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Modeling and Solving Line Planning with Mode Choice

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Abstract. We present a mixed-integer linear program (MILP) for line planning with mode and route choice. In contrast to existing approaches, the mode and route decisions of passengers are modeled depending on the line plan and commercial solvers can be applied to solve the corresponding MILP. The model aims at finding line plans that maximize the profit for the railway operator while estimating the corresponding passenger demand with aggregate choice models. Hence, the resulting line plans are not only profitable for operators but also attractive to passengers. By suitable preprocessing we are able to apply any aggregate choice model for mode choices using linear constraints. We provide and test means to improve the computational performance. Our experiments on the Intercity network of the Randstad, a metropolitan area in the Netherlands, show that models which assume a fixed passenger assignment to modes cannot attract the passenger numbers they were designed for and therefore lead to inferior profit compared with our new model.

Supplemental Material: The online appendices are available at https://doi.org/10.1287/trsc.2022.1171.

Keywords: public transport optimization • line planning • passenger demand estimation • mode choice • route choice • logit model

1. Introduction

Public transport operators face changes in travel demand due to a variety of reasons. Some of these changes are temporary in nature, like events attracting visitors, weather conditions that make passengers favor public transport over alternative modes, or temporary government measures, like home-working policies and travel restrictions as they were issued during the course of the COVID-19 pandemic. Permanent changes in demand have their origin in the development of regions, long-term policy measures, or changes in passenger interests and behavior. For example, in the Netherlands, it is expected that working at home will become more common also after the COVID-19 pandemic (*NL Times* 2021).

To adapt their service to small or temporary fluctuations, operators can make adjustments on the level of tactical planning, like, for example, small adaptions in the timetables and rolling stock schedules. However, such adjustments are not suitable to cope with greater and longer-term changes in demand. Instead, this issue needs to be approached from a strategic planning perspective and the line plan needs to be adjusted from time to time.

When designing a line plan, it is important to distinguish between passenger and traveler demand. With *traveler demand*, we refer to the total number of people who want to travel. Travelers may choose to use any available mode of transport, such as railway or car. *Passenger demand*, in our paper, includes only those travelers who choose to travel by railway. Throughout this paper, we refer to the public transport mode in question as *railway*, but the proposed model can be applied to public transport services of any kind. The decision of travelers for their mode of transport, and thus the number of passengers, depends to a certain extent on the quality of the railway service offered. This poses an interesting yet complex situation for operators: railway services must be designed to provide sufficient capacity for passenger demand, which in turn depends on the service.

In this paper, we consider the problem of finding a line plan and simultaneously estimating the corresponding passenger demand based on a prognosis for traveler demand. The aim is to estimate both the share of travelers deciding to use the railway (mode choice) and the passenger distribution in the network (route choice). The travelers' choices depend on the quality of the service offered: they value a service with fast, direct, and frequent connections. A line plan offering such connections between two stations will attract more passengers between those stations, while passengers between stations with slow and infrequent connections will be inclined to turn to other modes of transport. Considering travelers' decisions allows an accurate estimation of passenger demand during optimization, and the resulting line plans are aligned with the demand they generate.

Although many approaches state that passenger demand and line plans are interdependent, we identify two reasons why demand estimation is mostly not modeled accurately. First, the travelers' mode choices are in most cases neglected. Second, if a passenger distribution on routes is considered, usually one of the following two simplifications is applied: either all passengers traveling between two stations are required to use the same route, or the model can assign passengers to routes in favor of a system optimum rather than considering the choices that passengers would make. An imprecise demand estimate is obstructive to the search for efficient line plans and carries the risk of insufficient seating capacity. Only a few publications deal with the integration of passenger choice models in line planning. However, these approaches are usually not computationally tractable, and the quality of solutions found with heuristic approaches is hard to assess. In Section 2, we discuss the related line planning literature in detail.

In our paper, we present a mixed-integer linear programming (MILP) formulation for finding a line plan and corresponding passenger loads from given traveler demand. The passenger loads depend on the utilities of available routes provided by the line plan. To estimate the passenger mode choice, we apply a logit model, but our framework allows use of other aggregate choice models. We then assume that passengers distribute over the best available routes. By suitable preprocessing of the utilities for the passengers' mode and route decisions, the logit model can be linearized, and commercial solvers can be used to find solutions. Approaching the problem from the operator's perspective, our objective is profit maximization, where profit is equal to the revenue from serving passenger demand minus the operating costs. This yields line plans that attract many passengers while being efficient concerning operational costs.

We test and analyze the model in experiments on the Intercity network of the Randstad region that is operated by NS, the largest Dutch railway operator. Additional constraints and branching strategies are tested to improve the computational performance. The model is compared with a basic line planning model with predetermined passenger loads, which highlights the advantages of demand estimation during optimization. Furthermore, the integration of passenger decision models enables to conduct a sensitivity analysis of the service level and operator profit on fluctuations in traveler demand. This gives valuable insights for operators into their business models and the optimal solutions provide concepts for how the line plans should be adjusted in response to demand changes.

Summarizing, our main contributions are fourfold. First, we present a novel line planning model that considers both route and mode choice from a customer's perspective. In contrast to existing optimization approaches, passengers are not assigned to routes according to a system optimum but distribute on the best routes in our model. Second, we develop a linear formulation for this model that allows the use of commercial solvers, and we provide means to improve the computational performance. Third, we show in experiments that operators should include an estimation of mode and route choice *during* optimization to achieve the best possible profit and passenger shares. Fourth, we show the impact of drastic changes in travel demand on the modal share and the financial performance of the railway operator.

The remainder of this paper is structured as follows. In Section 2, related line planning approaches are summarized. The modeling of line planning and passenger demand estimation is described in Section 3. This section discusses the used choice models in detail and the assumptions made to linearize them. Section 4 gives information about the experimental setup, used data sets, and parameters. The experiments are described and discussed in Section 5, followed by practical insights for operators in Section 6. The paper concludes in Section 7 with a summary of the findings.

2. Related Literature

2.1. Line Planning

The goal of the line planning problem is to find a set of lines with corresponding frequencies such that conditions on operating costs and passenger service level are satisfied. In this context, lines are defined as a sequence of stations that are served by a vehicle. Schöbel (2012) summarizes different modeling approaches and solution methods for the line planning problem in public transport and identifies several variations: In some formulations, the task is to select lines from a given pool of lines (Gattermann, Harbering, and Schöbel 2017), whereas in others, the line routes are constructed during optimization (Borndörfer, Grötschel, and Pfetsch 2007). There also exist different objectives for line planning. On the one hand, cost-oriented objectives aim at minimizing operational costs while ensuring a certain passenger service level (Claessens, van Dijk, and Zwaneveld 1998, Goossens, van Hoesel, and Kroon 2006, Friedrich et al. 2017). On the other hand, passenger-oriented objectives mostly consider a budget constraint on operational costs and maximize passenger service level, represented by the share of passengers with direct connections (Bussieck, Kreuzer, and Zimmermann 1997, Bussieck 1998, Schöbel and Scholl 2006) or by passenger journey times (Schöbel and Scholl 2006, Goerigk and Schmidt 2017).

