PHILOSOPHICAL TRANSACTIONS A

royalsocietypublishing.org/journal/rsta

Discussion

Cite this article: Muller FA. 2023 Aristotle on identity: close enough!. *Phil. Trans. R. Soc. A* **381**: 20220102.
https://doi.org/10.1098/rsta.2022.0102

Received: 23 January 2023
Accepted: 5 June 2023

One contribution of 12 to a theme issue 'Identity, individuality and indistinguishability in physics and mathematics'.

Subject Areas:
mathematical logic

Keywords:
Aristotle, identity, philosophy, equality, Leibniz

Author for correspondence:
F. A. Muller
e-mail: f.a.muller@uu.nl

Aristotle on identity: close enough!

F. A. Muller[1,2]

[1]Erasmus School of Philosophy, Erasmus University Rotterdam, Burg. Oudlaan 50, Rotterdam 3062 PA, The Netherlands
[2]Descartes Centre for the History and Philosophy of Science, Faculty of Science, Utrecht University, Utrecht, The Netherlands

FAM, 0000-0002-7343-4649

We present what Aristotle wrote on identity in a leisurely manner, which is much more than is generally known, save among the *cognoscenti* (Aristotle scholars), and *mutatis mutandis* about the introduction of the identity-symbol (=). We add two codas, one on the so-called Leibniz' Law, which is different (but resembles) what passes for it in logic and philosophy, and one on the status of identity, as accepted by mathematicians and logicians, in physics.

This article is part of the theme issue 'Identity, individuality and indistinguishability in physics and mathematics'.

1. Preamble

Why a paper nearly entirely devoted to what Aristotle wrote on identity? Because what Aristotle wrote on identity is undeservedly not generally known, except among Aristotle scholars, and perhaps historians of antique Greek philosophy, and historical justice demands that it should be generally known (§3). Aristotle nearly completely nailed it! This will lead us to what incorrectly passes for *Leibniz' Law* among logicians and philosophers, which is again something not generally known, except among Leibniz scholars (§4). We finally reflect on whether the universally accepted characterization of identity by logicians and mathematicians is, or should, also be universally accepted by physicists. We then inquire into whether the characterization of identity in logic and mathematics fits physics too (§5). We say goodbye with a list of ☛ take home messages, the harvest of a loudmouth eye (§6). We begin with an *amuse-gueule*, on the sign that symbolizes identity (§2).

THE ROYAL SOCIETY PUBLISHING

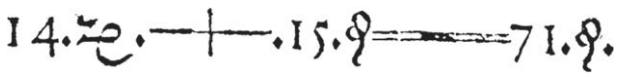

Figure 1. First equation printed ever with the symbol =.

2. Recorde on equality

The equality sign '=' (as well as the plus-sign '+', the minus-sign '–' and the word *algebra*) saw the light of day in the treatise *The Whetstone of Witte, whiche is the seconde parte of Arithmeteke: containe thextraction of Rootes; the Cossike practise, with the rule of Equation; and the workes of Surde Nombers* (London, 1557), written by Robert Recorde (1510–1558), in the days before Shakespeare:

> And to avoide the tediouse repetition of these woordes: is equalle to: I will sette as I doe often in woorke use, a paire of paralleles, of Gemowe (or twin) lines of one lengthe, thus: = = = = = =, bicause noe 2 thynges can be moare equalle.

The first equation printed ever, in Recorde's treatise, is this one ($14x + 15 = 71$, solve it):[1]

As Recorde points out, the form and shape of the *sign* '=' expresses the concept: two parallel bars of *equal* length and thickness. Brilliant. It took about no less than two centuries before this apt symbol was generally accepted. In the seventeenth century, Descartes and Leibniz also used the symbol '∞', and Newton wrote *eq.* to abbreviate *aequitatis* (so indeed Newton never wrote down his laws with this symbol: $\mathbf{F} = m\mathbf{a}$, $\mathbf{F}_{12} = -\mathbf{F}_{21}$, $F_g = Gm_1m_2/r^2$).

Robert Recorde was an Anglo–Welsh mathematician, entering Oxford University at the early age of 15, and 6 years later, in 1531, became a member of All Souls College. He climbed socially, eventually occupied various positions in the government, and became personal physician of King Edward VI and later of Queen Mary. After being sued for alleged defamation by political enemy Earl William Herbert, Recorde was arrested for debt, because he could not afford to pay Herbert financial compensation ($1000, about $250 000 today), as ordered by some local Tudor 'Court of Justice'. Recorde was locked up in the King's Bench Prison, Southwark, where he languished at the age of 48. That was the sad fate of the man who invented the identity-symbol (figure 1).

Robert Recorde, may you rest in peace.[2]

Recorde did not write anything *about* the concept of equality; he only introduced the sign that symbolizes the concept.

Since Frege's work, we consider equalities as identity statements applied to numbers; and we consider equations as identity statements applied to any mathematical objects. This makes equalities and equations identity statements. The symbol for equality and equation became, appropriately, also the symbol for identity in general.

