies or on an industry-by-industry basis, is not a realistic undertaking for the FASB.” See also Horwitz and Kolodny (1980).

28 Which reporting convention applies, i.e., equilibrium selection, is determined outside of our baseline model. On the one hand focussing on truthtelling equilibria in which actual and reported investment coincide could be justified by several empirical findings indicating that misreporting and earnings manipulation lead to lower reporting quality, more opacity and, thus, higher costs of capital since outsiders price in the information risk (see, e.g., Francis et al. 2005 or Bhattacharya et al. 2003). Formally, this could be captured by imposing some costs of misreporting borne by shareholders with our baseline setting corresponding to the limit as these costs go to zero. On the other hand, some firms may want to engage in classification shifting also in equilibrium, e.g., for reasons of competition as in Darrough et al. (2017).
thus, be based on observable measures of investment output rather than on direct but possibly manipulated input measures in the form of reported investment expenditures.

Accordingly, all subsequent empirical implications will be stated in terms of two measures of investment output, which are both observable and pinned down uniquely by the optimal contract. Investment output in our theory model comes in the form of investment successes and failures. The former become more, the latter less frequent, as actual investment expenditures increase according to the respective arrival rates of $\nu p(I)$ and $\nu (1 - p(I))$. The first measure of investment output we use, hence, is the success rate, i.e., the number of observed successes per unit of time, which in the context of our main application to R&D could be measured empirically by the number of patents filed or a count of successful product launches within each year. While the success rate essentially is a “counting measure” our second measure of investment output, firm profitability, $\mu$, further captures the value of an investment success or failure, corresponding, e.g., to a gain in market share, whenever the respective firm is at the research frontier, while falling behind the research frontier results in respective losses.

**Implementation and Cross-Sectional Averages.** In order to derive testable implications from our dynamic agency model, we next show how to interpret key variables and parameters of the optimal contract within an implementation.

While it is well known that such an implementation is not unique (see, e.g., DeMarzo et al. 2012 for a discussion), the robust feature shared by all common implementations is that the key state variable, the agent’s continuation payoff, is interpreted as a measure of the firm’s financial slack. To see this relation, recall that when the agent’s continuation payoff falls to zero, this triggers a restructuring of the firm, which is costly for its owners and involves the replacement of the incumbent management. As, for incentive reasons, the agent’s continuation payoff $w_t$ moves with cash flows with sensitivity $\lambda$, the maximum loss that a firm can sustain without triggering a restructuring, i.e., its financial slack, is equal to $M_t := w_t / \lambda$. While this insight can be formalized in a variety of ways, for illustrative purposes, we consider one particular implementation of the optimal contract based on

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29While we state the subsequent implications in terms of the success rate, they clearly also hold – with reverse sign – for the respective failure rate.

30See for instance Hall et al. (2005) or Kogan et al. (2017) for empirical estimates of the (market) value of innovative output.
cash reserves as a measure of financial slack (see Appendix B.3 for details). When cash holdings reach the payout boundary, \( M^{pb} := \bar{w}/\lambda \), the firm issues dividend payments to its shareholders and, thus, also the manager who holds fraction \( \lambda \) of the firm’s equity. If cash reserves fall to zero, which could be interpreted as insolvency, the firm needs to raise cash from the capital market, which involves a fixed cost of \( k \). These costs of raising external funds capture the key financing friction in this implementation of the optimal contract.

In the following, we will therefore interpret all state-dependent model variables as a function of cash holdings, e.g. for the case of investment, \( I_t := I(\lambda M_t) \), where \( M_t = w_t/\lambda \). Then, in order to derive cross-sectional implications we need to compute the cross-sectional distribution of cash holdings \( M \). For this, note that since the firm in our model operates forever, i.e., within the implementation, it is always optimal to refinance it when internal funds are depleted, the stationary (cross-sectional) distribution of the state variable \( M \) can be computed by simulating a sufficiently long path for a single firm.\(^{31}\) We then use the stationary distribution of \( M = w/\lambda \) to compute cross-sectional averages – denoted by a bar, e.g. for the case of investment, \( \bar{I} \) – for the subsequent implications.

The theoretical analysis in Section 3.3 discussed the distortions in and the dynamics of actual investment expenditures as a function of the agent’s continuation value, corresponding to the firm’s cash holdings within our implementation. In particular, we have shown that actual investment expenditures are distorted upwards and decreasing in cash holdings in firms with very profitable investment technologies (as captured by high first-best investment), while with less profitable investment technologies, investment is distorted downwards and increasing in cash holdings. In the same spirit, we formulate the empirical implications below in terms of cross-sectional averages of firms for which intangible investment is more important (such as in the pharma or the high-tech sector) and those for which it is less important (such as in oil, mining, or textile industries). For concreteness, we will parameterize the importance of intangible investment by varying the sensitivity of expected cash flows to the investment outcome, i.e., \( \Delta \mu = \mu^h - \mu^l \).\(^{32}\) This purely techno-

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\(^{31}\)That is, cross-sectional (“ensemble”) averages can be obtained by computing the corresponding path-wise long-term averages, e.g., \( \bar{M} := \frac{1}{T} \sum_{t=0}^{T} M_t \), with \( T \) sufficiently large. This ergodicity property is exploited also by Brunnermeier and Sannikov (2014) or Klimenko et al. (2016). We compute cross-sectional averages in this way by Monte-Carlo simulation with a total number of \( T = 10^7 \) periods with period length \( h = 10^{-3} \). We further specify \( p(I) = \phi \sqrt{I} \), with \( \phi > 0 \) such that \( I \in [0, 1/\phi^2] \). For more details and an outline of the employed algorithm, we refer to Online Appendix B.4.

\(^{32}\)For the numerical analysis, we vary \( \Delta \mu \) between 0 and an upper limit that guarantees an interior solution for the optimal investment level solving (16).
logical parameter does not affect the agency problem directly, which allows us to clearly isolate the impact of the returns to investment on optimal contracts (see also footnote 25).

### 4.1 Investment - Distortions and Dynamics

In this section we derive testable implications regarding the efficiency of investment, its dependence on firm’s cash holdings, and how this relation is affected by financial frictions. We further consider the relation between corporate governance, short-term compensation and investment. Throughout we will focus on cross-sectional implications for observable measures of investment output (success rate and profitability) as introduced above.

**Distortions in Intangible Investment.** Since (unobservable) investment input is monotonically related to investment output, as measured by either the success rate or profitability, it is immediate that they follow a similar pattern, which allows for empirical testing of investment distortions. In particular, comparing average investment output with the “owner-manager” (or contractible investment) benchmark, we obtain:

**Implication 1.** *Compared to otherwise similar owner-manager firms, average investment output is distorted*

- i) *downwards in firms with low returns to investment, and*
- ii) *upwards in firms with high returns to investment.*

Figure 3 plots the percentage deviation of average success rates (left panel) and average profitability (right panel) from the owner-manager benchmark as a function of the returns to investment $\Delta \mu$. When $\Delta \mu$ is low, the average success rate is lower than that of comparable owner-manager firms, which corresponds, e.g., to a lower patent count or fewer innovations in general. On the other hand, in firms with high returns to investment, the average success rate is higher than in comparable owner-manager firms (left panel). The same qualitative pattern of distortion holds for our valuation measure of investment output, namely firm profitability (see right panel). Notably, the distortions in investment (output) depicted in Figure 3 arise endogenously via the optimal contractual response to a multitask problem, without assuming any direct costs or benefits of investment on the manager’s side (such as, e.g., empire building preferences or pressure from the stock-market).
Distortions from Owner-Manager Benchmark

Figure 3: Plots percentage distortions from the owner-manager benchmark for the average success rate (left panel) and the average firm profitability (right panel), as a function of returns to investment $\Delta \mu$. All remaining parameters are set as in Example 1.

Pay for Short-Term Performance, Corporate Governance, and Investment Output. A common presumption in much of the literature on managerial short-termism is that if pay is more sensitive to short-term performance, managers will be biased towards improving the former, while neglecting (hard to verify) long-term investment. This would then lead to lower investment output in the long-run (see, e.g., Bebchuk and Fried 2004 or Edmans et al. 2017). Figure 3 illustrates that, under a long-term contract that optimally balances incentives over different horizons, the relation between pay for short-term performance and long-term investment output can in fact be positive. In particular, as the agent’s diversion benefit $\lambda$ increases, more compensation for short-term performance is needed to induce truth-telling according to incentive constraint (7). By incentive constraint (8), then also the exposure to investment output needs to increase to incentivize any given level of investment input. As a consequence, investment distortions are aggravated, which implies more severe overinvestment (underinvestment) in firms with high (low) returns to investment, $\Delta \mu$.

Implication 2. The relation between investment output and the sensitivity of pay to short-term performance is

i) negative in firms with low returns to investment, and

ii) positive in firms with high returns to investment.
Given that better corporate governance reduces the manager’s potential to divert funds for private consumption (lower $\lambda$), Implication 2 can also be interpreted as a statement on the relation between corporate governance and investment output.\textsuperscript{33} Our multitask model therefore describes a novel channel, relating the impact of corporate governance on investment output to industry characteristics, which does not assume any direct costs or benefits of investment on the manager’s side.

**Investment-Cash Sensitivities.** So far we considered distortions in investment output and associated comparative statics for a *given* level of cash holdings. Next, we derive implications for the dynamics of investment, in particular its sensitivity to cash flows. In doing so, we again focus on the two observable measures of investment output.

Figure 4 plots the average elasticity of investment output with respect to cash *holdings* implied by the optimal contract as a function of the returns to investment. Since higher cash flows ceteris paribus imply higher cash holdings, the sensitivity of investment to cash *flows* follows a similar pattern. To understand this hump-shaped pattern, note that, intuitively, the agency problem is mitigated when the manager’s “stake” in the firm increases. As a consequence, investment distortions relative to the constant first-best level decrease with positive firm performance, i.e., with higher cash flows, and, thus, higher cash holdings. Hence, average investment cash (flow) sensitivities have the opposite sign as the average investment distortions in Implication 1.