The impact of the service on passengers is often neglected. Most of the existing approaches have the assumption in common that (an estimate of) the passenger demand is known before the line plan is found. This means, the number of passengers between each station pair is assumed to be fixed. In addition, passengers are in many cases assigned a priori to routes in the network to estimate the required capacity between stations. However, both the number of passengers and the passenger routes depend on the line plan and the corresponding passenger service level (de Dios Ortúzar and Willumsen 2011).

2.2. Choice Modeling

Discrete choice theory provides many approaches to predict choices of humans based on the utility of alternatives (Ben-Akiva and Lerman 1985). There is a large variety of discrete choice models developed especially for transport related applications. In these models the choices of travelers are aggregated to estimate, for example, the number of trips of travelers, their destinations, or their mode and route choices (de Dios Ortúzar and Willumsen 2011). Because most research in discrete choice modeling for transport systems considers a high level of detail to obtain estimates that are as precise as possible, these models are not suitable to be directly applied within optimization approaches.

Nevertheless, many public transport optimization approaches deal with passenger demand modeling dependent on the solution. The first approaches assumed that all travelers choose the same objectively best alternative, for example, in a shortest path routing integrated into timetabling as done in Schmidt and Schöbel (2015b), Gattermann et al. (2016), Schiewe and Schöbel (2020). This all-or-nothing approach does not model passenger behavior perfectly and especially reaches its limitations in case multiple comparable best alternatives are available, but it accounts for the most pervasive choice dynamics at manageable increasing complexity.

To improve the estimate of passenger choices, some approaches incorporated a binary logit model, which serves as the basis for many discrete choice models used to estimate mode or route choices. We are aware of three ways to avoid the high complexity of integrating choice models into optimization models. First, it is iterated between the estimation of passenger choices and the optimization of public transport supply. For example, Nachtigall (1998) and Siebert and Goerigk (2013) experiment with repeatedly finding a public transport timetable for a fixed passenger assignment to routes and assigning passengers to routes given the new timetable according to any desired choice model. Although this approach avoids complex integration of choices, it is not clear whether the iterative process finds an optimum or converges at all. Second, the choice models are included in the optimization approach, but linearized choice models are used in the solution step. For example, Cordone and Redaelli (2011) replace the nonlinear logit function by a piecewise linear overestimate to find public transport timetables maximizing public transport ridership. Established linearizations of the logit model are summarized in Haase and Müller (2014). Third, simulation approaches can be used to efficiently estimate passenger choices according to choice models within optimization models (Train 2009). The basic idea is to consider multiple scenarios and modify the utility of the alternatives with some random error term in each scenario. By drawing the error terms from certain distributions and averaging passenger decisions over the scenarios, certain choice models can be approximated. For example, Hartleb and Schmidt (2021) propose a simulation approach for estimating passenger route choice according to a logit model within public transport timetabling and compare it with multiple other timetabling approaches.

2.3. Route and Mode Choice in Line Planning

Also for line planning, many models were developed that include passenger route choices. Most of these approaches either apply a single (shortest) route search (Guan, Yang, and Wirasinghe 2006, Nachtigall and Jerosch 2008, Liu et al. 2019) or a distribution according to a system optimum (Borndörfer, Grötschel, and Pfetsch 2007, Borndörfer and Karbstein 2012). Both strategies are unlikely to accurately estimate a passenger distribution, bearing the risk for operators of crowded or underused vehicles. In a cross-entropy heuristic for integrated line planning and timetabling presented by Kaspi and Raviv (2013), passengers are distributed on shortest paths for evaluation, which serves as the basis to refine the search for an updated solution in the next iteration. Schmidt and Schöbel (2015a) and Friedrich et al. (2017) present generic line planning models with integrated passenger route choice and discuss complexity and bounds. A passenger-optimal route search was introduced in Schmidt (2014) and Goerigk and Schmidt (2017), where sufficient seating capacity is ensured assuming that passengers distribute over the available shortest routes. This approach overcomes the problem that passengers are assigned to suboptimal routes by the model and prevents capacity conflicts during operation. Schiewe, Schiewe, and Schmidt (2019) propose a game-theoretical approach where passengers are individual players choosing their routes with the highest travel quality.

All the approaches discussed previously consider a flexible passenger to route assignment or search but assume the total passenger demand to be fixed. Only a few publications consider the mode choice of travelers during line planning to estimate the number of passengers attracted by the solution. The integrated stop location and line planning approaches discussed in

Laporte et al. (2005; 2007) aim at a maximum trip coverage. Similarly, Klier and Haase (2015) maximize the number of expected passengers and estimate the mode choice with the logit model as a traveler's decision between the best available route and an alternative mode of transport. Bertsimas, Ng, and Yan (2021) have the same objective to maximize ridership of the public transport system. In their model, each additional line increases the modal share by a predefined percentage, independent of which other lines are selected and their frequencies. De-Los-Santos et al. (2017) use the logit model for the mode choice as well and approximate it with a piecewise linear function. For the specific case of Intercity buses, Steiner and Irnich (2018) consider different passenger demand levels depending on departure and travel time in a combined optimization approach for stop selection on a line and timetabling. A comprehensive model integrating network design, line planning, traveler mode choice, passenger route choice, and fleet investment is discussed in Canca et al. (2016). Later, the authors provide in Canca et al. (2017) an adaptive large neighborhood search metaheuristic for this model, limiting the passenger route search to a shortest route. For a revised model including an integrated passenger distribution on routes, Canca et al. (2019) present a two-level local search matheuristic that was successfully used to find solutions for real-world sized instances.

These models include a modeling of travelers' mode choice during line planning, but they contain at least one of the two limitations. Either the passenger distribution on routes, although relevant for seat capacity estimation, does not reflect passenger route choices as modeled by a choice model, or the quality of the solutions found by the applied metaheuristics is in many cases hard to assess. In this paper, we aim at finding profit-optimal line plans and simultaneously estimating the passenger mode and route choices.