3. Aristotle on identity

(a) Sameness

Aristotle of Stagyra is the first thinker on record to have thought about identity.[3] Most results of his thought about identity one finds in *Topics*, one of the six books of his survived work on logic and method *Organon*, and which is considered as one of Aristotle's early works; in other works, notably *Categories* and *Metaphysics*, one finds an occasional scattered remark on identity; his attention is occupied with the problem of diachronic identity and different meanings of 'sameness'.

[1]Downloaded from Wikipeda's entry on Robert Recorde, on 31 December 2022.

[2]Moore [1].

[3]White [2], the index in Barnes [3], the Kneale & Kneale [4], Pelletier [5], Gobbo [6] and Mariani [7] were helpful to find where Aristotle writes about identity.

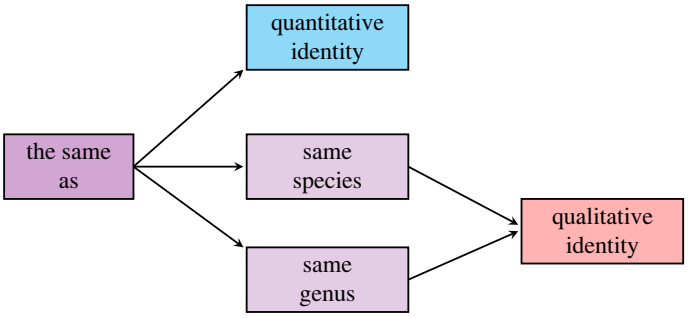

Figure 2. Aristotle's analysis of *the-same-as* in three types. Same-substance-as and same-genus-as fall under qualitative identity, i.e. sharing qualities.

Aristotle opens *Topics* (Book I, ch. 7, p. 171[4]) by distinguishing three meanings of *sameness* (our italics):

> First of all, we must determine the number of ways we talk of sameness. Sameness would be generally regarded as falling, roughly speaking, into three divisions. We generally apply the terms numerically or to species or to genus—*numerically in cases where there is more than one name but only one thing*, e.g. doublet and cloak.

Species identity is belonging to the same species, genus identity is belonging to the same genus, and numerical identity is the subject-matter of this paper. The italicized phrase expresses the idea of co-reference, which we shall turn to in §3b. Ch. 1, Book VII of *Topics* opens as follows (p. 255):

> Whether two things are the same or different, in the most strict meaning ascribed to 'sameness' (and we said that the same applies in the most strict sense to what is numerically one), may be examined in the light of their inflexions and coordinates and opposites.

In *Metaphysics* (Book V, ch. 15, p. 1612), Aristotle proposes to abjure the polynomous use of *the same*, and use *equal* for identity (numerically one):

> Those things are *the same* whose substance is one; those are *like* whose quality is one; those are *equal* whose quantity is one.

Each of the three meanings of 'the same' (figure 2) has its own opposite, or contrary. The relevant opposite of numerical identity, or quantitative equality as Aristotle puts it in the citation above, is *plurality* (also known as *numerical diversity*, and as *quantitative diversity*), which Aristotle mentions explicitly in *Metaphysics* (Book X, ch. 3, p. 1665, our italics):

> To the one belong, as we indicated graphically in our distinction of the contraries, the same and the like and the equal, and to *plurality* belong the other and the unlike and the unequal.

Aristotle gets even more refined about the different meanings of 'being one' in Book V, ch. 6 of *Metaphysics*, which we gloss over.[5]

[4]All page numbers referred to when citing Aristotle are to *The Complete Works of Aristotle*, 2 volumes, edited by J. Barnes (1995).

[5]Pelletier [5] argues that Aristotle revised some of his views on 'the same', as expounded in *Topics* (an early work), when he wrote *Metaphysics* (a later work).

(b) Co-reference

Back to *Topics*. In Book I, ch. 7 (p. 172), Aristotle points out that besides proper names, reference (Aristotle: indication) by means of contextual definite descriptions is also possible (again our italics):[6]

> For all these are meant to signify numerical unity. That what I have just said is true may best be seen where *one form of appelation is substituted for another*. For often when we give the order to call one of the people who are sitting down, indicating him by name, we change our *description*, whenever the person to whom we give the order happens not to understand us; he will, we think, understand us better from some accidental feature; so we bid him call to us *the man* who is sitting or who is conversing—clearly supposing ourselves to be indicating the same object by its name and by its accident.

That *substitution* of co-referring terms does not make a difference points in the direction of Leibniz' Substitution *Salva Veritate* Principle—pointing in the right direction should not be confused with arriving at a destination. That sentences of the form where the identity-sign is flanked by singular terms (proper names, Russellian definite descriptions) mean that these terms refer to one and the same object (recall also Aristotle's phrase 'there is more than one name but only one thing') has stood the test of time shiningly

$$t = s \text{ iff there is some object } o \text{ such that } t \text{ and } s \text{ both refer to } o. \tag{3.1}$$

Here, 'object' has to be taken in the logical sense of the word, devoid of any metaphysical commitments, and therefore encompassing every metaphysical category provided it makes sense to quantify over the members of the category. Elswhere in *Topics* (Book I, ch. 3, p. 1665), we find it again, tersely worded, and *perhaps* more general:

> Things whose names are many, but the thing is one, are the same in number.