**Implication 3.** *The sensitivity of investment output to internal funds (cash) is*

\begin{itemize}
  \item[i)] *positive in firms with low returns to investment, and*
  \item[ii)] *negative in firms with high returns to investment.*
\end{itemize}

There is a large empirical literature (starting with Fazzari et al. 1988) analyzing the relationship between firms’ investment and cash flows. While initially, the literature focused on capital investment, recent papers have explicitly considered also intangible investment, such as R&D spending, showing that the distortions and dynamics of intangible investment

\textsuperscript{33}In line with this implication, Brav et al. (2017) find a negative relation between hedge fund activism and R&D expenditures in their sample of high-tech firms, while Aghion et al. (2013) report a strong positive relation between institutional ownership and R&D. Also David et al. (2001) find a positive relation between institutional investor activism and R&D. The role of incentive pay as a substitute for corporate governance is documented, e.g., in Core et al. (1999) or, more recently, Fahlenbrach (2009).
turn out to differ fundamentally from that of capital investment. For instance, Peters and Taylor (2017) find small, or even negative investment-cash flow sensitivities in industries with a high intensity of intangible investment, such as the high-tech and health sector, and a positive sensitivity in industries with a low intangible investment intensity, such as the consumer and the manufacturing sector. Hence, while they use reported investment input to measure intangible investment, their results are in line with our model’s predictions regarding investment output in Implication 3 above. That intangible investment may both increase as well as decrease in cash flows – depending on firms’ returns to such investment – may also explain recent empirical findings documenting the decline or even disappearance of investment-cash flow sensitivities in a broad cross-section of firms over the last decades (see, e.g., Brown and Petersen 2009 or Chen and Chen 2012).

**Financial Constraints.** Much of the empirical investment-cash flow sensitivity literature has evolved around the question of whether these sensitivities are a good proxy for financial constraints. While this literature has focussed mostly on capital investment, our model’s predictions regarding the dependence of intangible investment on cash flow suggest a novel perspective on this debate. We find that more severe financial constraints as captured by higher costs of external financing, $k$, imply a higher sensitivity of intangible investment (output) to cash holdings and, thus, also cash flows. As illustrated in Figure 4, the sensitivities of both measures of investment output with respect to cash holdings increase in absolute terms with financing frictions. Importantly, however, the sign and strength of this relation depends on the respective returns to investment.

**Implication 4.** The sensitivity of investment output to cash flows is

i) positive and increasing in the costs of external financing in firms with low returns to investment, and

ii) negative and decreasing in the costs of external financing in firms with high returns to investment.

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34 Similarly, Hovakimian (2009) finds that firms with negative investment-cash flow sensitivities have higher R&D expenditures, higher market-to-book ratios, and lower asset tangibility than firms with positive investment-cash flow sensitivities.

35 This argument directly applies to results for intangible investment such R&D, for which Chen and Chen (2012) find no significant relation to cash flows. Further, given the growing importance of intangible capital over the last decades, it may also explain related results regarding the disappearance of total, i.e., capital + intangible, investment-cash flow sensitivities (see, e.g., Brown and Petersen 2009).
In light of Implication 4, empirical results documenting the decline and disappearance of investment-cash flow sensitivities in the cross-sectional average over the last decades as discussed above, need not imply that investment-cash flow sensitivity is a bad measure of financial constraints (see, e.g., Chen and Chen 2012 for an argument along these lines). Instead, when investment input is not perfectly contractible, our model suggests that empirical tests of the dependence of intangible investment on cash flows in financially constrained firms should explicitly take into account that its sign may depend on the returns to intangible investment.

4.2 Pay for Investment-Performance

Given the considerable discretion managers have when deciding on how much to spend on investment in intangibles, executive compensation needs to provide incentives for them to invest in shareholders’ best interest. In our model, the resulting agency costs create the distortions in and dynamics of intangible investment discussed above. However, our theoretical analysis further provides clear predictions about how to optimally structure these investment incentives that ensure the firm’s long-term profitability while also incentivizing the manager to meet short-term performance targets. In particular, we obtain novel empirical implications for the dependence of executive pay on the performance of intangible investment projects as captured by the sensitivities $\beta^q_t$ and $\beta^b_t$ with respect to...
Figure 5: Plots the average reward and punishment as a function of the returns to investment $\Delta \mu$. All remaining parameters are set as in Example 1.

investment success and failure respectively.

Notably, in order to incentivize a given level of investment input at lowest costs, the optimal contract conditions executive pay mainly on the more informative investment outcome resulting in an asymmetric response of compensation to investment success and failure.\(^{36}\) Since investment success is the less (more) informative performance signal when the implemented level of investment is high (low) – see the discussion in Section 3.3 – firms with high returns to investment stipulate an average reward for investment success, $\overline{\beta^g}$, that is smaller than the average punishment for investment failure, $|\overline{\beta^b}|$, while in firms with low returns to investment, this relation is reversed. This is illustrated in the upper right panel in Figure 5.

\(^{36}\)In contrast, as is standard also in single-task cash-flow diversion models (see, e.g., DeMarzo and Sannikov 2006), the sensitivity of pay to short-term performance ($\alpha_t = \lambda$) in our multitask model is symmetric, i.e., gains and losses of equal size imply an equal size reward or punishment.
Implication 5. The average sensitivity of pay to investment output is asymmetric with pay being more sensitive to

i) investment success in firms with low returns to investment, and to

ii) investment failure in firms with high returns to investment.

To get some more intuition for Implication 5, note first that with higher returns to investment, the manager’s compensation becomes more sensitive to investment output in order to incentivize higher investment expenditures (upper left panel of Figure 5), i.e., the total sensitivity of pay to investment performance, $\beta_I + |\beta_b|$ increases in $\Delta\mu$.\(^{37}\) However, not only the size but also the structure of incentive compensation changes as investment becomes more profitable. In particular, as investment expenditures increase investment failure becomes more and more informative. To provide investment incentives, the optimal contract, thus, stipulates on average stronger punishment following an investment failure (lower right panel of Figure 5). In contrast, the average reward for investment success is hump shaped in the returns to intangible investment as illustrated in the lower left panel of Figure 5. That is, although the total sensitivity of compensation to investment output needs to increase in $\Delta\mu$ (upper left panel of Figure 5), incentive provision via punishment for failure eventually becomes so efficient that the average reward for investment success decreases even in absolute terms and not only relative to the average punishment for failure.

The above implications on the sensitivity of pay with respect to intangible investment output focus on the part of the manager’s incentive compensation that ensures the firm’s long-term profitability. However, in our multitask setting, optimal incentive compensation needs to balance incentives for short- and long-term performance. More concretely, if a manager is harder to incentivize in his short-term task, which in our model is captured by higher benefits of cash flow diversion $\lambda$ reflecting, e.g., weaker corporate governance, the sensitivity of pay with respect to short-term earnings has to increase. However, this also requires a higher sensitivity of pay with respect to the long-term investment outcome (see upper left panel Figure 5), in order to avoid the manager from inflating short-term earnings

\(^{37}\)This is in line with empirical evidence that CEO compensation in high-tech firms is often tied to indicators of innovation success (such as project milestones or number of patents), but not so in traditional firms (Balkin et al. 2000). More generally, Balkin and Gomez-Mejia (1990) provide evidence that compensation in high-tech firms differs systematically from that in non-high-tech firms.
by reducing long-term investment. Hence, under the optimal contract the sensitivity of
pay to short- and long-term performance are intimately linked.

**Implication 6.** *The average sensitivity of pay to investment output is increasing in the
sensitivity of pay to short-term performance. This increase is asymmetric with pay becoming relatively more sensitive to*

1. *investment success in firms with low returns to investment, and to*
2. *investment failure in firms with high returns to investment.*

While a more severe cash flow diversion problem, thus, calls for a higher *total* sensitivity
of pay to investment performance, the *relative* increase in rewards for investment success
and punishments for investment failure again depends on the relative informativeness of
these two investment outcomes. For firms with low returns to investment, incentivizing
investment through rewards is more efficient than through punishment. Therefore, when
an increase in diversion benefits $\lambda$ (e.g., via a deterioration in corporate governance) re-
quires steeper incentives also for investment, these firms increase the sensitivity of pay to
investment success more than that with respect to investment failure. For firms with high
returns to investment, the opposite applies. Overall, steeper short-term incentives, thus,
call not only for steeper long-term incentives but also for a more asymmetric exposure to
investment outcomes (upper right panel Figure 5).

### 4.3 Endogenous Volatility and Liquidity Management

In the implementation of our model, financial frictions, which arise endogenously from
the agency problem, provide a precautionary motive for corporate cash holdings or, more
generally, liquidity management. In particular, fluctuations in corporate cash holdings via
their dependence on both short-term cash flows as well as risky investment outcomes may
require costly refinancing when cash reserves are depleted. One implication is that higher
risk in a firm’s cash position makes “financial slack” more valuable and, hence, higher
cash buffers are accumulated before the firm pays any dividends. Notably, and in contrast
to standard single-task models such as DeMarzo and Sannikov (2006), the level of risk in
a firm’s cash position is *endogenous* in our multitask model with delegated investment.
It depends, in particular, on the returns to intangible investment via the implemented
Volatility and Payout Boundary

Figure 6: The figure plots the cross-sectional average instantaneous volatility in cash holdings due to providing investment incentives (left panel) and the payout boundary (right panel) as a function of the returns to investment $\Delta \mu$. All remaining parameters are set as in Example 1.

investment profile as well as the pay for performance sensitivities used to incentivize it. As a consequence also corporate liquidity management strategies such as payout policies will vary systematically with the returns to intangible investment.\(^\text{38}\)

To see these relationships, consider the instantaneous volatility of cash holdings, $M$, that stems from the exposure to investment output, $\sigma_M := (\beta^g - \beta^b) \sqrt{\nu p(1-p)/\lambda}$.\(^\text{39}\) The left panel of Figure 6 plots the cross-sectional average of this endogenous volatility component under the optimal contract as a function of the returns to intangible investment. The endogenous volatility disappears for very low returns to investment, since (i) only little exposure to the risky investment outcome is needed to incentivize investment expenditures close to zero, i.e., $\beta^g - \beta^b \to 0$ as $I \to 0$ and (ii) the volatility of the investment outcome $p(1-p)$ vanishes as the probability of an investment success approaches zero. Now, as returns to investment increase – and with it optimal investment – also $\sigma_M$ increases at first. Intuitively, both the investment outcome becomes more risky (second term of $\sigma_M$ increases) and the sensitivity of total pay to investment performance increases as well (first term of $\sigma_M$ increases; see also upper left panel of Figure 5). However, as returns to investment and with it the implemented investment level get sufficiently large, the volatility of the risky

\(^{38}\)Note that this is not the case in the benchmark setting with contractible investment considered in Appendix B.1, highlighting again the crucial differences between non-contractible intangible investment and, e.g., contractible capital investment.

\(^{39}\)Thereby, we abstract from the remaining volatility, which is due to the exposure to short-run performance and which is independent of the returns to intangible investment.
investment outcome, the second term of $\sigma_M$, decreases again since an investment failure becomes less and less likely. Eventually, this decrease will outweigh the increase in the pay for investment performance sensitivity, such that the average volatility of firms’ cash holdings is hump-shaped in returns to investment (see left panel of Figure 6).