3. Modeling

We develop a MILP model for finding a line plan that is tailored to the corresponding passenger demand. Lines are selected from a pool of potential lines. The passenger demand is estimated with a discrete choice model and the distribution on routes is modeled in accordance with passengers' choices. This modeling of passengers provides a basis for an accurate estimation of the required seat capacity and the expected revenues from ticket sales. The objective is to find a profit-optimal line plan, which means the difference between revenue and cost is maximized. In the following sections, we discuss all components of the line planning problem, the underlying assumptions made, and how we model it.

3.1. Network Structure and Line Selection

We consider a railway network with a set of *stations* $\mathcal{S}$ as nodes, a set of *tracks* $\mathcal{T}$ as direct connections between the stations, and symmetric traveler demand between the stations. Let k denote an *origin-destination (OD) pair*, that is, an unordered pair of stations s_1 and $s_2 \in \mathcal{S}$, and let $\mathcal{OD}$ be the *set of OD pairs*. The *traveler demand*, that is, the number of persons wanting to travel between OD pair $k \in \mathcal{OD}$, is denoted by δ_k. We assume a *line pool* $\mathcal{P}$ to be given, where each *line* $l \in \mathcal{P}$ is an undirected sequence of stations $(s_1, s_2, \ldots, s_{n_l})$.

The aim is to find a *line plan* $\mathcal{L} \subseteq \mathcal{P}$, a subset of lines from the line pool, for a periodic and symmetric service. When selecting a line from the pool $\mathcal{P}$, it is assumed to operate in both directions.

The selection of lines is modeled using binary decision variables

$$z_l \in \{0,1\} \text{ with } z_l = \begin{cases} 0, & \text{Line } l \text{ is not selected,} \\ 1, & \text{Line } l \text{ is selected.} \end{cases}$$

This constitutes the basis of the line planning problem: Lines have to be selected from a line pool to meet passenger demand. Higher line frequencies can be achieved by selecting multiple lines with the same itinerary. This indirect modeling of frequencies is adapted to unambiguously link lines and passenger routes in Section 3.3.3. In the following, the objective of the line planning model is defined, the procedure to estimate the passenger demand is explained, and based on that, the required capacity is calculated.

3.2. Profit Maximization

Our objective is to find a line plan that is optimal with respect to profit, which means, the difference of revenue generated by passenger fares and costs for installing and operating lines should be as high as possible. The *costs* c_l for installation and operation of line l comprise acquisition and maintenance of rolling stock and personnel and energy expenses. We assume an OD pair-dependent passenger pricing as it is applied in the Netherlands. There, passengers are charged based on the locations of their origin and destination only, and ticket prices are independent of the chosen route. Let p_k denote the *ticket price* for a passenger of OD pair k and let m_k denote the *share of travelers* of OD pair k that choose to travel by train. Then, the following function describes the profit associated with a line plan $\mathcal{L}$.

$$\sum_{k \in \mathcal{OD}} p_k \cdot \delta_k \cdot m_k - \sum_{l \in \mathcal{P}} c_l \cdot z_l \tag{1}$$

The total cost is calculated as the sum of costs for all selected lines. The product of the total number of travelers δ_k, the share of travelers by train m_k, and the ticket price p_k gives the total revenue from passengers of OD pair k. Next to the cost, the revenue is affected by the selected lines. With a better supply, more passengers decide to travel by train and generate more revenue. Therefore, the share of railway passengers m_k is a variable in this context. How the value of $m_k \in [0, 1]$ is estimated based on the selected lines as described in the next section.

3.3. Demand Estimation

3.3.1. Assumptions Made in Modeling Demand. In this section, we present and discuss our modeling assumptions related to passenger demand. We use these assumptions to model mode choice and route choice in our MILP model in Sections 3.3.2 and 3.3.3.

We assume that traveling demand is given in the form of pairs of origin and destination *railway stations*. Only travelers that decide to travel by railway generate revenue for the railway operator, whereas travelers that choose to travel with an alternative mode, such as private car, do not contribute to the objective function (1). To compute the share of passengers that make use of railway transportation and to distribute this demand over several routes between origin and destination of an OD pair, we make the following assumptions.

1. We assume that traveler demand is given and the share of travelers using railway can be estimated using a logit model based on the utility of rail and the utility of other alternatives.

2. We assume that changes in the line plan do not affect the utility of alternative modes. This is not completely realistic: In general, changes in the railway system affect the modal share, and can therefore affect, for example, the congestion on roads which may change the utility of traveling by car. However, as long as the modal share is not affected substantially, the impact on the utility of the alternative mode is negligible.

3. In the remainder of this paper, we assume that rail is competing with only one other mode. This is motivated by Assumption 2. Because utilities of competing modes are independent of the line plan, different alternative modes can be aggregated to just one competing "not rail" mode.

4. We assume that passengers do not consider multimodal journeys in the sense that they do not combine rail with other modes of transport.

5. Within the railway network, several *routes* may be available to a passenger. A *route* is a sequence of consecutive line segments, where each line segment corresponds to a track in the network operated by a certain line. Routes may differ geographically, or just in the sequence of lines that passengers take to reach their destination. We assume that for an OD pair the utility of traveling by rail depends both on the number of *reasonably good* routes and on the journey time of these routes.

(a) We compute the *journey time* j_r of a route to be the approximate driving time from origin to destination including dwell times, plus a transfer penalty for each transfer. For routes that contain a transfer, the transfer time is not accounted for in the journey time. Although long transfer times can increase the journey time, short transfer time may be perceived as stressful by passengers, as transfer may be missed in case of delays, even if they are small. As the timetable, and consequently the transfer times will only be determined in a later planning step, we have decided to include a transfer *penalty* for the calculation of the route journey times, capturing these aspects as a generalized *inconvenience* of transferring. This is in line with the observation of de Keizer, Kouwenhoven, and Hofker (2015) that the time equivalent of a transfer is significantly higher than the transfer time, and it is a commonly made assumption in line planning (Schöbel and Scholl 2006, Schöbel 2012, Schmidt and Schöbel 2015a, De-Los-Santos et al. 2017, Goerigk and Schmidt 2017).

(b) The number of *reasonably good* railway routes for an OD pair k plays a significant role for k's utility in using the mode rail, because more frequent departures decrease the expected time difference between desired departure time and actual departure time for passengers of k that travel by railway. We call this time difference *adaption time*. With a more frequent service, the adaption time decreases inversely proportional. Therefore, we include the adaption time in the definition of the utility of the mode rail in Section 3.3.2. There are different ways to model the adaption time. The modeling we use in the computational studies of this paper is described in the experimental setup in Section 4.3.