Paraphrase:

> Suppose we have several things, and every thing has many names, then there are as many things as there groups of co-referring names.

As a general definition of identity, (3.1) has to be amended, because it leads to the by now very familiar trouble of making all non-referring identity sentences false, which does not make sense, in e.g. fiction—the rider of Pegasus = the victor of the monster Chimaera = Bellerophon *né* Hypponeus; who slayed the one-eyed giant Cyclops = who heard the Sirens sing and was not subjected to them = the King of Ithaca who married Penelope = who let the Greeks to victory by hiding in a gigantic wooden horse and opening the city gates of Troy in the dark of night = Odysseus; *etc.* We gloss over this issue, which was not any concern of Aristotle in his works where identity is discussed.

With his three meanings of 'the same' (§3a), Aristotle indisputably demonstrates awareness of the distinction between 'qualitative identity' and 'quantitative identity'. *Qualitative identity* is sharing a number of qualities (properties, attributes, accidents, belonging to the same species, to the same genus, even to the same substance if you like), by objects, as in when we say: These portraits are the same; whereas *quantitative identity* is essentially stating there is a single object, as in when we say: These are portraits of the same woman. Therefore, we can ascribe to Aristotle one of the most important insights concerning identity, namely the distinction between achronic quantitative identity and qualitative identity.

[6]Cf. Mariani [7], fn. 3, p. 108.

royalsocietypublishing.org/journal/rsta Phil. Trans. R. Soc. A 381: 20220102

(c) Properties of identity

We return to *Topics* (Book VII, ch. 1, p. 256), where Aristotle mentions (what we would now call) a conceptual truth about identity:

> Again, look and see if, (*a*) supposing the one to be the same as something, the other also is the same as it: (*b*) for if they be not both the same as the same thing, clearly neither are they the same as one another.

Let u, v and w be nominal variables. Rephrasing (*a*): suppose u is identical to some w, the other (which is also identical to w), say v, then v is identical to u

$$(u = w \wedge v = w) \longrightarrow v = u. \tag{3.2}$$

This is almost the same as Euclid of Alexandria's *Common Notion 1* in *The Elements*:[7]

> Things that are equal to the same thing are also equal to each other.

Using the same nominal variables

$$(u = w \wedge v = w) \longrightarrow (v = u \wedge u = v). \tag{3.3}$$

Kneale & Kneale [4, p. 42] see in (*a*) an expression of the transitivity of identity

$$(u = w \wedge w = v) \longrightarrow u = v. \tag{3.4}$$

Certainly the transitivity and the symmetry of identity,

$$\text{Symmetry}: \quad u = v \longrightarrow v = u, \tag{3.5}$$

jointly imply (3.3). In *Prior Analytics* (Book II, ch. 21), Aristotle also states Transitivity:[8]

> Similarly too in respect of being; for we have seen that if C is B, and B is A, then C is A.

In the same passage, we read something that strongly smells of the symmetry of identity (3.5):[9]

> Then since he thinks that B and C to be identical, he will also think that C is B.

Rephrasing (*b*) of Aristotle's citation above from *Topics*: if not both u and v are the same as w, then u and v are not both the same as w

$$\big((u = w \wedge v \neq w) \vee (u \neq w \wedge v = w)\big) \longrightarrow \neg(u = w \wedge v = w). \tag{3.6}$$

Or perhaps more succinctly, with appropriate quantifiers inserted: for every u and v, if u is identical to some w and v is not, then u is not:[10]

$$\forall u \forall v \exists w \big[(u = w \wedge v \neq w) \longrightarrow u \neq w\big]. \tag{3.7}$$

[7] Heath [8, p. 155].

[8] Barnes [3, p. 108].

[9] Hadgopoulos [9, p. 113] comments on this passage that it 'suggests to us the symmetry of identity, and that Aristotle apparently presupposed it, although what he writes above does not amount to a statement of the principle.' because Aristotle is indeed writing here about essences, not arbitrary objects; he seems to take symmetry for granted, yet he does mention it explicitly.

[10] Cf. White [2, p. 178].

(d) Substitutivity

In *Topics*, Aristole proceeds as follows (*ibid.*):

> Moreover, examine them in the light of their accidents or of the things of which they are accidents: (i) for any accident belonging to the one must belong also to the other, and if the one belong to anything as an accident, so must the other also. (ii) If in any of these respects there is a discrepancy, clearly they are not the same.