**Implication 7.** The average volatility of cash holdings is hump-shaped in returns to investment.

Over the past decades, cash holdings of US corporations have increased substantially, and this increase has gone hand in hand with a similar increase in the importance of intangible investment, in particular R&D (see e.g. Pinkowitz et al. 2016). While this joint development has been attributed to the lower collateral value of intangible investments, our model highlights another, orthogonal channel, which operates via the agency problem between firm insiders and outside investors: The need to incentivize intangible investment requires exposure of the firm’s cash position to additional risk, thus, creating additional endogenous volatility strengthening the precautionary motive for corporate cash holdings relative to an all-tangible investment firm. As a consequence, optimal corporate payout policies require a higher cash balance before distributing dividends to shareholders. In light of Implication 7, however, the relationship between the importance of intangible investment and precautionary cash holdings is not monotonic but follows the same shape as the endogenous volatility component $\sigma_M$. Hence, as returns to investment become sufficiently high, the target level of cash holdings at which dividends are distributed eventually decreases again. This pattern is illustrated in the right panel of Figure 6, which shows the optimal payout barrier $M^{pb}$, i.e., the level of cash holdings at which firms start paying dividends.

**Implication 8.** The average cash holdings of dividend paying firms are hump-shaped in returns to investment.

5 **Conclusion**

In this paper, we analyze the dynamics of corporate investment under endogenous financial constraints in a dynamic agency model with multitasking. The manager privately observes

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40See Bates et al. (2009) or Falato et al. (2013).
the firm’s cash flows which he can divert for private consumption and, in addition, use to finance non-contractible investment in the firm’s future profitability. Thus, when reported cash flows are low (or even negative), the principal does not know whether this is because of stealing or because of investment. To ensure truth-telling and investment in the interest of firm owners, the optimal compensation scheme ties the manager’s compensation both to reported cash flows as well as to imperfect performance signals indicating investment success or failure. Exposing the manager to compensation risk is, however, costly such that incentivizing investment causes additional agency costs.

The optimal investment profile trades off these agency costs of investment with the potential efficiency gains. As a result, investment will be history dependent and distorted away from its first-best level. Investment distortions decrease (in absolute terms) with good performance, as the manager’s stake in the firm increases mitigating the agency problem. Our model predicts overinvestment and negative investment-cash flow sensitivities in industries where returns to non-contractible intangible investment such as R&D are high (e.g., as being in a position of technological leadership is important like in the pharmaceutical industry). By contrast, in industries where this is not the case, investment will be distorted downwards and increasing in cash flows.

If raising external funds is more expensive, investment will be both distorted more severely and more sensitive to past performance; i.e., in firms with high returns to investment, investment increases and the relation between financial slack and investment becomes more negative, while investment decreases and its sensitivity to financial slack becomes more positive in firms with low returns to investment. Our multitask theory, thus, offers a new perspective on the interpretation of investment-cash flow sensitivities as a measure of financial constraints, taking into account the increased importance of alternative forms of investment such as process innovation, R&D, or intangible investment in general. Furthermore, we provide new, testable implications on how the structure of executive compensation, as well as level and volatility of corporate cash holdings differ between industries in which intangible investment is of high, respectively low importance.
References


Electronic copy available at: https://ssrn.com/abstract=3164471


Appendix A  Omitted Proofs

Proof of Lemma 1. For \( S = S^* \), we define the manager’s expected lifetime utility evaluated conditional on time \( t \) information by:

\[
v_t = \int_0^t e^{-\gamma s} dU_s + e^{-\gamma t} w_t, \tag{A.1}
\]

which, by construction, is a martingale with respect to the filtration generated by \( (Z, N^j) \), \( j \in \{g, b\} \) under the probability measure \( Q^{S^*} \). By the martingale representation theorem (cf., Theorem III 4.34 in Jacod and Shiryaev 2003), we can express \( v_t \) for some predictable processes \( \alpha \) and \( \beta^j \), \( j \in \{g, b\} \) as

\[
v_t = v_0 + \int_0^t e^{-\gamma s} \alpha_s \left( d\hat{Y}_s - (\mu_s - I_s^*) \right) ds + \int_0^t e^{-\gamma s} \beta^g_s (dN^g_s - \nu p(I_s^*) ds)
+ \int_0^t e^{-\gamma s} \beta^b_s (dN^b_s - \nu (1 - p(I_s^*))) ds,
\]

where, under \( Q^{S^*} \), \( \hat{Y}_t - \int_0^t (\mu_s - I_s^*) ds \) is a standard Brownian motion and \( N^g_t - \int_0^t \nu p(I_s^*) ds \) as well as \( N^b_t - \int_0^t \nu (1 - p(I_s^*)) ds \) are compensated Poisson processes. Differentiating both this representation and the definition of \( v_t \) in (A.1) yields (6). Q.E.D.

Proof of Lemma 2. Consider any feasible policy of the agent, \( S = \{\hat{Y}_t, I_t : 0 \leq t \leq \tau\} \), with \( dY_t - d\hat{Y}_t \geq 0 \) and \( I_t \geq 0 \). The associated expected lifetime utility is given by

\[
v_t = v_0 + \int_0^t e^{-\gamma s} \alpha_s \left( d\hat{Y}_s - (\mu_s - I_s^*) \right) ds + \int_0^t e^{-\gamma s} \lambda \left( dY_s - d\hat{Y}_s \right)
+ \int_0^t e^{-\gamma s} \beta^g_s (dN^g_s - \nu p(I_s^*) ds) + \int_0^t e^{-\gamma s} \beta^b_s (dN^b_s - \nu (1 - p(I_s^*))) ds, \tag{A.2}
\]

where \( d\hat{Y}_t - (\mu_t - I_t^*) dt = \sigma dZ_t \) for \( S = S^* \), and \( dN^g_t \) \( (dN^b_t) \) is a Poisson process with intensity \( \nu p(I_t) \) \( (\nu (1 - p(I_t))) \). Differentiating (A.2) and taking expectations gives

\[
e^{\gamma t} E [dv_t] = (\lambda - \alpha_I) E \left[ dY_t - d\hat{Y}_t \right] + \alpha_I (I_t^* - I_t) dt + \nu (\beta^g_t - \beta^b_t) (p(I_t) - p(I_t^*)) dt.
\]

Clearly, \( v_t \) is a martingale for \( S = S^* \). For \( S = S^* \) to be incentive compatible, the drift of \( v_t \) has to be non-positive for all possible deviations, i.e., \( v_t \) has to be a supermartingale for any feasible \( S \neq S^* \). Consider, first, a deviation from truth-telling, i.e., underreporting for consumption \( dY_t - d\hat{Y}_t > 0 \). This deviation is suboptimal for the agent if \( \alpha_I \geq \lambda \). Second, increasing \( I_t \) marginally above \( I_t^* \), is suboptimal for the agent if \( \alpha_I \geq \nu p(I_t) (\beta^g_t - \beta^b_t) \).
while, third, decreasing $I_t$ marginally below $I_t^*$, is suboptimal if $\alpha_t \leq \nu p^2(I_t) (\beta_q^2 \beta_q^b)$. So, incentive compatibility requires (7) and (8) to hold. \textbf{Q.E.D.}

\textbf{Proof of Proposition 1.} Consider the process $\kappa$ counting the number of replacements and the associated time points $\tau(\kappa)$ where we normalize $\tau(0) = 0$. Then we can write the principal’s value as

$$f^i(w) = \max_{I, U, \tau} \left\{ E \left[ \int_0^\infty e^{-rs} (dY_s - dU_s) - k \sum_{\kappa=1}^\infty e^{-r\tau(\kappa)} \big| w, \mu^i \right] \right\},$$

where maximization is subject to incentive compatibility in (4), limited liability as well as promise-keeping $w_0 = w$. Now denote the (stochastic) time of the next technology shock by $T$ and recall that technology shocks arrive with exogenous rate, such that $T$ is clearly beyond both the agent’s and the principal’s control. Thus, substituting from (1), we have

$$f^i(w) = \mu^i + \max_{I, U, \tau} \left\{ E \left[ \int_T^\infty e^{-rs} \mu_s ds - \int_0^\infty e^{-rs} (dU_s + I_s ds) - k \sum_{\kappa=1}^\infty e^{-r\tau(\kappa)} \big| w \right] \right\},$$

where the maximization problem determining the optimal contract is clearly independent of $\mu_0 = \mu^i$ for any $w$. \textbf{Q.E.D.}

\textbf{Proof of Lemma 3.} Consider the process $\kappa$ counting the number of replacements and the associated time points $\tau(\kappa)$ where we normalize $\tau(0) = 0$. Then we can write the principal’s value as

$$f^i(w) = \mu^i + \max_{I, U, \tau} \left\{ E \left[ \int_T^\infty e^{-rs} \mu_s ds - \int_0^\infty e^{-rs} (dU_s + I_s ds) - k \sum_{\kappa=1}^\infty e^{-r\tau(\kappa)} \big| w \right] \right\},$$

\textbf{Concavity.} Consider the function $f(w)$ that, for a given $w$ and $\mu$, solves

$$\mu + \nu p(I(w)) \Delta - I(w) + \frac{1}{2} (\alpha(\mu))^2 \sigma^2 f''(w)$$

$$= (r + \nu) f(w) = \gamma w - \nu (\beta_q^2 w \mu(I(w)) + \beta_q^b (w) (1 - p(I(w)))) + \nu (1 - p(I(w))) f(w + \beta_q^b (w))$$

for $w \in [0, \bar{w}]$ and $f(w) = f(\bar{w}) - (w - \bar{w})$ for $w > \bar{w}$, with boundary conditions $f(0) = \max_w \{ f(\bar{w}) - k \}$, $f'(\bar{w}) = -1$, and $f''(\bar{w}) = 0$.

Investment $I(w)$ satisfies the first order condition

$$1 = \left[ \frac{\nu p'(I(w)) (\beta_q^2 (w) - \beta_q^b (w)) f_w(w)}{+ \nu p'(I(w)) \left[ f(w + \beta_q^b (w)) + \Delta - f(w + \beta_q^b (w)) \right] + \alpha(w) p(I(w)) \frac{p''(I(w))}{p'(I(w))^2} \left[ f'(w) - f'(w + \beta_q^b (w)) \right]} \right],$$

46
\[ f'(w) = p(I(w))f'(w + \beta^g(w)) + (1 - p(I(w)))f'(w + \beta^b(w)) \tag{A.5} \]

for \( \beta^i(w) \geq -w \) while \( \beta^i(w) = -w \) otherwise.

Differentiating (A.3) and using (A.4) and (A.5) yields

\[ - (\gamma - r) f'(w) = \left[ \gamma w - \nu \left( \frac{\alpha(w)p(I(w))}{\nu p'(I(w))} + \beta^b(w) \right) \right] f''(w) + \frac{1}{2} \sigma^2 (\alpha(w))^2 f'''(w). \]

From the boundary conditions at \( \bar{w} \) we get \( f'''(\bar{w}) = 2 \frac{\gamma - r}{\sigma^2 \alpha(w)^2} > 0 \), such that \( \exists \varepsilon > 0 \) with \( f'''(\bar{w} - \varepsilon) > 0 \) and \( f'''(\bar{w} + \varepsilon) < 0 \).