6. We assume that passengers distribute uniformly over routes of *reasonably good and similar quality*. We compare the quality of routes r based on their journey time j_r, which depends on the driving time as well as the number of transfers per route. In a preprocessing step, we determine for each OD pair k a choice set of routes that are among the journey-time shortest routes and consider only those as alternative routes for the passengers. In this paper, we use the journey time to derive a choice set of *reasonably good and similar routes*, but in principle, any definition of quality that is independent of the timetable could be used.

The following situation exemplifies our selection of reasonably good and similar routes. Imagine there are three available routes for an OD pair k with respective journey times of 20, 21, and 40 minutes. We consider the route with a journey time of 40 minutes as unattractive for passengers and exclude it from the choice set, and we assume that the passengers distribute uniformly over the other two routes with reasonably good and similar journey times of 20 and 21 minutes. The used conditions for exclusion of a route from the choice set is explained in detail in Assumption 8.

There are limitations to the validity of our assumption that passengers distribute uniformly over routes of similar journey time. First, the distribution of passengers on routes also depends on the temporal spread of departure and arrival times of the available routes. However, it is not possible to include that information since the timetable is not known yet at the stage of line planning. Hence, it is only possible to estimate the passenger distribution based on the quality of the available routes.

Another issue occurs when some routes in the choice set are partially overlapping with each other. This can be best illustrated in an example. Imagine there are three available routes for an OD pair k: two transfer routes that are geographically identical and both use the same line l up to a station s; they differ only in the line that passengers transfer to at station s; one geographically longer route that contains no transfer. Assuming that the journey time of the third route is similar to the journey time of the geographically shorter routes, passengers will be distributed equally among the three routes by our model. However, what we would expect to observe in reality is that among the two transfer routes, the one with the shorter transfer time is chosen predominantly. It can even be expected that passengers will not perceive the transfer route with longer transfer time as a real alternative and will split equally among the two alternatives "direct route" and "transfer route starting with line l." This is a well-known phenomenon in discrete choice theory, often referred to as *red bus, blue bus problem* (de Dios Ortúzar and Willumsen 2011, section 6.5.3).

7. Regarding capacity decisions, we take the standpoint that in a strategic planning level, passengers should not be expected to change to inferior routes to compensate for capacity shortages. This implies that the model may be forced into the costly decision of adding another line if the capacity is just not sufficient for the expected passenger demand. An additional line might not always be necessary because passengers often react to crowded vehicles and change to less crowded routes. However, we believe that, on a strategic level, we should aim at providing enough capacity to transport all passengers, without relying on the assumptions on congestion-based rerouting. Modeling capacity considerations of passengers in route choice is often done in later planning stages, such as timetabling in the tactical level since the seating capacity has been fixed.

8. Finally, depending on the journey time, we either do or do not include a route in our choice set. More precisely, in our experiments, prior to solving the model, we decide on a tolerance coefficient α and a tolerance addend ε: if the journey time j_r of a route r does not exceed $\alpha \underline{j}_k + \varepsilon$, where $\underline{j}_k$ is the journey time of the journey-time shortest route for OD pair k, it is included in the choice set and receives the same passenger load as other available routes in the choice set. Otherwise, it receives no passenger load at all. Thus, in contrast to, for example, a logit model, routes whose journey time falls outside of the tolerance do not receive decreased passenger numbers but no passengers.

The decision for a cutoff value for inclusion of a route in the choice set was motivated by tractability considerations. This will allow us to formulate the route choice constraints in Section 3.3.3 as linear constraints and to solve our problem as a MILP, whereas constraints based on other choice models would require different, probably nonlinear and intractable constructions here.

Despite the mentioned shortcomings that are intrinsic to our modeling approach, we believe that this modeling comes closer to an actual distribution than the common assumptions that all passengers of an OD pair use a *single* (shortest) route or can *be assigned* to routes according to a system optimum. Hence, we expect this distribution to give a better estimate of required seating capacity than existing approaches. At the same time, the simplifications made are sufficient to be able to solve the problem for real-world sized instances, which is an advantage over models that use a more realistic, yet more complicated modeling of passengers' route choice.

3.3.2. Including Mode Choice in the MILP Model.

The average journey time $\bar{j}_k$ of OD pair k is determined as the average journey time of all routes for OD pair k in the choice set $\mathcal{C}_k$:

$$\bar{j}_k = \frac{1}{|\mathcal{C}_k|}\sum_{r\in\mathcal{C}_k} j_r \qquad \forall k \in \mathcal{OD}.$$

Remember that the journey time j_r of a route r is measured as the approximate driving time from origin to destination including dwell times, plus a transfer penalty for each transfer, and that *reasonably good* routes for each OD pair are collected in a choice set $\mathcal{C}_k$. Because we design the choice sets $\mathcal{C}_k$ in such a way that all routes have a very similar journey time, the journey time $\bar{j}_k$ for OD pair k is very close to the route journey time j_r for any route $r \in \mathcal{C}_k$ and does not depend on which routes are available.

Using this, we define the *utility* of mode rail for OD pair k as the sum of the average journey time $\bar{j}_k$ of OD pair k and the adaption time a_k of k.

$$u_k = \bar{j}_k + a_k \qquad \forall k \in \mathcal{OD} \tag{2}$$

Because $\bar{j}_k$ is determined as the average journey time over all routes in the choice set $\mathcal{C}_k$ (independent of whether they are established in the line plan or not) and all routes in the choice set have a very similar journey time, it can be predetermined prior to the computation of the line plan. That means, the impact of the line plan on the utility of the mode rail for OD pair k is constrained to its effect on the adaption time, which depends exclusively on the number of available routes.

In this paper, we use a logit model to estimate the travelers' mode choice and to derive the modal share. The logit model is a discrete choice model that is commonly used to estimate travelers' choices. The probability that an alternative is chosen depends on the utilities of all available alternatives. For the mode choice of travelers, we consider two alternatives: traveling by railway and traveling by an alternative mode.