Let us begin with (ii). If u has an attribute (say A) that v lacks, so that there is a discrepancy with respect to A, then u and v are not the same. In the language of second-order logic (with 'A' as an attribute-variable)

$$\exists\, A[A(u) \wedge \neg A(v)] \longrightarrow u \neq v. \tag{3.8}$$

Which is logically equivalent to

$$u = v \longrightarrow \forall\, A\big[A(u) \longrightarrow A(v)\big]. \tag{3.9}$$

Aristotle's (i) adds the converse of the succedent of this necessary condition for identity to the necessary condition:

$$u = v \longrightarrow \forall\, A\big[A(u) \longleftrightarrow A(v)\big]. \tag{3.10}$$

Further on in *Topics*, Aristotle generalizes (ii) appropriately (*ibid.*):

> Speaking generally, one ought to be on the look-out for *any discrepancy anywhere in any sort of predicate* of each term, and in the things of which they are predicated. For all that is predicated of the one should be predicated also of the other, and *of whatever the one is a predicate, the other should be a predicate of it as well*.

When we let variable A in (3.10) range over *all monadic predicates* (including defined ones), we call (3.10) *Aristotle's Law (of Identity)*.

In *Physics* (Book III, ch. 3), Aristotle limits sharing all attributes to things ('in any way the same', hence *numerically* the same, hence identical) that exist:[11]

> For it is not things which are in any way the same that have all their attributes the same, but only those to be which is the same.

In *Topics* (Book V, ch. 4, p. 224), Aristotle harkens to a necessary condition for the identity of accidents (our italics):

> This commonplace rule is useful also in dealing with accident; for *the same attributes* ought either to belong or not to belong to *the same things* in so far as they are the same.

If 'they' refers to the attributes, we have co-extensiveness of predicates as a necessary condition for identity of predicates

$$A = B \longrightarrow \forall\, u\big[A(u) \longleftrightarrow B(u)\big]. \tag{3.11}$$

(e) Diachronic identity

Another concept of identity Aristotle is aware of is (what we now call) *diachronic identity*. In *Categories* (ch. V, p. 7), we read

> A substance, however, numerically one and the same, is able to receive contraries. For example, an individual man—one and the same—becomes pale at one time and dark at another, and hot and cold, and bad and good.

[11] Barnes [3, p. 345].

royalsocietypublishing.org/journal/rsta *Phil. Trans. R. Soc. A* **381**: 20220102

> For the same statement seems to be both true and false. Suppose, for example, that the statement that somebody is sitting is true; after he has got up, this statement will be false.

What makes Socrates who is wet at one moment in time and dry in another, at home in the morning and at the market place in the afternoon, drinking wine yesterday and hemlock today, the very same man? What if he were to lose both of his legs? Sharing accidental attributes obviously is not enough. Sharing essential attributes, like having the capacity of reason or being a biped, is not enough either. What else is there to share? Aristotle's Law (3.10) seems to fail. Aristotle wrestles with this problem in *Metaphysics* (Book VII) and solves it by an appeal to continuity (p. 1604):

> A thing is called continuous that has by its own nature one movement and cannot have any other; and the movement is one when it is indivisible, and indivisible in time.

In Book X, ch. 1 (p. 1662):

> A thing is of this sort because its movement is one and indivisible in place and time.

Charitably interpreted, according to Aristotle, enduring concrete objects, say u and v, at different times, say t and t', and occupying (not necessarily) different locations, say l and l', are *diachronically identical*, $\mathrm{DiaId}(u,l,t,v,l',t')$, iff u at l and t is connected to v at l' and t' by a continuous motion, presupposing that co-located concrete objects are a metaphysical impossibility (although a quantum-mechanical possibility). This criterion for diachronic identity can be made rigorous in the language of differential geometry; its core concept of a differentiable manifold is the standard mathematical representant of space–time in Physics. Since different manifolds represent different space–times, the diachronic identity predicate becomes septatic, with a differentiable-manifold-variable $\mathcal{M}$ added: $\mathrm{DiaId}(u,l,t,v,l',t',\mathcal{M})$. (Concerning quantum mechanics, the following. If two non-interacting particles of equal mass, spin and other super-selected properties, have the same state at some time, say $t=0$, their states coincide at all times, provided no measurements are performed on them; then their spatial probability measures are always equal, and there is a positive probability to find them upon measurement at exactly the same location. Co-located concrete objects are a quantum-mechanical possibility.) Currently, we call such enduring concrete objects *continuants*.[12] Yet note that this criterion for diachronic identity forces one to say that the melted lump of iron and the forged sword are identical concrete objects, to the sorrow of the knight, the squire and the blacksmith.[13] Then *synchronic identity* is by definition diachronic identity when the two times are set equal: $t=t'$. Synchronic identity is taken to be sufficient and necessary for *achronic* identity ($=$), in which the concept of time does not occur.