The proof then is by contradiction. So assume that \( \exists \bar{w} := \sup \{ w < \bar{w} : f''(w) \geq 0 \} \), where it holds by continuity that \( f''(\bar{w}) = 0 \) and \( f'''(\bar{w}) < 0 \), implying that \( f'(\bar{w}) = -f'''(\bar{w}) \frac{1}{2} \sigma^2 \alpha(w)^2 > 0 \). Now, consider two points \( w^1 < \bar{w} < w^2 \) close to \( \bar{w} \), such that \( f''(w^1) > 0 \) > \( f''(w^2) \) and \( w^1 f'(w^1) = w^2 f'(w^2) \) and observe that \( f(w) \) can be written as

\[ rf(w) = \gamma w f'(w) + \frac{1}{2} \sigma^2 (\alpha(w))^2 f''(w) + g(w), \]

with

\[ g(w) = \mu - I(w) + \nu \left[ p(I(w)) \left( f(w + \beta^b(w)) + \frac{\alpha(w)p(I(w))}{\nu p'(I(w))} + \Delta \right) \right] \]

\[ + (1 - p(I(w))) f(w + \beta^b(w)) - f(w) \]

\[ - \left( \frac{\alpha(w)p(I(w))}{\nu p'(I(w))} + \beta^b(w) \right) f'(w) \]

Next, compute the differential of \( g(w) \) around \( \bar{w} \), \( dg(w)_{w=\bar{w}} = g'(\bar{w})dw \), and observe that

\[ g'(\bar{w}) = I_w(w, \mu) \left[ \begin{array}{c} -1 + \nu p'(I(w)) \left[ f(\bar{w} + \beta^g(\bar{w})) + \Delta - f(\bar{w} + \beta^b(\bar{w})) \right] \\ -\alpha(w) f'(\bar{w}) \\ + \alpha(w)p(I(w))p''(I(w)) \left[ f'(\bar{w}) - f'(\bar{w} + \beta^g(\bar{w})) \right] \\ + \nu \left( 1 + \beta^b_w(\bar{w}) \right) \left[ p(I(w))f'(\bar{w} + \beta^g(\bar{w})) + (1 - p(I(w))) f'(\bar{w} + \beta^b(\bar{w})) - f'(\bar{w}) \right] \\ - \nu \left( \frac{\alpha(w)p(I(w))}{\nu p'(I(w))} + \beta^b(\bar{w}) \right) f''(\bar{w}) \end{array} \right] = 0, \]

which follows from \( f''(\bar{w}) = 0 \) together with (A.4) and either (A.5), or, in case the limited liability constraint binds, \( \beta^b_w(\bar{w}) = \partial \beta^b(\bar{w}) / \partial w = -1 \). Thus, evaluating \( f(w) \) in \( w^1 \) and
w^2, we get
\[ r \left[ f(w_1) - f(w_2) \right] = \frac{1}{2} \sigma^2 \alpha(w)^2 \left[ f''(w_1) - f''(w_2) \right] > 0, \]
where we have used that the effect of a change in \( g(w) \) around \( \bar{w} \) is of second order and will thus be dominated by the change in \( f''(w) \). However, this directly contradicts \( f'(\bar{w}) > 0 \).

**Verification.** For any incentive compatible contract \((S, U, \tau)\), define
\[ G_t = \int_0^t e^{-rs} (dY_s - dU_s) + e^{-rt} f(w_t, \mu_t) \]
and recall that \( w_t \) evolves according to
\[ dw_t = \gamma w_t dt - dU_t + \alpha_t \sigma dZ_t + \beta_t^g (dN_t^g - \nu p(I_t) dt) + \beta_t^b (dN_t^b - \nu (1 - p(I_t)) dt), \]
where \( \alpha_t \geq \lambda \) and \( \beta_t^g - \beta_t^b = \frac{\alpha_t^0}{\nu p(I_t)} \). Recall that \( f(w) = f(w, \mu^1) = f(w, \mu^b) - \Delta \) so that from differentiating \( G \) using Itô’s lemma and the change in variables formula for point processes as well as (A.3), we get
\[ e^{rt} dG_t = \nu \left( p(I_t) \left[ f(w_t + \beta_t^g + \Delta) + (1 - p(I_t)) f(w_t + \beta_t^b) \right] \right. \]
\[ \left. - \left[ \beta_t^g p(I_t) + \beta_t^b (1 - p(I_t)) \right] f'(w_t) - I_t \right) dt \]
\[ - \nu \left( p(I(w)) \left[ f(w + \beta_t^g (w)) + \Delta] + (1 - p(I(w))) f(w + \beta_t^b (w)) \right] \right. \]
\[ \left. - \left[ \beta_t^g (w) p(I(w)) + \beta_t^b (w) (1 - p(I(w))) \right] f'(w) - I(w) \right) dt \]
\[ + \frac{1}{2} \left( \alpha_t^2 - \lambda^2 \right) \sigma^2 f''(w_t) dt - [1 + f'(w_t-)] dU_t + [\sigma + \alpha_t \sigma f'(w_t-)] dZ_t \]
\[ + f(w_t + \beta_t^g + \Delta] dM_t^g + f(w_t + \beta_t^b) dM_t^b - f(w_t-) (dM_t^g + dM_t^b) \]

for \( w = w_t \in [0, \bar{w}] \), where \( M_t^i \) denote the compensated point processes associated with \( N_t^i, i \in \{g, b\} \). Now observe that the sum of the first two lines is less than or equal to zero, because \( \beta^i(w), i \in \{g, b\} \) and \( I(w) \) are the solution to
\[ \max_{\beta' \geq -w, I} \left[ p(I) \left[ f(w + \beta^g) + \Delta] + (1 - p(I)) f(w + \beta^b) \right] \right. \]
\[ \left. - \left( \beta_t^g p(I) + \beta_t^b (1 - p(I)) \right) f'(w) - I \right]. \]

Now turn to the third line of (A.6). The first term is non positive as \( f'' \leq 0 \) and \( \alpha_t \geq \lambda \) for any \( t \geq 0 \). The second term is non positive as \( f' \geq -1 \) and \( dU \geq 0 \) while \( Z \) and \( M_t^i \) are martingales. Hence, \( G_t \) is a supermartingale and a martingale if and only if for \( t > 0 \), \( \beta_t^i = \beta^i(w), I_t = I(w), \alpha_t = \lambda \) and \( dU_t > 0 \) only when \( w > \bar{w} \).

Now consider the principal’s expected payoff under any incentive compatible contract
\((S, U, \tau)\):

\[
E \left[ \int_0^\tau e^{-rs} (dY_s - dU_s) + e^{rt} L_\tau, \right]
\]

where \(\tau\) denotes the time when the incumbent manager is replaced and \(L_\tau\) the principal’s expected profits from restarting with a new agent, net of replacement costs \(k\). We then have that

\[
E \left[ \int_0^\tau e^{-rt} (dY_s - dU_s) + e^{rt} L_\tau, \right] = E \left[ G_{t \land \tau} + E \left[ \mathbf{1}_{t \leq \tau} \left( \int_t^{\tau} e^{-rs} (dY_s - dU_s) + e^{-rt} L_\tau \right. \right. \right] = f (w_0) + E \left[ \mathbf{1}_{t \leq \tau} \left( \int_t^{\tau} e^{-r(s-t)} (dY_s - dU_s) + e^{-r(\tau-t)} L_\tau | \mathcal{F}_t \right) \right] - f (w_t),
\]

where the inequality follows since \(G_{t \land \tau}\) is a supermartingale and \(G_0 = f (w_0)\). Now note that

\[
E \left[ \int_t^{\tau} e^{-r(s-t)} (dY_s - dU_s) + e^{-r(\tau-t)} L_\tau | \mathcal{F}_t \right] < \mu_h - \frac{w_t - \mu_h}{r} - \frac{w_t}{r}.
\]

and, as \(f' \geq -1\), we have that \(f (w_t) + \frac{w_t}{r} \geq L_\tau\), which yields

\[
E \left[ \int_0^\tau e^{-rt} (dY_s - dU_s) + e^{rt} L_\tau, \right] \leq f (w_0) + e^{-rt} E \left[ \mathbf{1}_{t \leq \tau} \left( \frac{w_t}{r} - L_\tau \right) \right].
\]

Taking \(t \to \infty\) yields

\[
E \left[ \int_0^\tau e^{-rt} (dY_s - dU_s) + e^{rt} L_\tau, \right] \leq f (w_0).
\]  

Finally, under the contract stated in Proposition 1, \(G_{t \land \tau}\) is a martingale and, hence, (A.7) holds with equality. Q.E.D.

**Proof of Lemma 4.** Whenever the limited liability constraint (9) does not bind, for which it is sufficient that \(w \geq \frac{\lambda}{\nu' (I)}\), first order condition (19) holds. The first claim then follows immediately from rewriting (19) as

\[
\frac{f' (w + \beta (w)) - f' (w)}{f' (w) - f' (w + \beta (w))} = \frac{p (I (w))}{1 - p (I (w))},
\]

and observing that \(p (I (w))|_{\psi'} < p (I (w))|_{\psi''}\) since \(I (w)|_{\psi'} < I (w)|_{\psi''}\) and \(p' (\cdot) > 0\). Next,
when \( I(w)|_\phi \to \bar{T} \) and the limited liability constraint is slack, strict concavity of \( f(w) \) implies that (19) can only be satisfied for \( \beta^g(w) \to 0 \). In case \( p(I) \to 0 \), (19) similarly requires that \( \beta^b(w) \to 0 \), while, from incentive compatibility, \( \beta^g(w) \to \frac{\lambda}{\nu p'(I)} \). For bounded \( p'(0) \), it then trivially follows that \( -\beta^b(w)/\beta^g(w) \to 0 \). Now, if \( p'(I) \to \infty \) as \( I \to 0 \), both \( \beta^g(w) \) and \( \beta^b(w) \) go to zero as \( I \to 0 \). Then, up to a first-order approximation, we have \( f'(w + \beta^b(w)) = f'(w) + \beta^b(w)f''(w) \), and \( f'(w + \beta^g(w)) = f'(w) + \beta^g(w)f''(w) \). Hence, there exists an \( \varepsilon > 0 \) such that, from (A.8), we have \( p(\varepsilon) = \frac{p(\varepsilon)}{1-p(\varepsilon)} = \frac{-\beta^b(w)}{\beta^g(w)} \).

Finally, the last claim follows from strict concavity of \( f(w) \) for \( w < \bar{w} \), implying that (19) can only be satisfied for \( \beta^g > 0 > \beta^b \). Q.E.D.