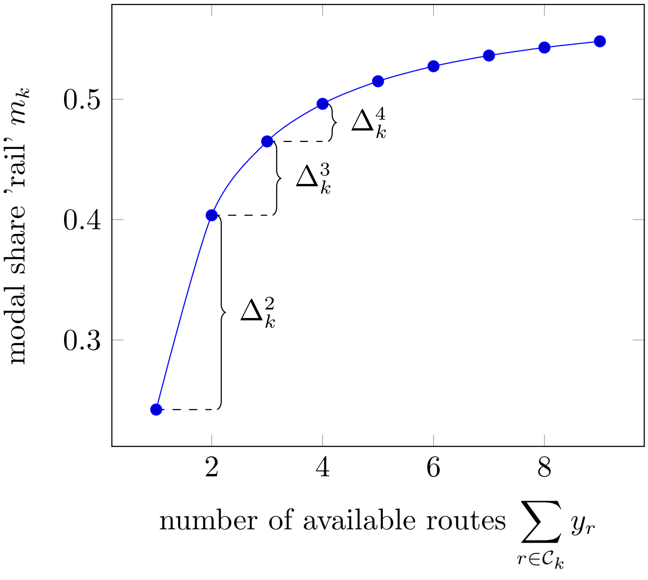
Let u_k be the utility of mode rail for OD pair k from Equation (2) and $\hat{u}_k$ be the utility of the alternative mode. Then, the logit model estimates the modal share for rail as

$$m_k = \frac{e^{\beta u_k}}{e^{\beta u_k} + e^{\beta \hat{u}_k}}, \tag{3}$$

where $\beta > 0$ is the logit coefficient to tune the model.

Figure 1 shows the expected share of travelers using mode rail for different numbers of routes, exemplified for one OD pair. The *increments in modal share* Δ_k^i indicated in Figure 1 express the additional share of travelers of OD pair k deciding to use rail if i instead of $(i-1)$ routes are available. For higher numbers of available routes, the increments of modal share decrease in size and the curve approximates a maximum modal share. This implies that extremely high modal shares are often not reachable by optimizing the line plan. Even if it was possible to chose infinitely many lines and thus eliminate the adaption time in theory, the utility of the mode rail is bound by the rail travel times. Hence, the alternative mode will always get a certain modal share which depends on its travel time.

Figure 1. (Color online) Estimated Modal Share for One OD Pair k as a Function of the Number of Available Routes



The values Δ_k^i can be precomputed for each OD pair k and each possible number n of established routes of reasonable quality i in $\{0, 1, 2, \ldots, |\mathcal{C}_k|\}$. Using these values, the mode choice of travelers can be modeled linearly with the constraints

$$m_k = \sum_{i=1}^{|\mathcal{C}_k|} \Delta_k^i \cdot b_k^i \qquad \forall k \in \mathcal{OD}. \tag{4}$$

Here, we use the binary variables

$$b_k^i \in \{0,1\} \text{ with } b_k^i = \begin{cases} 0, & \text{OD pair } k \text{ has less than } i \text{ available routes,} \\ 1, & \text{OD pair } k \text{ has at least } i \text{ available routes} \end{cases} \tag{5}$$

to count the number of available routes of OD pairs is introduced. Hence, Constraints (4) set the modal share of rail m_k for OD pair k dependent on the number of available routes.

This framework allows the integration of travelers' mode choice according to any aggregate choice model using the linear constraints (4). Our model is thus not limited to the logit model that is used in this paper to derive the values for Δ_k^i.

3.3.3. Including Route Choice in the MILP Model. For each OD pair, routes of *reasonably good quality* are contained in a precomputed choice set $\mathcal{C}_k$. (See Section 4.2 for a description on how we compute these choice sets in our experiments.) We assume that passengers of OD pair k distribute uniformly over all routes from their choice set $\mathcal{C}_k$ that are *available* in the chosen line plan. The availability of a route $r \in \mathcal{C}_k$ is modeled with the binary auxiliary decision variable

$$y_r \in \{0,1\} \text{ with } y_r = \begin{cases} 0, & \text{Route } r \text{ is not available,} \\ 1, & \text{Route } r \text{ is available.} \end{cases}$$

A route is *available* if all lines that are used on the route are selected. The selection of lines and availability of routes can be linked with the following set of constraints:

$$y_r \le z_l \qquad \forall k \in \mathcal{OD},\ r \in \mathcal{C}_k,\ l \in \mathcal{P}_r, \tag{6a}$$

$$y_r \ge \sum_{l \in \mathcal{P}_r} z_l - |\mathcal{P}_r| + 1 \qquad \forall k \in \mathcal{OD},\ r \in \mathcal{C}_k, \tag{6b}$$

where $\mathcal{P}_r$ is the set of lines in the pool used on route r.

Let the variable w_r denote the *share of travelers using route r*. Then,

$$w_r \in [0,1] \text{ with } w_r = \begin{cases} 0, & y_r = 0, \\ \dfrac{m_k}{\sum_{r' \in \mathcal{C}_k} y_{r'}}, & y_r = 1 \end{cases} \qquad \forall k \in \mathcal{OD},\ r \in \mathcal{C}_k. \tag{7}$$

The values of w_r are the same for all available routes for OD pair k, and they sum up to the share of travelers using mode rail m_k. Using binary variable b_k^i that was introduced in (5) to count the number of available routes, the route choice constraints (7) can be linearized with the following set of constraints:

$$w_r \le y_r \qquad \forall k \in \mathcal{OD}, r \in \mathcal{C}_k, \tag{8a}$$

$$w_r \le \frac{m_k}{i} - b_k^i + 1 \quad \forall k \in \mathcal{OD}, r \in \mathcal{C}_k, i \in \{1, \dots, |\mathcal{C}_k|\}, \tag{8b}$$

$$\sum_{r \in \mathcal{C}_k} w_r = m_k \qquad \forall k \in \mathcal{OD}. \tag{8c}$$

Constraints (8a) ensure that w_r is positive only if route r is available, that is, if $y_r = 1$. Constraints (8b) impose an upper bound of $\frac{m_k}{i}$ to w_r if at least i routes are available for OD pair k, that is, if $b_k^i = 1$. For increasing i, $\frac{m_k}{i}$ decreases and these constraints get tighter. If less than i routes are available and $b_k^i = 0$, the right-hand side is greater than one, and the constraints are redundant. Constraints (8c) ensure that the shares w_r of passengers of OD pair k using route r sum up to the share of travelers using mode rail m_k. Together, Constraints (8) model that the passengers of OD pair k distribute uniformly on all available routes from the set $\mathcal{C}_k$. Both the number of passengers and the number of available routes depend on the line plan.

The number of available routes can be counted within the model with the following constraints:

$$\sum_{r \in \mathcal{C}_k} y_r \ge i \cdot b_k^i \qquad \forall k \in \mathcal{OD}, i \in \{1, \dots, |\mathcal{C}_k|\}, \tag{9a}$$

$$b_k^i \ge \frac{1}{|\mathcal{C}_k|}\left(\sum_{r \in \mathcal{C}_k} y_r - i + 1\right) \qquad \forall k \in \mathcal{OD}, i \in \{1, \dots, |\mathcal{C}_k|\}. \tag{9b}$$

Constraints (9a) force b_k^i to zero if less than i routes are available for OD pair k, and Constraints (9b) ensure that b_k^i equals to one if at least i routes are available.