Currently, we take the failure of Aristotle's Law (3.10) to be the hallmark of extensionality, as opposed to intensionality. Sentences with propositional attitudes, like knowing-that, are one example of a type of intensional sentences. The struggles of Frege, Russell, Hintikka and others of how to deal with such intensional sentences is well-known. In an Appendix to *Topics*, called *Sophistical Refutations* (ch. 24, p. 305), Aristotle expresses awareness, dimly or clearly, that his law fails for this type of intensional sentences:

> For only to things that indistinguishable and one in substance does it seem that all the same attributes belong; (. . .) nor in the case of a man approaching, or wearing a mask, is to approach the same thing as to be Coriscus, so that I know Corsicus, but do not know the man who is approaching, it still isn't the case that I both know and do know the same man. (. . .) To know and not to know the same thing is thought to be possible, e.g. that one knows

[12]Cf. Miller [10, pp. 488–490].

[13]Cf. Wiggins [11, p. 9].

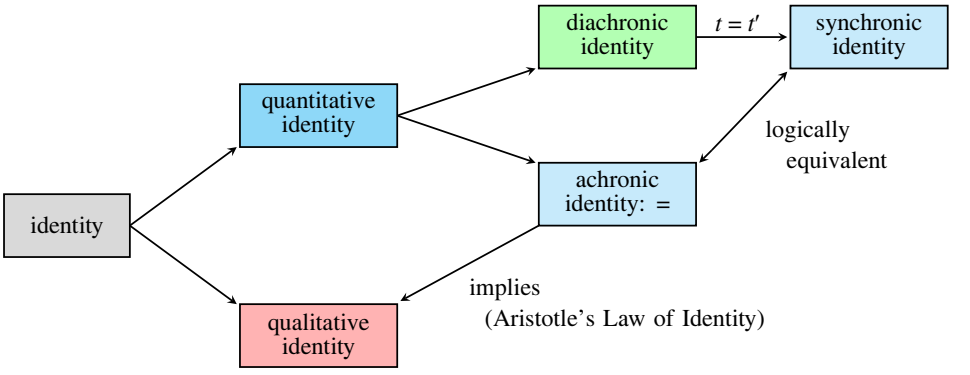

Figure 3. Extracted from Aristotle's works, save the nowhere explicitly mentioned synchronic identity, which is a special case of diachronic identity; synchronic identity is logically equivalent to achronic identity (=).

> that he is what but does not realise that he is musical; for in that way he does know and does not know the same thing though not in the same respect.

The restriction that Aristotle proposes on the scope of his law (3.10), in terms of substances, is however no longer our solution.[14]

We summarize Aristotle's remarkable insights into identity in figure 3.

(f) Identity in logic

Frege provided the canonical characterization of identity in *Begriffschrift* (transformed here into the familiar language of first-order logic):[15]

$$\left.\begin{aligned} \text{reflexivity}: &\quad \vdash \forall u[u = u]. \\ \text{substitutivity}: &\quad \vdash \forall u \forall v\big[\big(u = v \wedge \varphi(u)\big) \longrightarrow \varphi(v)\big]. \end{aligned}\right\} \tag{3.12}$$

Let binary relation R be a *congruence relation* iff the following schema holds for R:

$$\forall u \forall v\big[R(u, v) \longrightarrow \big(\varphi(u) \longrightarrow \varphi(v)\big)\big]. \tag{3.13}$$

Axioms (3.12) imply that identity is (up to logical equivalence) the one and only binary relation that is both an equivalence relation and a congruence relation (Exercise). Identity is the unique relation that relates *every* object to itself and to no other object—which makes a curious *relation*, never *relating two things*. These axioms also imply Euclid's Common Notion 1 (3.3) and Aristotle's other claims (3.8) and (3.9) about identity.

Substitutivity (3.12), which essentially is Aristotle's Law of Identity (3.10), is frequently called 'Leibniz' Law'. Add Aristotle's symmetry (3.5) and transitivity (3.4) and we come very close to the canonical characterization (3.12). Yet rarely is Aristotle mentioned for this. More than half a century ago, Kneale & Kneale [4, p. 42] deplored that Aristotle generally does not get credit for his insights about identity. I continue deploring it. Any logician out there listening?

[14]Cf. White [2, pp. 179–180]), Barnes [12, §5]), Pelletier [5].

[15]In Frege [13], formulae (52) and (57) express Substitutivity, and formula (54) expresses Reflexivity; formula (55) expresses Symmetry, and, to the best of my knowledge, no formula in *Begriffsschrift*, expresses Transitivity. Frege deduces all these from more basic principles.

4. Coda on Leibniz

Rather than Aristotle, Leibniz is frequently credited for Substitutivity (3.12), sometimes put in ungrammatical English as 'the principle of indiscernibility of identicals':

$$\vdash \forall u \forall v \big[u = v \longrightarrow \big(\varphi(u) \longleftrightarrow \varphi(v)\big)\big], \tag{4.1}$$

which also follows from identity axioms (3.12). Indeed, all over the place in Leibniz's works one finds statements connecting identity to substitutivity. *Salva veritate!* In *A Specimen of the Universal Calculus* (1657–1686), we find:[16]

> (7) Two terms are 'the same' of which one can be substituted everywhere for the other without loss of truth, such as 'trilateral' and 'triangle'.