**Proof of Lemma 5.** See main text. Q.E.D.

**Proof of Proposition 2.** Denote the right-hand side of the HJB in (12) by \( \mathcal{L} \) and take the first derivative with respect to \( I \), noting that \( \beta^g = \beta^b + \frac{\lambda}{\nu p'(I)} \) from incentive constraint (8), to obtain

\[
\frac{\partial \mathcal{L}}{\partial I} = \nu p'(I(w))\Delta - 1 - \left[ \left( \frac{\lambda - p''(I(w))}{\nu p'(I(w))} \right) \delta^b(w) + \nu p'(I(w)) \left( \delta^g(w) - \delta^b(w) \right) \right],
\]

(A.9)

with

\[
\delta^b(w) := p(I(w)) \left( 1 - p(I(w)) \right) \nu \left[ f'(w + \beta^b(w)) - f'(w + \beta^g(w)) \right] \geq 0,
\]

(A.10)

\[
\delta^g(w) := f(w) + \beta^g(w) f'(w) - f(w + \beta^g(w)) \geq 0,
\]

(A.11)

\[ j \in \{ g, b \} \]

where we have used (19), which holds whenever the limited liability constraint does not bind, and the inequalities in (A.10) and (A.11) follow from strict concavity of \( f(w) \) for \( w < \bar{w} \). The term in square brackets in (A.9) are the marginal agency costs of investment which we denote by \( \phi(w) \).

To show the overinvestment result for sufficiently high levels of \( I_{FB} \) it is sufficient to show that the marginal agency costs of investment are strictly negative as \( I_{FB} \) approaches \( \bar{T} \), such that \( \partial \mathcal{L} / \partial I|_{I(w)=I_{FB}} > 0 \) for \( I_{FB} > \bar{T} \). The result then follows from continuity and the fact that \( I(w) = I_{FB} \) is the uniquely optimal investment level for \( I_{FB} = \bar{T} \). So, note that \( p(\bar{T}) = 1 \) implies that \( \delta^b(w) \to 0 \) and, by Lemma 5, \( \beta^g(w), \delta^g(w) \to 0 \), such that \( \phi(w)|_{I(w)=I_{FB}} \to \nu p'(I_{FB}) \delta^b(w) \to 0 \), where the inequality follows again from the concavity of \( f(w) \) and the fact that \( \beta^b(w) > 0 \) by incentive constraint (8).\footnote{Note also that from \( \nu p'(I_{FB})\Delta - 1 = 0 \) we must have that \( p'(I_{FB}) > 0 \), which is also a necessary}
As for the remaining case, underinvestment if $I_{FB}$ is sufficiently low, we will show that $\partial L/\partial I|_{I(w)=I_{FB}} \to 0$ from below as $I_{FB} \searrow 0$. To see this note that, in the limit, $\delta^g$ dominates $\delta^b$ from Lemma 5, such that $\phi(w)$ is strictly positive. The result then follows from continuity and the fact that $I(w) = I_{FB}$ is the uniquely optimal investment level for $I_{FB} = 0$. Q.E.D.

**Proof of Lemma 6.** For part i) we first show that there exists $\hat{w}_\beta \in (0, \bar{w})$ such that $\beta^b(w) = -w$ for $w \in [0, \hat{w}_\beta]$. To see this, note that, for any $I(w) > 0$, we must have $\beta^g(w) - \beta^b(w) > 0$, which together with the strict concavity of $f(w)$ for $w \in [0, \bar{w})$ implies that the first-order condition in (19) cannot be satisfied in a neighborhood of $w = 0$. Thus, we get the stated result, with $\beta^b(w) = -w$ and $\beta^g(w) = \beta^b(w) + \lambda/\nu p'(I(w)) > 0$ on $w \in [0, \hat{w}_\beta]$, where

$$\hat{w}_\beta := \min \left\{ w > 0 : p(I(w)) f'(\frac{\lambda}{\nu p'(I(w))}) + (1 - p(I(w))) f'(0) = f'(w) \right\}.$$ 

Next, let us show the underinvestment result for small $w$. With the limited liability constraint binding, marginal agency costs of investment $\phi(w)$ in (A.9) can be rewritten to obtain

$$\phi(w) = \nu p'(I(w)) \left[ f\left(\frac{\lambda}{\nu p'(I(w))}\right) - \left(f(0) + \frac{\lambda}{\nu p'(I(w))} f'(w)\right)\right] + \lambda p(I(w)) p''(I(w)) \left[p'(I(w))^2 + f'(w) - f'(w + \beta^g(w))\right].$$

It remains to show that this expression is negative for small $w$. To see this, note first, that the expression in the second line is negative for all $w$ by strict concavity of $f(w)$. As for the remaining right-hand-side expression, it also follows from concavity of $f(w)$ that this is negative for $w$ small enough.

For part iii), note that as $f(w)$ extends linearly for $w > \bar{w}$, we have that $f(\bar{w} + \beta^g(\bar{w})) = f'(\bar{w})$ and (19) can only be satisfied if $\beta^b(\bar{w}) = 0$. It remains to show that $I(\bar{w}) = I_{FB}$. Since $\beta^g(\bar{w}) > 0$, but $f'(\bar{w} + \beta^g(\bar{w})) = f'(\bar{w})$, and $\beta^b(\bar{w}) = 0$, the marginal agency costs of investment $\phi(w)$ in (A.9) are equal to zero, implying first-best investment.

Part ii) then is immediate as $\beta^b(\bar{w}) = 0$ and continuity imply that the limited liability constraint (9) is slack for $w$ close to $\bar{w}$ such that Lemma 4 and Proposition 2 apply. Q.E.D.
Appendix B Additional Material (Online)

This Appendix provides some supplementary material, including a complete characterization of the optimal contract in the benchmark case with contractible investment expenditures (Appendix B.1), as well as an analysis of the extension of the baseline model to an arbitrary number of investment outcomes (Appendix B.2). Further, in Appendix B.3 we provide an implementation of the optimal contract, and Appendix B.4 contains an outline of the algorithm used for solving for the optimal contract numerically.

B.1 Contractible Investment Benchmark

When investment expenditures are contractible, the problem reduces to a standard single-task cash flow diversion problem in which the principal directly controls investment. Formally, he then also solves the boundary value problem in (12)-(15), but does not have to respect the incentive constraint for investment (8). We denote the respective value function with contractible investment by \( f_{CI}(w) \) and index contractual parameters, such as the compensation boundary \( \bar{w}_{CI} \). The solution to this benchmark case is summarized as follows:

**Proposition B.1.** Assume investment expenditures are contractible, then, under the optimal truth-telling contract, investment is given by the constant first-best investment level \( I_{FB} \) in (20) \( \forall t \). The incumbent agent’s continuation payoff evolves according to (6) with \( \alpha_t = \lambda, \beta_t^h = \beta_t^l = 0 \) and \( I_t = I_{FB}, \forall t \). When \( w_t^- \in [0, \bar{w}_{CI}) \), \( dU_t = 0 \); when \( w_t^- \geq \bar{w}_{CI} \) payments \( dU_t \) cause \( w_t^- \) to reflect at \( \bar{w}_{CI} \). The incumbent agent is replaced when \( w_t^- = 0 \). The principal’s expected payoff at any point in time is given by \( f_{CI}(w_t), i \in \{h,l\} \), which satisfies \( f_{CI}(w) := f_{CI}^h(w) = f_{CI}^l(w) - \Delta \), where \( f_{CI}(w) \) is concave, strictly so for \( 0 \leq w < \bar{w}_{CI} \) and solves, for \( w \in [0, \bar{w}_{CI}] \), the HJB equation

\[
rf_{CI}(w) = \mu^t - I_{FB} + \nu p(I_{FB}) \Delta + \gamma w f'_{CI}(w) + \frac{1}{2} \sigma^2 \lambda^2 f''_{CI}(w) \quad \text{B.1}
\]

with boundary conditions \( f_{CI}(0) = f_{CI}(w_{CI}^*) - k \), where \( w_{CI}^* \in \arg \max_w \{f_{CI}(w)\} \), \( f_{CI}'(\bar{w}_{CI}) = -1 \) and \( f_{CI}''(\bar{w}_{CI}) = 0 \).

**Proof of Proposition B.1.** This result is a straightforward extension of Hoffmann and Pfeil (2010), who study a dynamic cash-flow diversion models with exogenous shocks to profitability; we therefore will be brief. Note, first, that \( f_{CI}(w) \) is strictly concave, which follows from the same arguments as in Hoffmann and Pfeil (2010). Further, the incentive
constraint binds, i.e., $\alpha = \lambda$. Interior solutions for $I$ are then given by
\[
\Delta + \left[ f_{CI}(w + \beta^g) - f_{CI}(w + \beta^b) \right] - (\beta^g - \beta^b) f'_C(w) = \frac{1}{\nu p'(I)},
\]
while the $\beta^j, j \in \{g, b\}$, are determined from $f'_C(w + \beta^g) = f'_C(w) = f'_C(w + \beta^b)$. Due to the strict concavity of $f_{CI}$ this immediately implies that $\beta^g = 0$ and $\beta^b = 0$, and, plugging into (B.2), we find that optimal investment is equal to the first-best level as characterized in (20), such that the proposed solution achieves efficient investment. Verification is then standard (cf., Hoffmann and Pfeil 2010) and therefore omitted. Q.E.D.

As in the case with non-contractible investment, the agent’s incentive constraint with respect to cash flow diversion (7) binds under the optimal contract because it is costly to provide incentives (formally the instantaneous volatility of $w$ increases in $\alpha$ and $f_{CI}(w)$ is concave). A similar argument implies that, when investment expenditures are contractible and there is, thus, no need to provide incentives based on the investment outcome, it is optimal to choose $\beta^g_{CI} = \beta^b_{CI} = 0$. As a consequence, the agent’s continuation value is insensitive to the investment outcome and the principal’s investment problem is, thus, independent of the cash-flow diversion problem. Hence, the optimal investment policy is equal to first-best.

### B.2 General Model

In this Appendix we show how our analysis can be extended beyond the binary state structure allowing firm profitability to take on values $\{\mu^i\}_{i=1}^{M}$ for any $M \geq 2$, where $\mu^1 < \mu^2 < \ldots < \mu^M$.\textsuperscript{43} As in our baseline model the industry is subject to (rare) exogenous technology shocks governed by a Poisson process $N$ with intensity $\nu$, indicating the availability of a new technology. Investment $I \in [0, \bar{I}]$ determines the probability distribution over future profitability in the event of a technology shock, now according to $p_i(I_t) := \Pr (\mu_{t+} = \mu^i | I_t)$ for $i = 1, \ldots, M$.\textsuperscript{44} It is then again convenient to define Poisson processes $N^i$ with arrival rate $\nu p_i(I_t)$ for $i = 1, \ldots, M$ that capture the investment outcome

\textsuperscript{42}Formally, this follows from the fact that the costs of compensating the agent, as reflected in the slope $f'_{CI}(w)$, are independent of current profitability. If the costs of compensating the agent differed across states $i \in \{h, l\}$, e.g., due to state-dependent hiring costs, it would also be efficiency enhancing to specify $\beta^g_{CI}, \beta^b_{CI} \neq 0$ (cf., Hoffmann and Pfeil (2010) for a formal model of this “reward for luck” effect).