3.4. Operational Requirements

3.4.1. Capacity Constraints. The strength of the demand modeling in Section 3.3 is that the number of passengers can be estimated on each available route. It should be ensured by the operator that there is sufficient seating capacity on each available route r for the number of passengers that is expected to choose route r. We model this as one capacity constraint per *line segment*, the part of a line l traversing a track t. Let κ_l be the *seating capacity* of line l, $\mathcal{P}(t)$ be the set of lines in the line pool that operate on track t, and $\mathcal{C}_k(l,t)$ be the choice set of routes for OD pair k using line l on track t. The following constraints ensure that on each line segment sufficient capacity is provided by the line plan for all passengers on their chosen routes.

$$\sum_{k \in \mathcal{OD}} \sum_{r \in \mathcal{C}_k(l,t)} \delta_k \cdot w_r \le \kappa_l \cdot z_l \qquad \forall t \in \mathcal{T}, l \in \mathcal{P}(t) \tag{10}$$

The presented model uses individual capacity constraints for each line segment and considers a passenger distribution on the best available routes. The combination of individual capacity constraints and a distribution on routes that is in accordance with passenger interests is important for accurate capacity estimation. It achieves that passengers use similar routes in the model as they would choose in real life, thus avoiding potential conflicts with capacity constraints. Existing line planning models (see the overview given in Schöbel 2012) often assign passengers on a single route or according to a system optimum, or they use only one capacity constraint per track t, aggregated over all lines operating on that track. Both can cause capacity conflicts unless passengers accept additional transfers to make space for other passengers, which is unrealistic.

3.4.2. Minimal Service Requirement. In addition to their aim to meet the passenger demand, most operators are required to offer a minimal service in certain parts of the networks. This ensures that all passengers have access to railway transport, also in sparsely populated areas. We model this as an additional set of constraints ensuring that at least f_t vehicles service track t.

$$\sum_{l \in \mathcal{P}(t)} z_l \ge f_t \qquad \forall t \in \mathcal{T} \tag{11}$$

3.5. Line Planning Model with Mode Choice

In this section, we give the MILP for line planning with mode choice (*LPwMC*). As described in Section 3.2, the objective is to maximize profit, defined as revenue minus cost in Equation (1). The first part deals with the demand estimation explained in Section 3.3 including auxiliary modeling constraints. The traveler mode choice is estimated according to any aggregate choice model with Equations (4). The linking constraints between route and line variables are discussed in Equations (6), and the uniform passenger distribution on available routes is modeled with Equations (8). The number of available routes is counted within the model with Equations (9). The second part deals with the operational requirements from Section 3.4. It is ensured by Equations (10) that sufficient seating capacity is available for the expected number of passengers on each line segment. This links the line selection and estimation of passenger demand in the model. Furthermore, the minimal service requirement in Equations (11) ensures a minimal frequency on each part of the network. The last part defines the domains of the variables.

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(**LPwMC**)

max profit = revenue − cost

$$\max \quad \sum_{k\in\mathcal{OD}} p_k \cdot \delta_k \cdot m_k - \sum_{l\in\mathcal{P}} c_l \cdot z_l$$

traveler mode choice

$$\text{s.t.} \quad m_k = \sum_{i=1}^{|\mathcal{C}_k|} \Delta_k^i \cdot b_k^i \quad \forall k\in\mathcal{OD}$$

link between line and route variables

$$y_r \le z_l \quad \forall k\in\mathcal{OD}, r\in\mathcal{C}_k, l\in\mathcal{P}_r$$

$$y_r \ge \sum_{l\in\mathcal{P}_r} z_l - |\mathcal{P}_r| + 1 \quad \forall k\in\mathcal{OD}, r\in\mathcal{C}_k$$

passenger route choice

$$w_r \le y_r \quad \forall k\in\mathcal{OD}, r\in\mathcal{C}_k$$

$$w_r \le \frac{m_k}{i} - b_k^i + 1 \quad \forall k\in\mathcal{OD}, r\in\mathcal{C}_k, i\in\{1,\ldots,|\mathcal{C}_k|\}$$

$$\sum_{r\in\mathcal{C}_k} w_r = m_k \quad \forall k\in\mathcal{OD}$$

counting of available routes

$$\sum_{r\in\mathcal{C}_k} y_r \ge i \cdot b_k^i \quad \forall k\in\mathcal{OD}, i\in\{1,\ldots,|\mathcal{C}_k|\}$$

$$b_k^i \ge \frac{1}{|\mathcal{C}_k|}\left(\sum_{r\in\mathcal{C}_k} y_r - i + 1\right) \quad \forall k\in\mathcal{OD}, i\in\{1,\ldots,|\mathcal{C}_k|\}$$

capacity constraints per line segment

$$\sum_{k\in\mathcal{OD}} \sum_{r\in\mathcal{C}_k(l,t)} \delta_k \cdot w_r \le z_l \cdot \kappa_l \quad \forall t\in\mathcal{T}, l\in\mathcal{P}(t)$$

minimal service requirement

$$\sum_{l\in\mathcal{P}(t)} z_l \ge f_t \quad \forall t\in\mathcal{T}$$

domains of variables

$$z_l \in \{0,1\} \quad \forall l\in\mathcal{P}$$

$$y_r \in \{0,1\} \quad \forall k\in\mathcal{OD}, r\in\mathcal{C}_k$$

$$b_k^i \in \{0,1\} \quad \forall k\in\mathcal{OD}, r\in\mathcal{C}_k, i\in\{1,\ldots,|\mathcal{C}_k|\}$$

$$w_r \in [0,1] \quad \forall k\in\mathcal{OD}, r\in\mathcal{C}_k$$

$$m_k \in [0,1] \quad \forall k\in\mathcal{OD}$$

3.6. Valid Inequalities

We describe two kinds of valid inequalities that can be added to model (*LPwMC*) to support the solution finding process. Both valid inequalities are tested for their effectiveness in Section 5.1.