In *General Inquiries about the Analysis of Concepts and of Truths* (1686):[17]

> That *A* is the same as *B means* that one can be substituted for the other in any proposition without the loss of truth.

In *A Study in the Plus-Minus Calculus* (1690):[18]

Definition 4.1. Those terms are 'the same' of which one can be substituted for the other without loss of truth.

When t and r are singular terms, it seems we have here the following:

$$t = r \text{ iff } \big[\mathrm{Tr}(\varphi(t)) \text{iff } \mathrm{Tr}(\varphi(r))\big]. \tag{4.2}$$

Indeed, White [2, p. 178] calls Aristotle's Law (3.10) 'a version of Leibniz's Law'. Feldman [15, p. 15] provides the following citation from Tarski's *Introduction to Logic* (1941) as the possible origin of identifying Aristotle's Law for Leibniz's Subsitution *Salva Veritate* Principle:

> Among the logical laws concerning the concept of identity the most fundamental is the following:
>
> $x = y$ if, and only if, x has every property which y has, and y has every property which x has.
>
> This law was first stated by LEIBNIZ (although in somewhat different terms) and hence may be called LEIBNIZ' LAW.

In Van Dalen's *Logic and Structure* [16, p. 149], we encounter the following second-order definition of identity, going back to Whitehead and Russell's *Principia Mathematica*:

> **Definition 4.6 (Leibniz-identity).** $x = y := \forall X(X(x) \longleftrightarrow X(y))$.

In *Logic for Philosophy* [17, p. 159], Sider mentions that the conjunction of Aristotle's Law (3.10) and its converse (Leibniz's Principle of the Identity of Indisernibles) 'are sometimes collectively called "Leibniz' Law".' The same holds for Pelletier [5], Gobbo [6] and Mariani [7].[19] All this terminology is however historically erroneous.

To begin with, Leibniz speaks of *salva veritate*, whereas in Aristotle's Law the concept of truth does not occur. Minor detail. Decisive difference is that what Leibniz means by 'term' is not what we currently understand by that term in Logic.

[16] Parkinson [14, p. 34].

[17] Parkinson [14, p. 52].

[18] Parkinson [14, p. 122].

[19] In my PhD thesis, *Structures for Everyone* [1998, p. 21], I called it 'Leibniz' Principle'.

Leibniz held, as one of the first, that all propositions are of the subject-predicate form: saying something (predicate) about something else (subject). In *De Arte Combinatoria* (1666), we read:[20]

> A proposition is composed of subject and predicate; all propositions, therefore, are com2nations.

For Leibniz, terms were often concepts. In *Specimen*, we find:[21]

> (10) By 'term' I do not understand a name, but a concept, that which is signified by a name; you could also call it a notion, an idea.

An unrestricted Substitutivity *Salve Veritate* Principle is what Leibniz advances as an identity criterion for *concepts*, not for objects:

> Two concepts are identical iff they can be substituted for each other in *every* proposition without loss of truth.

If we replace 'concepts' with 'singular terms', the ensuing principle has to be restricted to extensional propositions, for a singular-term version notoriously fails for intensional propositions (modal propositions, propositional attitudes); the ensuing principle can even be used as a criterion to distinguish between extensional and intensional propositions. (We have seen that Aristotle, remarkably, was aware of this.)

That Leibniz' Subtitution *Salve Veritate* Principle has never been propounded by Leibniz for singular terms but for concepts may too not be as widely known as it deserves to be known. Among Leibniz scholars this is however common and well-trodden ground.[22] Worth mentioning is that Feldman [15, pp. 520–521] has shown that a 'restricted version of' Aristotle's Law and its converse do follow from Leibniz's Substitution *Salva Veritate* Principle in combination with his Principle of Complete Concepts.

5. Coda on physics

Equality, equation, identity was logically characterized with eyes wide open to mathematics. Frege argued successfully that logic is the proper home for identity. The assumed universal applicability and topic-neutrality of logic then implies the universal applicability and topic-neutrality of the concept of identity. Provided identity conditions can be found for objects of a given category, the identity relation applies to all things belonging to that category. The recent wave of papers and PhD theses in philosophy of physics about indiscernibility and the status of the Principle of the Identity of Indiscernibles (PIdIn) have *stricto sensu* nothing to do with identity *an sich*, which is taken for granted and considered taken care of by logic, but the wave has everything with indiscernibility and PIdIn.

Since in the current paper, we are riding the high horse Historical Justice, we better point out that not Leibniz was the first to state PIdIn, but perhaps the Roman Pyrronic sceptic Sextus Empiricus (*ca.* 150–220) or perhaps some Roman Stoic, about 15 centuries before Leibniz saw the light of day. In Book VII *Against the Logicians* of *Principles of Pyrronism*, Empiricus raises the issue of whether indiscernibles, which thus make indiscernible sensory impressions on us, exist or can exist:

> For instance, if there are *two eggs exactly alike*, and I give them to the **Stoic** one after the other, will the wise person, after fastening upon them, have the capacity to say infallibly

[20]Parkinson [14, p. 3]). The '2' in 'com2nations' is not a typo: Leibniz signals thereby that a proposition is composed of *two* concepts.