\textsuperscript{43}For expositional clarity we assume that it is profitable to run the firm in each state, which requires that $\mu^1$ is sufficiently large.

\textsuperscript{44}Clearly, we must have $p_i(I) \geq 0$ as well as $\sum_{i=1}^{M} p_i(I) = 1$. Further, to ensure identifiability, the probability distribution $p_i(I)$ must vary with investment $I$, in particular, assuming differentiability of $p_i(I)$ for $I \in (0, \bar{I})$, there must for each $I$ exist an $i$ such that $\frac{d}{dI} p_i(I) \neq 0$. 

53
in the event of a technology shock with \( dN_t^i = 1 \) if \( \mu_{t+} = \mu^i \) and zero else. In between two technology shocks, profitability remains unchanged. Hence, first-best investment, which corresponds to optimal investment when investment expenditures are contractible (cf., Proposition B.1), solves

\[
I_{FB} = \text{arg max} \left\{ \nu \sum_{i=1}^{M} p_{i}(I) \Delta^i - I \right\},
\]

where \( \Delta^i := \frac{\mu^i - \mu^1}{r + \nu} \) captures the gain from increasing profitability above the minimal level properly accounting for the Markov switching structure. In order to obtain a tractable analysis with non-contractible investment, we now impose the following assumptions on the investment technology which are straightforward generalizations of the assumptions made in the main text to the case with \( M > 2 \).

Assumption B.2. The investment technology has the following properties

(i) \( p_i'(I)/p_i(I) > p_j'(I)/p_j(I) \) for \( i > j \) and all \( I \), i.e., the monotone likelihood ratio property (MLRP) holds.

(ii) \( p_M(0) = 0 \) and \( p_1(\bar{I}) = 0 \), i.e., the best (worst) possible outcome cannot occur if \( I = 0 \) (\( I = \bar{I} \)).

(iii) \( p_i'(I)p_i''(I) < 0 \) for all \( i, I \), i.e., the first-order approach (FOA) applies.

Part (i) of Assumption B.2 (MLRP) ensures that the realization of higher profitability is more indicative of the manager having invested a lot. It further implies a first-order stochastic dominance (FOSD) ranking on the cdf \( F(\mu^i|I) = \sum_{j=1}^{i} p_{j}(I) \), i.e., conditional on a technology shock, investment increases the chances of high future profitability according to \( \frac{d}{dI} F(\cdot|I) < 0 \), thus, shifting probability mass from low to high levels of profitability. Part (ii) of Assumption B.2 ensures that this FOSD shift is sufficiently strong in the sense that the realization of extremely bad (good) outcomes becomes very unlikely if investment is sufficiently high (low). A reasonable chance at realizing the highest possible outcome, thus, requires some minimal investment, which constitutes a limit to good luck, while high investment protects the firm from the worst possible outcome, thereby limiting bad luck. While parts (i) and (ii) of Assumption B.2 are not necessary for the subsequent characterization of the optimal contract, they will play a role in establishing the sign of investment distortions, in particular they are sufficient (but not necessary) to show that

\[45\]In a slight abuse of notation \( p_i'(I) \) and \( p_i''(I) \) for \( I = 0 \) and \( I = \bar{I} \) refer to the respective right-/left-hand side derivatives.
investment is distorted upwards for highly profitable and distorted downwards for less profitable investment technologies. We comment more on their relevance after establishing this result. Finally, as is common in moral hazard problems with continuous actions (see e.g., Holmström 1979), we impose a technical condition to ensure validity of the first-order approach, such that we can replace the incentive constraint for investment at each \( t \) by the respective first-order condition. Part (iii) of Assumption B.2 provides a sufficient (but not necessary) condition. We note that Assumption B.2 is clearly satisfied for the investment technology in our baseline model with binary states.

Apart from the richer investment technology all other aspects of the model are as described in Section 2. Hence, the contracting problem is to find an incentive compatible truth-telling contract \((S^*, U, \tau)\), maximizing the principal’s expected profit \( f_0 \) from (3) for given initial profitability \( \mu_0 \in \{\mu^1, \ldots, \mu^M\} \), subject to incentive compatibility (4) and limited liability, while delivering expected payoff \( w_0 \) as defined in (2) to the agent. Accordingly the following derivation is a straightforward extension of the analysis in Section 3 of the main text and we will therefore be brief.\(^{46}\)

**Continuation Payoff and Local Incentive Compatibility.** Again, the contract can be written in terms of the agent’s continuation payoff \( w_t \) as defined in (5) as the single state variable. In particular, analogous to Lemmas 1 and 2 the agent’s continuation payoff evolves as

\[
   dw_t = \gamma dw_t - dU_t + \alpha_t \left( dY_t - (\mu_t - I^*_t) \, dt \right) + \sum_{i=1}^{M} \beta_i \left[ dN_i - \nu p_i(I^*_t) \, dt \right].
\]

Truth-telling and following the prescribed investment \( I^*_t \in (0, \bar{I}) \) is incentive compatible if and only if \( \alpha_t \geq \lambda \) and\(^{47}\)

\[
   \nu \sum_{i=1}^{M} p_i(I^*_t) \beta_i^2 - \alpha_t = 0, \quad \forall t.
\]

\(^{46}\)Formal proofs of all steps leading to the subsequent results are available from the authors upon request.

\(^{47}\)Condition (B.4) requires \( I^*_t \) to solve the first-order condition of the agent’s maximization problem over investment \( I_t \) given the contract. It is then easy to see that under the optimal contract we must have \( p_i(I^*_t) \beta^i \geq 0 \), i.e., if the probability of investment outcome \( i \) is increasing (decreasing) in investment at \( I_t = I^*_t \) the agent should be rewarded (punished) if \( i \) realizes. Hence, using part (iii) of Assumption B.2, also the agent’s second-order condition is satisfied

\[
   \nu \sum_{i=1}^{N} p_i''(I^*_t) \beta_i^2 \leq 0.
\]
Further, limited liability requires, as before, that $\beta^i_t \geq -w_t^i$ for all $i$.

**Optimal Contract**  As in the baseline model in the main text, the optimal contract can be derived using the dynamic programming approach where we denote the principal’s value function for given profitability $\mu^i$, $i \in \{1, ..., M\}$ and agent’s continuation value $w$ by $f^i(w)$. As the agency problem for given $w$ is independent of $\mu^i$, it follows from the same arguments as in the main text that the optimal contract is independent of current profitability. Hence, we can conveniently characterize the optimal contract using the principal’s value function in the lowest profitability state $\mu^1$, which, in analogy to the notation in the main text, is denoted by $f(w) := f^1(w)$. Then, extending Lemma 3 to more than two profitability states, we obtain that $f^i(w) = f(w) + \Delta^i$ for all $i \in \{2, ..., M\}$ with $\Delta^i := \frac{\mu^i - \mu^1}{r + \nu}$.

The optimal compensation policy is then again characterized by a threshold $\overline{w}$ solving $f'(\overline{w}) = -1$. Compensation is deferred until $\overline{w}$ is reached, where the contract starts paying out cash compensation. For $w \in [0, \overline{w}]$ the principal’s problem can then be written as

$$ (r + \nu) f(w) = \max_{\beta^i \geq -w, i} \left\{ \begin{array}{l} \mu^1 - I + \nu \sum_{i=1}^{M} p_i(I) \Delta^i + \frac{1}{2} \sigma^2 \lambda^2 f''(w) \\ + \left[ \gamma w - \nu \sum_{i=1}^{M} p_i(I) \beta^i \right] f'(w) \\ + \nu \sum_{i=1}^{M} p_i(I) f(w + \beta^i) \end{array} \right\} \tag{B.5} $$

s.t. (B.4)

where we already substituted the optimally binding truth-telling constraint $\alpha = \lambda$. The optimal contract is then characterized as the solution to (B.5) with the relevant boundary conditions as given by (13) to (15) in the main text.

**Optimal Investment.** In order to provide incentives for investment the contract has to specify some reward/punishment ($\beta^i$) depending on the investment outcome to satisfy incentive constraint (B.4). Taking first-order conditions in (B.5) and denoting by $\eta(w) \geq 0$ the Lagrange multiplier on (B.4), interior optimal values of $\beta^i(w)$ solve

$$ \frac{f'(w) - f'(w + \beta^i(w))}{p_i'(I)/p_i(I)} = \eta(w), \tag{B.6} $$

for $I = I(w)$, which is exactly condition (18) in the main text. Hence, the need to provide incentives (see (B.4)) then implies that the shadow costs of delegated investment $\eta(w)$ are strictly positive as long as signals are not perfectly informative – i.e., the likelihood ratio $p_i'(I)/p_i(I)$ is bounded for all $i$ – and rewarding or punishing the agent is costly, which for
$w < \overline{w}$ is reflected in the concavity of the value function. As the principal is risk averse with respect to variation in the agent’s compensation, $f''(w) < 0$, it is then optimal to reward the agent ($\beta^i(w) > 0$) for all $i$ with strictly positive likelihood ratio and punish ($\beta^i(w) < 0$) for all $i$ with strictly negative likelihood ratio, where the size of the respective reward and punishment is increasing in the informativeness of the respective performance signal as measured by the absolute value of the likelihood ratio. From MLRP (see Assumption B.2 part (i)) the agent’s compensation is, thus, increasing in the investment outcome (it is negative for $\mu^i$ small and positive for $\mu^i$ large).

In order to understand the subsequent analysis of investment distortions, it is now instructive to compare how the distribution of the (marginal) costs of providing incentives $|f'(w) - f'(w + \beta^i(w))|$ across investment outcomes $i = 1, \ldots, M$ changes with the implemented level of investment. To do so, note that from (B.6) the optimal incentive scheme for investment equalizes the expected (marginal) costs of providing incentives through rewards and punishment (akin to (19) for the two state case):

$$
\sum_{i \in \{j = 1, \ldots, M: p'_j(I) > 0\}} p_i(I) \left[ f'(w) - f'(w + \beta^i(w)) \right] = \sum_{i \in \{j = 1, \ldots, M: p'_j(I) \leq 0\}} p_i(I) \left[ f'(w + \beta^i(w)) - f'(w) \right]. \tag{B.7}
$$

Now, as investment increases, the implied FOSD shift in the outcome distribution (which follows from Assumption B.2 part (i)) directly implies that, under the optimal compensation policy, the costs of providing incentives upon realization of a low outcome triggering punishment (right-hand side) have to increase faster on average than the costs upon realization of a high outcome warranting a reward (left-hand side).\(^{48}\) Hence, the costs of providing incentives are optimally shifted from high towards low investment outcomes, which – while becoming less and less likely – are, thus, more and more costly upon realization. This insight will be crucial in understanding distortions in optimal investment to which we turn next.