The first set of valid inequalities links the values for the auxiliary variables b_k^i, which are used for counting the number of available routes for each OD pair. By definition, b_k^i equals one if for OD pair k at least i routes are available and zero otherwise. This implies that b_k^i can only be one if b_k^{i-1} equals one. The other way around, b_k^i can only be zero if b_k^{i+1} equals zero. This relation can be modeled with the order constraints

$$b_k^i \ge b_k^{i+1} \quad \forall k \in \mathcal{OD}, \; \forall i \in \{1, \ldots, |\mathcal{C}_k| - 1\}. \tag{12}$$

The second set of valid inequalities are symmetry breaking constraints for the binary decision variables z_l, which model whether line l is selected in the line plan or not. To model higher frequencies of a line, duplicates are considered in the line pool that can be selected independently. Let $\mathcal{P}_l \subset \mathcal{P}$ be the subset of all lines in the line pool with the same itinerary as line l. To break the symmetry implicated by this setting, we consider an arbitrary but fixed order of the indices of lines in $\mathcal{P}_l$. Then, constraints

$$z_{li} \ge z_{li+1} \quad \forall i \in \{1, \ldots, |\mathcal{P}_l| - 1\} \tag{13}$$

enforce an order of selection of a line and its duplicates.

4. Experimental Setup

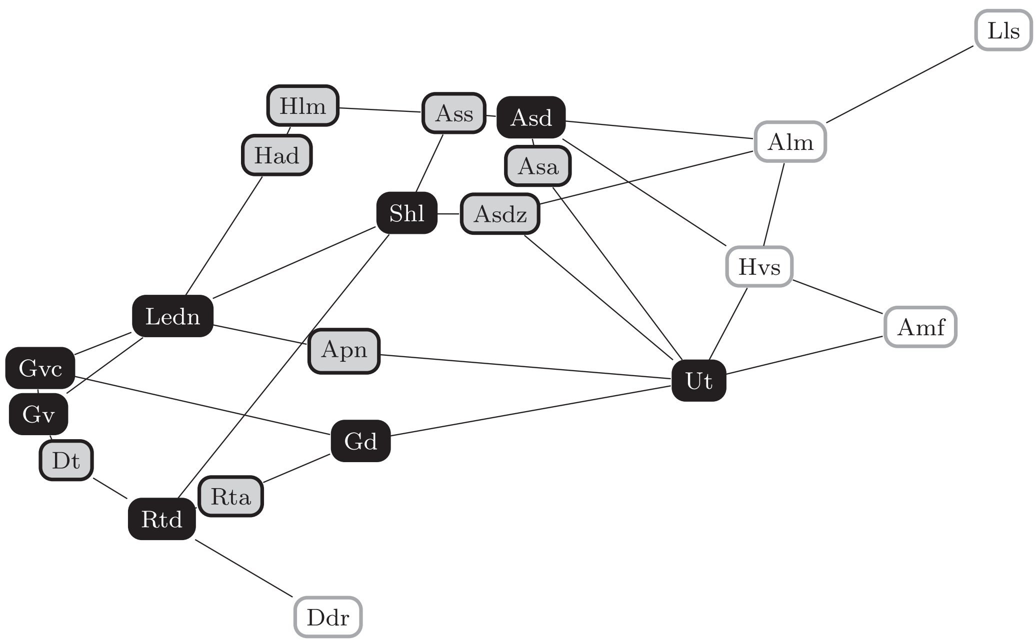
We solve the model (*LPwMC*) with the branch and bound method implemented in the Fico[textregistered] Xpress Optimizer version 35.01. All experiments are conducted on the Lisa cluster (SURFsara 2021) operated by SURFsara with a time limit of one hour per model run. In the following sections, the instances are introduced, the derivation of passenger routes is explained and the choices for parameters described.

4.1. Instances

We consider the intercity network of the Randstad, a metropolitan area in the Netherlands. This is a partial network of the network operated by the largest Dutch railway operator, Netherlands Railways (NS). The network contains 21 stations connected by 31 direct tracks between them. The network is depicted in Figure 2 and denoted by *IC*21, indicating the number of stations in the network. The line pool $\mathcal{P}$ contains 107 lines, 43 of which are duplicates to be able to model higher line frequencies. The pool contains all relevant lines that currently operate in the considered area and is given in Online Appendix B. A reference line plan is available that is used as a feasible start solution. The reference line plan is a solution of a line planning model with fixed passenger assignment. Based on this reference line plan, a competing mode such as driving by car, and passenger count data from NS, traveler demand was estimated with the logit model. This resulted in 174 undirected OD pairs with positive traveler demand.

To obtain a variety of instances in some experiments, we consider two additional instances *IC*08 and *IC*16 and randomized demand situations for all networks. These instances have 8 and 16 stations, respectively, and are subnetworks of network *IC*21. In Figure 2, the stations in network *IC*08 are marked by a black node color scheme. The stations that are additionally in network *IC*16 have a grey color scheme. The remaining stations with the lightest color scheme are only contained in network *IC*21. The *randomized demand situations* are generated by multiplying the number of travelers δ_k with a random number between 0.5 and 1.5 for each OD pair k.

Figure 2. IC Network of Randstad Region in the Netherlands



4.2. Passenger Routes

In a preprocessing step, a choice set of passenger routes δ_k for each OD pair k is determined and used as input to the model (*LPwMC*). As described in Section 3.3.3, we consider the journey-time shortest passenger routes for the route choice sets. In our experiments, we model this with a tolerance coefficient α and a tolerance addend ε to limit the maximally acceptable journey time. First, we derive the shortest possible journey time j_k for each OD pair based on an extensive line pool for each network. Then, a route r is in the set $\mathcal{C}_k$, if and only if its journey time j_r is at most $\alpha j_k + \varepsilon$. Only routes that are at most 5% ($\alpha = 1.05$) and $\varepsilon = 10$ minutes longer than the shortest possible journey time were accepted. The journey time of a route comprises drive and dwell times and a transfer penalty, if applicable. Average driving times per track are used and at all stations a dwell time of 4 minutes is assumed. A penalty of 20 minutes is added to the journey time of routes including a transfer. In the experiments, we restrict the route choice sets to routes with at most one transfer.

This yields a set $\mathcal{C}_k$ of passenger routes of comparable journey time for each OD pair k. For the intercity network of the Randstad *IC*21 from Figure 2, a total of 6,391 routes are considered, which is on average 36.7 routes per OD pair.

4.3. Parameters

The monetary values for the OD pair k dependent ticket prices p_k for passengers, and cost c_l related to line operation were chosen to represent a simplified situation for the Dutch railway operator NS. All lines are operated by trains with a capacity of κ_l = 1,000 seats.

For estimating the mode choice, the logit model (3) is used. Finding a precise value for the logit parameter β for the considered network and demand situation requires an extensive study on its own. For example, Calastri et al. (2019) determine a value of $\beta = -0.137$ for buses as the considered public transport option in a revealed preference study where travelers track their trips with a GPS logger on their smartphone. We expect a value of around $\beta = -0.1$ for the considered network size and use this as a reasonable value for the logit parameter in our experiments. We estimate the adaption time based on the assumption that demand is uniformly distributed over the period, and that route departures are spread evenly over the hour. We therefore arrive at an adaption time of half of the considered period of 60 minutes, divided by the number of available routes. The alternative mode resembling individual transport does not have an adaption time. Hence, the utility $\hat{u}_k$ of the alternative mode is quantified by the journey time only. We use the SAQ method (FGSV 2008) to estimate the journey time based on the Euclidean distance between stations.