[21]Parkinson [14, p. 39].

[22]Parkinson [18], [14, Introduction], Ishiguro [19, ch. II], Castañeda [20], cf. Feldman [15].

> whether the egg he is being shown is a single one, or the one and then the other? The same argument also applies in the case of twins. For the superior person will grasp a false appearance, even though he has the appearance as from a real thing, and stamped and impressed in accordance with just that real thing, if he gets an appearance of Castor as if from Polydeuces.[23]

Historian of philosophy Ierodiakonou [21, p. 37] ascribes a version of PIdIn to the Stoics:

> That is to say, the Sceptics' thought experiment asked the Stoics to imagine what would happen, if they were first shown one egg and then another identical one, or one man and then his identical twin brother; would they be able to distinguish them as two different eggs or two different brothers? The Sceptics suggested that it would not be possible to distinguish them, and thus the Stoic wise man may have false impressions though he considers them as true; for instance, it may be the case that he thinks he has a cognitive impression of Polydeuces, when in reality he looks at Castor. The Stoics, however, replied that, on the basis of their *doctrine of indiscernibility*, according to which *there are no two things that are exactly alike*, it is always possible for the wise man, at least in principle, to distinguish between two eggs or between twin brothers.

Yet no one in the history of thought employed and applied PIdIn with such fervour and consequence as Leibniz. Rodriguez-Peyera [22, pp. 15–20] has listed no less than 36 finding places of PIdIn in Leibniz's labyrinthian *opera*.

If there are physical entities that violate PIdIn, then there are no identity conditions for these entities, and then the provision is not met and the application of the identity relation to those physical entities is balderdash. In *quasi-set theory*, this is indeed what has happened.[24] The identity relation does not apply to 'micro-physical objects', with the indiscernible elementary particles and gauge bosons of quantum theory as the prime candidates for the micro-physical objects of quasi-set theory. The formation rules of the formal language of quasi-set theory turn $a = b$ into a badly formed formula, a sequence of symbols that is not a syntactically well-formed expression. For these micrio-physical objects, quasi-set-theory employs a primitive indiscernibility relation ($\equiv$). The identity relation does apply to macro-physical objects. In other logical approaches, e.g. Schrödinger logics, identity is no longer reflexive, one has $x \neq x$. Identity then does apply to elementary particles of the same kind, but the negated self-identity statements are always false.[25]

These very possibilities do not yet answer the question whether the identity relation as characterized by the Frege axioms (3.12) is appropriate whenever it applies to physical objects, although both quasi-set theory and non-reflexive logics have renounced identity.

Physics is however stuffed with equations, hence stuffed with identity statements. But these are all identity statements between mathematical entities, not between physical entities. Identity statements about physical entities hardly occur in physics. Suppose physicist Francesca says she considers two electrons (numerical diversity). Frequently physicists called 'identical particles', by which is meant indiscernible or indistinguishable particles. They do not mean 'identical' in the sense of '$=$' (no numerical diversity), and Francesca will not say this, for otherwise she would contradict herself in a single breath of air. Francesca will also not say that the two electrons are not identical ($\neq$), because why utter this banality? She has already said to consider *two* electrons. If Francesca were to say she considers a single electron, then she will, again, not go on to say that this single particle is identical to itself. For, again, uttering banalities is not what physicists do. But let us, in spite of contradiction and banal tautology, ponder the question whether the Frege axioms are appropriate for the identity relation between physical objects to which it meaningfully applies.

[23]Translation quoted in Ierodiakonou [21, p. 37].

[24]See French & Krause [23], §7.2 for quasi-set theory.

[25]See French & Krause [23], ch. 8 for such non-reflexive logics.

The answer to the question whether physicists will subscribe to the Frege axioms (3.12) for physical objects is in the affirmative: for every single physical object is identical to itself (banal Reflexivity), and, as soon as one says something about some physical object, then we must say the same about an identical object, because that object is the very same object we started with (substitutivity). Physics can make do perfectly with Frege's characterization of identity (3.12), unless there are indiscernibles, which is something that no physical theory implies, but which is something that can be chosen for on metaphysical grounds, due to French's second UnderDetermination Thesis: the physics underdetermines the metaphysics.

6. Take home messages

If this contribution is going to have any effect, then by taking home the following messages.