\(^{48}\)Formally, we have

$$
\sum_{i \in \{j = 1, \ldots, M: p'_j(I) \leq 0\}} p_i(I) \frac{\partial [f'(w + \beta^i(w)) - f'(w)]}{\partial I} > \sum_{i \in \{j = 1, \ldots, M: p'_j(I) > 0\}} p_i(I) \frac{\partial [f'(w) - f'(w + \beta^i(w))]}{\partial I}.
$$
From (A.3) the optimal interior investment policy solves the following first-order condition:

$$\nu \sum_{i=1}^{M} p'_i(I) \Delta^i - 1 = \frac{\partial}{\partial I} \Phi(w, I(w)),$$

(B.8)

where the marginal agency costs of delegated investment are given by

$$\frac{\partial}{\partial I} \Phi(w, I) := -\eta \nu \sum_{i=1}^{M} p''_i(I) \beta^i(w)$$

$$+ \nu \sum_{i \in \{j=1, \ldots, M: p'_j(I) > 0\}} \int_{0}^{\beta^i} [f'(w) - f'(w + x)] dx$$

$$+ \nu \sum_{i \in \{j=1, \ldots, M: p'_j(I) \leq 0\}} \int_{0}^{\beta^i} [f'(w + x) - f'(w)] dx.$$  \hspace{1cm} (B.9)

When $\frac{\partial}{\partial I} \Phi(w, I(w)) = 0$, first-best investment obtains. From $\eta(w) \geq 0$ and (B.3) the first term in (B.9) is unambiguously positive. Intuitively, as long as there is a relevant incentive problem, more investment requires more costly incentives. By concavity of the value function the second term is also positive while the third term is negative. 49 Increasing investment increases the probability of the states for which costly rewards are paid, but it reduces the probability of those states for which the optimal compensation policy requires costly punishment. Now, recall from the discussion of the optimal compensation policy above that, as the implemented investment level increases, costs of providing incentives are more and more concentrated in low investment outcomes triggering punishment. Thus, the third term becomes increasingly important for more profitable investment technologies and eventually may even dominate such that $\frac{\partial}{\partial I} \Phi(w, I(w)) < 0$. In this case the optimal investment level is distorted upwards relative to first-best, while we have underinvestment else. The following Proposition formalizes these results and provides a straightforward extension of Proposition 2 to the case of an arbitrary number of states.

**Proposition B.3.** Fix $w \in (0, \bar{w})$ and suppose that Assumption B.2 holds. Then, as long

49In order to interpret these terms, note that the expected total costs of providing incentives through reward or punishment can be expressed as

$$p_n(I) \int_{0}^{\beta^i} |f'(w + x) - f'(w)| dx.$$

While optimal compensation $\beta^i$ is chosen to equalize expected marginal compensation costs (see B.7), optimal investment reflects the change in expected total compensation costs, arising from the investment-dependent FOSD shift in the outcome distribution (see B.9).
as the limited liability constraint is slack, optimal investment is distorted
(i) downwards, \( I(w) < I_{FB} \), for sufficiently low values of \( I_{FB} > 0 \),
(ii) upwards, \( I(w) > I_{FB} \), for sufficiently high values of \( I_{FB} < T \).

**Proof of Proposition B.3.** The proof is a straightforward extension of the proof of Proposition 2 and we therefore are brief. Assume for simplicity that the optimal investment \( I(w) \) solving (B.8) is unique. Clearly, by setting \( \beta^i(w) = 0 \) for all \( i \) such that the agent chooses \( I(w) = 0 \), we have \( \Phi(w, I) = 0 \), whereas \( \Phi(w, I) > 0 \) whenever interior \( I(w) > 0 \) has to be incentivized. It then is a direct implication of the continuity of \( \Phi(\cdot, I) \),\(^{50}\) that the marginal agency costs of investment \( \phi(w, I) \) are positive at \( I_{FB} \) whenever \( I_{FB} \) is sufficiently small and part (i) follows from (B.8). To show part (ii), observe, first, that by Assumption B.2 part (i), \( p_1(I) \) is strictly increasing as we approach \( \bar{I} \) such that together with Assumption B.2 part (ii) an investment level equal to \( \bar{I} \) can be incentivized via punishing the agent off-equilibrium for the realization of \( \mu^1 \). Hence, \( \Phi(w, \bar{I}) = 0 \) and the result follows from \( \Phi(w, I) > 0 \) for interior investment levels and continuity. **Q.E.D.**

As Proposition B.3 is a straightforward extension of Proposition 2 in the main text to more than two investment outcomes, it also follows the same robust economic intuition: Recall that investment induces a first order stochastic dominance shift in the output distribution (see Assumption B.2 (i)) such that high (low) investment outcomes are unlikely under the first-best investment policy when \( I_{FB} \) is sufficiently low (high). From (B.7), incentives for investment are then optimally provided via high reward (severe punishment) for these unlikely and, thus, in a likelihood-ratio sense very informative outcomes. As a consequence the realization of these outcomes induces a high variation in the agent’s continuation value which is costly to the principal, who therefore distorts investment downwards (upwards) in order to make the outcomes – which are particularly costly in terms of incentive provision – less likely.

We next comment on the sufficient conditions imposed to show the results in Proposition B.3. In doing so, we focus on the overinvestment case (part (ii) in Proposition B.3) for expositional clarity, the discussion of the underinvestment case (part (i) in Proposition B.3) is symmetric. Intuitively, overinvestment obtains for high \( I_{FB} \) as long as at this investment level bad investment outcomes are more informative in a likelihood ratio sense than good investment outcomes such that incentives are mainly provided via punishment for poor performance. Now, the conditions we impose ensure that the informativeness of the *worst* possible investment outcome – as captured by its likelihood ratio \( |p'_1(I)/p_1(I)| \) – becomes *arbitrarily large* as \( I \) approaches \( \bar{I} \) and eventually dominates that of all favourable

\(^{50}\)Continuity follows from Assumption B.2 (i) and (ii).
outcomes sufficiently such that incentives are optimally provided exclusively via punishment for bad performance. Accordingly, overinvestment obtains for $I_{FB}$ sufficiently close to $\bar{I}$. Clearly, whether the informativeness of one or multiple low outcomes (with $p'_i(I) < 0$) grows large as $I \to \bar{I}$ does not matter for the argument, so, we chose the conditions to illustrate that it is sufficient for one low outcome to become highly informative. Under MLRP, i.e., Assumption B.2 part (i), this is always the worst possible outcome, but for the overinvestment result to hold it could be any $i$ with $p'_i(I) < 0$. The key economic channel underlying the results in Proposition B.3 is that higher investment shifts probability mass from low to high investment outcomes (which is implied by Assumption B.2 part (i)). Assumption B.2 (ii) ensures that this shift is sufficiently strong at the extreme in that the worst possible outcome does no longer occur with positive probability if investment is sufficiently high, $p_1(\bar{I}) = 0$, such that the associated likelihood ratio goes to infinity in absolute terms.\(^{51}\) While this limit argument is needed to obtain analytical results,\(^{52}\) it is easy to show numerically that overinvestment for highly profitable investment technologies holds more generally. This is already apparent from our analysis with two investment outcomes $M = 2$ in the main text. In particular, as we established there in Example 1, overinvestment is not restricted to the limit as $p_1(I_{FB}) \to 0$, but instead holds as long as the low state is sufficiently unlikely given first-best investment, i.e., for all $I_{FB}$ with $p(I_{FB}) > 0.74$. That is, for overinvestment to occur, the probability of failure has to fall below 0.26.

This last observation points at a limit to the result that firms with high (low) returns to investment overinvest (underinvest) in intangible capital (see Proposition B.3). That is, the underlying, economically robust effect that higher investment expenditures shift probability mass from low to high investment outcomes, may not be strong enough to warrant a sufficiently asymmetric provision of investment incentives. In the following, we formalize this intuition and quantify a boundary to our result by imposing a lower bound on the probabilities of the most extreme investment outcomes which, in turn, restricts their relative informativeness.\(^{53}\) For simplicity, we assume that the problem is symmetric.

---

\(^{51}\)Note that Assumption B.2 (ii) is much weaker than the analogous condition for $M = 2$ (see page 9), as it does not restrict any moment of the outcome distribution, other than the probabilities of the 2 most extreme investment outcomes.

\(^{52}\)The reason is that the cost of providing incentives as captured by the curvature of the principal’s value function changes with the agent’s continuation value. This together with the fact that a closed-form solution to the principal’s value function does not exist already in much simpler settings without delegated investment, and, thus, has to be characterized implicitly based on the solution to a boundary value problem, makes an analysis away from the limit analytically intractable.

\(^{53}\)That is, we consider a violation of part (ii) of Assumption B.2, which captures the economically robust effect underlying our results. On the other hand, we refrain from relaxing part (i), which would most likely not yield any interesting insights, or part (iii), which is crucial to keep the problem tractable.
in the sense that \(|p'_i(I)| = p'(I) \forall i \in \{1, \ldots, M\}\).\footnote{Hence, we abstract from the numerator in the likelihood ratios \(p'_i/p_i\).}

**Example B.1.** Assume that \(M = 4\) and the respective probabilities are given by

\[
\begin{align*}
p_1(I) &= (a + \phi) - \phi \sqrt{I} \\
p_2(I) &= (a + \phi + x) - \phi \sqrt{I} \\
p_3(I) &= (b - \phi + x) + \phi \sqrt{I} \\
p_4(I) &= (b - \phi) + \phi \sqrt{I}
\end{align*}
\]

thus, implying that \(I = 1\), with \(p_1(I) = a \geq 0\) and \(p_M(0) = b \geq 0\). We further assume that \(x = 1/2 - (a + b)\), such that \(\sum_{i=1}^M p_i(I) = 1\), and \(a + b < 1/2\) for strict MLRP to hold. We stipulate that \(\phi = 1/6\) and all other parameters are as in the baseline specification in Example 1 in the main text. Then the following statements hold whenever the limited liability constraint is slack:

(i) If part (ii) of Assumption B.2 is satisfied, i.e., \(a = b = 0\), investment is distorted upwards (downwards) when \(I_{FB}\) is higher (lower) than the threshold 0.32.

(ii) Assume \(b = 0\). Then, if \(a > 0.18\), investment is distorted downwards for all \(I_{FB} > 0\).