- ☛ Aristotle is not only the creator of Syllogistic Logic, and thereby one of the founding fathers of Logic, but also the first to acknowledge the concept of identity and think about it.
- ☛ Aristotle distinguished quantitative, achronic identity from diachronic, qualitative identity.
- ☛ Aristotle understood statements of numerical identity as expressing that two singular terms co-refer (3.1).
- ☛ Aristotle advanced, as the first on record, a number of conceptual truths about identity, among which Euclid's common notion 1 (3.3) of his *The Elements*, symmetry (3.5), transitivity (3.4), and (3.8) and (3.9).
- ☛ Aristotle held that identity statements imply the substitutivity of the terms in propositions; we propose to call Substitutivity (3.10): *Aristotle's Law of Identity*.
- ☛ Aristotle was aware that his Law of Identity does not hold unrestrictedly: in sentences that contain epistemic propositional attitudes, it fails—this casts a long shadow forward to the discovery of the distinction between extensional and intensional propositions.
- ☛ Aristotle advanced co-extensiveness of predicates as a necessary condition for their identity (3.11).
- ☛ Leibniz's Substitutivity *Salve Veritate* Principle (4.2) is about *concepts*, not about *objects*, like Aristotle's Law of Identity (3.10).

Admittedly Aristotle did not expound every one of these ☛ insights with the generality and clarity as Logic presents them nowadays, let alone that he shared the same background assumptions about predicates and possessed the same concepts as we currently have in Logic (quantifiers, variables, definite descriptions), as the subtle scholarly papers of Miller [10], Barnes [12], Pelletier [5], Gobbo [6] and Mariani [7] bear testimony. True enough. Yet, based on what Aristotle mostly wrote in *Topics*, and moreover based on various conclusions drawn in the papers just mentioned, I loudly say: *Close enough!*

Data accessibility. This article has no additional data.

Author's contributions. F.A.M.: conceptualization, writing—original draft, writing—review and editing.

Conflict of interest declaration. I declare I have no competing interests.

Funding. No funding has been received for this article.

Acknowledgements. Thanks to Humber van Straalen and two critical referees for suggestions and corrections.

References

1. Moore D. 1990 Robert recorde: the Welshman who invented the equals sign. *The International for all of Wales*, 5 February 1990.
2. White NP. 1971 Aristotle on sameness and oneness. *Phil. Rev.* **80**, 177–197. (doi:10.2307/2184029)
3. Barnes J (ed.). 1995 *The complete works of Aristotle*, vol. 2. Princeton, NJ: Princeton University Press. 1984, sixth printing with corrections.
4. Kneale W, Kneale M. 1962 *The Development of Logic*. Oxford, UK: Oxford University Press.

5. Pelletier FJ. 1979 Sameness and referential opacity in Aristotle. *Noûs* **13**, 283–311. (doi:10.2307/2215102)
6. Gobbo E. 2000 Identity and the predicables in Aristotle's topics. *Topoi* **19**, 83–98. (doi:10.1023/A:1006464706810)
7. Mariani M. 2000 Numerical identity and accidental predication in Aristotle. *Topoi* **19**, 99–110. (doi:10.1023/A:1006472923648)
8. Heath TL. 1968 *The thirteen books of Euclid's elements*, vol. 1. Cambridge, UK: Cambridge University Press. Translated from the text of Heiberg with Introduction and Commentary.
9. Hadgopoulos DJ. 1974 A note on Aristotle's theory of identity. *Phil. Phenomenol. Res.* **35**, 113–114. (doi:10.2307/2106606)
10. Miller FD. 1973 Did Aristotle have the concept of identity? *Phil. Rev.* **82**, 483–490. (doi:10.2307/2183712)
11. Wiggins D. 2012 Identity, individuation and substance. *Eur. J. Phil.* **20**, 1–25. (doi:10.1111/j.1468-0378.2012.00516.x)
12. Barnes KT. 1977 Aristotle on identity and its problems. *Phronesis* **22**, 48–62. (doi:10.1163/156852877X00173)
13. Frege G. 1972 *Conceptual notation and related articles*. Oxford, UK: Clarendon Press. Translated and Edited with a Bibliography and an Introduction by T.W. Bynum.
14. Parkinson GHR (ed.). 1966 *Leibniz' logical papers. A selection translated and edited with an introduction*. Oxford, UK: Clarendon Press.
15. Feldman F. 1970 Leibniz and Leibniz' law. *Phil. Rev.* **79**, 510–522. (doi:10.2307/2184291)
16. van Dalen D. 2004 *Logic and structure*. Berlin, Germany: Springer.
17. Sider T. 2009 *Logic for philosophy*. Oxford, UK: Oxford University Press.
18. Parkinson GHR. 1965 *Logic and reality in Leibniz' metaphysics*. Oxford, UK: Clarendon Press.
19. Ishiguro H. 1990 *Leibniz's philosophy of logic and language*, 2nd edn. Cambridge, UK: Cambridge University Press.
20. Castañeda H-N. 1974 Leibniz's concepts and their coincidence *Salva Veritate*. *Noûs* **8**, 381–398.
21. Ierodiakonou K. 2018 The triple life of ancient thought experiments. In *Routledge companion to thought experiments* (eds MT Stuart, Y Fehige, JR Brown), pp. 31–43. London, UK and New York, NY: Routledge.
22. Rodriguez-Peyera R. 2014 *Leibniz's principle of identity of indiscernibles*. Oxford, UK: Oxford University Press.
23. French S, Krause D. 2008 *Identity in physics: a historical, physical and formal analysis*. Oxford, UK: Clarendon Press.