(iii) Assume \(a = 0\). Then, if \(b > 0.23\), investment is distorted upwards for all \(I_{FB} < 1\).

Example B.1 nicely illustrates the robust intuition underlying our predictions on investment distortions. Investment is distorted upwards (downwards) if optimal incentives are predominately provided via costly punishment following unfavourable investment outcomes (reward following favourable investment outcomes). The choice of how to optimally provide incentives is in turn driven by the relative informativeness of favourable and unfavourable investment outcomes as captured by the respective likelihood ratios. Part (i) of Example B.1 corresponds to the benchmark case in which Assumption B.2 (ii) is satisfied and, thus, the worst (best) investment outcome will become extremely informative under sufficiently high (low) investment levels. Accordingly, if the returns to investment are sufficiently high (low), investment will be distorted above (below) first-best. Next, part (ii) of Example B.1 shows that if the probability of the worst investment outcome under the maximum possible investment level, \(p_1(I)\), is greater than 0.18, overinvestment will never arise under the optimal contract, no matter how profitable the investment technology is. The reason is that with \(p_1(I) > 0.18\), the relative informativeness of the lowest state (as
captured by its likelihood ratio relative to that of the highest state), remains sufficiently low, such that it is optimal to provide costly incentives mainly via rewards for favorable outcomes – even at very high levels of investment expenditures. This pushes the principal towards underinvestment in order to reduce the probability of favorable states and the associated incentive costs. Finally, part (iii) of Example B.1, considers the opposite case, in which the highest possible investment outcome occurs with a strictly positive probability even if the firm does not invest at all, i.e., \( p_M(0) > 0.23 \). In this case, the informativeness of favorable investment outcomes remains so low (relative to unfavorable outcomes), that incentives are mainly provided by costly punishment, even at low levels of investment. As a result, it is optimal to distort investment always above first-best (as long as the limited liability constraint is slack).

### B.3 Implementation of the Optimal Contract

In this Appendix we illustrate one particular implementation of the optimal contract as characterized in Section 3 of the main text. The implementation follows DeMarzo et al. (2012) and is based on cash reserves as a measure of financial slack, equity and a portfolio of derivative securities or insurance contracts.

In particular, the firm is equity financed and uses cash reserves to cover its short-term liquidity needs. Let \( M_t \) denote the level of cash reserves, earning interest \( r \). Cash reserves grow if cash flows are positive and they are used to cover operating losses, which corresponds to negative cash flows. Further, equity holders’ require a minimum dividend, given by

\[
dD_t = [\mu_t - I_t - (\gamma - r) M_t] dt, \tag{B.10}
\]

which is paid out of cash reserves \( M_t \). This minimum dividend comprises of the expected free cash flow \( \mu_t - I_t \) minus an adjustment factor that reflects the discounting difference \( \gamma - r \). If the firm fails to meet the minimum payout rate (B.10), or the cash holdings are exhausted, the manager is laid off, which is critical in providing incentives for the manager not to divert cash flows. Incentives for investment can then be provided by creating exposure of the firm’s financial slack \( M_t \) to the investment outcome, which we formalize through a portfolio of derivative securities contingent on the investment outcome.\(^{55}\) Derivatives are

\(^{55}\)Note that we stipulate that the investment outcome is a verifiable event. In our interpretation of investment into absorptive capacity (see footnote 10) an investment success could, e.g., be a patent granted to the firm and an investment failure a patent granted to a competitor. In this case, successes and failures would be reflected in the firm’s stock price, implying that the required exposure could be created by derivatives based on this underlying. For an alternative implementation in a setting with only downside risk see Biais et al. (2010). There the firm is requested to maintain an insurance contract against accident risk.
fairly priced given investors’ beliefs on the level of investment $I_t$. Concretely, we stipulate that holding a state-price security that pays one unit in case of an investment success (failure) incurs flow costs of $P^g = \nu p(I_t)$ ($P^b = \nu (1 - p(I_t))$), so that the instantaneous net payoff from holding such a security is given by $dS_t^j = dN_t^j - P_t^j dt$, for $j \in \{g, b\}$. We denote the number of securities of type $j \in \{g, b\}$ held by the firm at time $t$ by $n_t^j$, and require the firm to hold at any time a portfolio of size

$$n_t^j = \frac{\beta^j (\lambda M_t)}{\lambda}, \quad j \in \{g, b\},$$

(B.11)

of the respective security.\(^{56}\) Else, the manager is replaced. Other than that, the manager is free to choose investment and to distribute cash in form a special dividend $X$ at any time. The manager receives compensation in form of fraction $\lambda$ of this special dividend. Overall, the firm’s cash reserves, thus, follow

$$dM_t = rM_t dt + d\hat{Y}_t + n_t^g dS_t^g + n_t^b dS_t^b - dD_t - dX_t.$$  

(B.12)

When $M_t$ hits zero for the first time, the firm can no longer pay the minimum dividend $dD$ and, thus, goes into restructuring. In this process the incumbent manager is fired and equity holders realize a payoff corresponding to $L_\tau$, where $\tau$ denotes the first time at which $M_t$ falls to zero. The value of the firm’s equity claim is then given by

$$P(M_t, \mu_t) = E_t \left[ \int_t^\tau e^{-r(s-t)} (dD_s + (1 - \lambda) dX_s) + e^{-r(\tau-t)} L_\tau \right],$$

(B.13)

and we have the following result:

**Proposition B.4.** Suppose the firm has initial cash reserves $M_0$ and can operate as long as $M_t \geq 0$. When the manager is fired unless he maintains the minimum payout rate $dD_t$ and holds a derivative security portfolio $n_t^g$, $n_t^b$, it is optimal for him to refrain from diverting funds and to implement the optimal investment profile as characterized in Proposition 1. The firm accumulates cash $M_t$ until the threshold $M_{pb} := \overline{w}/\lambda$, and pays out cash in excess of this amount. Given this policy, the manager’s payoff is $w_t = \lambda M_t$, which coincides with the continuation value of Proposition 1, and the equity value satisfies $P(M_t, \mu_t) = f(\lambda M_t, \mu_t) + M_t$.

**Proof of Proposition B.4.** Under the proposed implementation, cash reserves evolve

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\(^{56}\)Note from $\beta^b < 0$ (see (19)), the firm holds a short position in security $b$, i.e., $n_t^b < 0$. 

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\[ dM_t = \gamma M_t dt + \left( d\hat{Y}_t - (\mu_t - I_t) dt \right) + \frac{\beta^g (\lambda M_t)}{\lambda} (dN^g_t - \nu p (I_t) dt) \\
+ \frac{\beta^b (\lambda M_t)}{\lambda} (dN^b_t - \nu (1 - p (I_t)) dt) - dX_t. \]

Now, define \( w_t = \lambda M_t \) to get

\[ dw_t = \lambda dM_t = \gamma w_t dt + \lambda \left( d\hat{Y}_t - (\mu_t - I_t) dt \right) + \frac{\beta^g (w_t)}{\lambda} (dN^g_t - \nu p (I_t) dt) \\
+ \frac{\beta^b (w_t)}{\lambda} (dN^b_t - \nu (1 - p (I_t)) dt) - \lambda dX_t. \]

Letting \( dU_t = \lambda dX_t \), incentive compatibility under the proposed implementation then follows from incentive compatibility of the optimal contract characterized in Proposition 1 and the agent’s value is given by \( w_t \). Note further, that the agent is indifferent as to when to issue the special dividend.

Next, consider the valuation of the equity claim, which follows from arguments similar to those in DeMarzo et al. (2012), in particular their Proposition 2. Substituting from (B.12), (B.13) can be written as

\[ P(M_t, \mu_t) = E_t \left[ \int_t^\tau e^{-r(s-t)} ((\mu_s - I_s) ds - dU_s) + e^{-r(\tau-t)} L_\tau \right] \\
+ E_t \left[ \int_t^\tau e^{-r(s-t)} (r M_s ds - dM_s) \right] \\
= f(W_t, \mu_t) + M_t = f^i(\lambda M_t) + M_t, \]

where we have used integration by parts. Q.E.D.

Note that in the implementation given in Proposition B.4, the state-price securities are not used to hedge against investment failure. By contrast, although firm value is a concave function of financial slack, the firm’s derivative position deliberately creates exposure of its financial position to the uncertain investment outcome in order to provide the appropriate incentives for investment. Let us also comment a bit more on the interpretation of the restructuring process triggered when \( M_t = 0 \). In this event, which could be interpreted as insolvency, the firm needs to raise cash from the capital market, which involves a fixed cost of \( k \). With the equity value net of the required cash injection given by \( f(\lambda M_t, \mu_t) \), the firm continues to operate with a new manager and initial cash reserves of \( M^* = w^*/\lambda \). Hence, we have \( P(0, \mu) = P(M^*, \mu) - k \), where we interpret \( k \) as the cost of raising external funds, which captures the key financing friction in our model.
B.4 Numerical Implementation

To solve numerically for the optimal contract of Proposition 1, we take the following iteration steps.

1. Solve for the principal’s value function \( f^{(0)} \), the replacement value \( L^{(0)} = \max_w f^{(0)}(w) - k \), and the free boundary \( \pi^{(0)} \) without technology shocks. That is, we solve the ODE in (12) with \( \nu = 0 \) (thus the initial investment is \( I^{(0)} = 0 \) and the initial rewards and punishments are \( \beta^{g(0)} = \beta^{b(0)} = 0 \)).

2. Given \( f^{(0)}, L^{(0)}, \pi^{(0)}, \) and \( \beta^{b(0)} \), update the optimal investment scheme \( I^{(1)} \) according to (16) and subject to incentive compatibility, i.e., \( \beta^g (I^{(1)}) = \beta^{b(0)} + \lambda / (\nu p'(I^{(1)})) \).

3. Given \( \beta^{g(0)}, \beta^{b(0)}, \) and \( I^{(1)} \), update the principal’s value function \( f^{(1)} \), the replacement value \( L^{(1)} \), and the free boundary \( \pi^{(1)} \).

4. Given \( f^{(1)}, L^{(1)}, \pi^{(1)}, \) and \( I^{(1)} \), update the optimal rewards and punishments \( \beta^{g(1)} \) and \( \beta^{b(1)} \) according to the first order condition (19), subject to the (binding) incentive constraint (8) and the limited liability constraint (9). That is, we solve (19) for \( \beta^{b(1)} \), such that \( \beta^{g(1)} = \beta^{b(1)} + \lambda / (\nu p'(I^{(1)})) \) and \( \beta^{b(1)} \geq w \).

5. Repeat steps 2 to 4 until the problem converges. The convergence criterion is

\[
\max \left[ \sup_w |I^{(i+1)} - I^{(i)}|, \sup_w |f^{(i+1)} - f^{(i)}| \right] < 10^{-5}.
\]