For the minimal service requirement (11), a minimal track frequency of $f_t = 2$ for all tracks $t \in \mathcal{T}$ is used. This is in line with the requirements for the Dutch railway operator NS.

5. Comparison and Analysis

To test the model for line planning with mode choice, we conduct experiments on the Randstad network. We test means to improve the computational performance and compare our model (*LPwMC*) with a standard line planning model with fixed passenger demand to investigate differences in solution quality.

5.1. Improvement of Computational Performance

We test the impact of adding order constraints (*order*), adding symmetry breaking constraints (*sym*), and setting priorities for branching (*prio*) on the solution-finding process. The settings *order* and *sym* add the valid inequalities (12) and (13), respectively, to the model (*LPwMC*). The branching priorities *prio* are motivated by the different modeling impact of the binary variables. The solution of the line planning problem is uniquely determined by the line plan $\mathcal{L}$, that means, the solution values for the line selection variables z_l. The corresponding solution values of all other variables can be reconstructed from the solution values of z_l. However, model (*LPwMC*) uses three different sets of binary variables, z_l, y_r, and b^i_k, where y_r and b^i_k are auxiliary variables to model the availability and the number of routes. By default, any of these variables can be used for branching. We test whether branching first on the variables z_l is preferable to the standard branching strategy of Fico Xpress. The Xpress Optimizer offers the option to set the branching priority of a variable between 0 and 1,000, where always a variable with a lower priority number will be selected for branching. We set a high branching priority (1) for variables z_l, medium branching priority (500) for variables y_r, and low branching priority (999) for variables b^i_k.

The tests are conducted on the three networks *IC*21, *IC*16, and *IC*08 with 10 randomized demand situations per network. We observe the CPU time until an optimal solution is found and the gap to the best bound found in case the time limit of one hour is exceeded. Table 1 gives the CPU times in seconds and the gaps to the best bound for solving the model (*LPwMC*). The CPU times in Table 1 are averaged over 10 randomized demand situations on each network. Only those runs that exceed the time limit are included for computing the average gap. The number in parentheses behind the average gap shows how often the time limit of one hour is exceeded. The first and last row show results for the settings where *none* (reference case) or *all* three options are used, respectively.

Adding order constraints (setting *order*) reduces the average CPU time for the smallest network and the average gap for the midsize network. However, it increases the average gap for the largest network compared with the reference case (setting *none*). The reason might be that the additional constraints increase

Table 1. CPU Times (Seconds) and Gaps (Percentage) to the Best Bound for Model (*LPwMC*) in Different Settings

	Average CPU time (s)			Average gap (%)		
	*IC*08	*IC*16	*IC*21	*IC*08	*IC*16	*IC*21
None	166	3,600	3,600	—	30.5 (10)	8.9 (10)
Order	144	3,600	3,600	—	11.4 (10)	17.2 (10)
Sym	31	303	3,194	—	—	1.6 (5)
Prio	1,034	3,600	3,600	—	2.5 (10)	9.3 (10)
All	32	248	1,305	—	—	—

Note. The number in parentheses gives the number of cases where the time limit of one hour is exceeded.

the problem size. This could initially make the search for a feasible solution more difficult but accelerate the solution process at a later point in time. The symmetry-breaking constraints (setting *sym*) significantly reduce the CPU times and gaps for all instance sizes. For the largest instance with 21 stations, the time limit is exceeded in only 5 of 10 cases, and the resulting average gap is with 1.6% very small. Setting branching priorities (setting *prio*) drastically increases the CPU time for *IC*08, showing that the solver was able to find better branching strategies for the small network. In contrast to that, the average gap for the medium network size is significantly reduced by setting the branching priorities, and the found solutions were close to the optimum. For the large instance, no improvement is found with the setting *prio*. The gaps are slightly higher than in the reference case without branching priorities. Tests with all strategies combined (setting *all*) yield by far the best CPU times and all instance sizes can be solved to optimality within the time limit of one hour. The largest instance considered with 21 stations is solved within an average CPU time of less than 22 minutes. Therefore, we keep this setting with all options (*order*, *sym*, and *prio*) for further experiments.

5.2. Comparison of (*LPwMC*) with Line Planning Without Mode Choice

In this section, we investigate the importance of using a good demand estimation in line planning. For this purpose, we compare our model (*LPwMC*) that integrates the demand estimation with a simple two-step approach (*LP*) that first preassigns demand to railway tracks and then covers this demand by lines in a cost-optimal way.

Because for model (*LP*), the passenger demand is predetermined in the load-assignment step, the revenue assumed by (*LP*) is constant and the objective of finding a cost-optimal line plan corresponds with finding a profit-optimal line plan in model (*LPwMC*). Similar to model (*LPwMC*), model (*LP*) considers a minimal service requirement and seating capacity constraints. In contrast to model (*LPwMC*), in (*LP*) the capacity constraints are aggregated per track, as it is not possible to take individual constraints per line segment into account when passengers are assigned to the network before the line plan is found. The MIP formulation for the line planning model (*LP*) used in the experiments can be found in Online Appendix C.

The number of passengers for model (*LP*) is obtained by assigning a fixed percentage of travelers to mode rail and distribute them uniformly on the routes in the choice set $\mathcal{C}_k$. To obtain an assignment of passengers to tracks, for each track, we aggregate loads over all routes that use the track. We test model (*LP*) with an assigned modal share for mode rail ranging from 40% to 90%.

Table 2 shows the modal share *MS* for rail in decimals, and the revenue *R*, cost *C*, and profit *P* of the found line plans. The values are given for model (*LP*) with different assigned passenger shares (shown in parentheses in the first column) and for model (*LPwMC*). The column $P^{(LP)}$ gives the objective value of model (*LP*), that is, the profit assuming the assigned passenger demand used as input. This value is only available for model (*LP*) with assigned passenger shares. All monetary values are given in relation to the profit of the line plan found by model (*LPwMC*), which is normalized to the value 100. That means a value of 110 implies that the corresponding monetary value is 10% higher than the profit of the solution of model (*LPwMC*).

To give a fair comparison with model (*LPwMC*), based on the found solution, for each OD pair we estimate the expected share of travelers that *decide* to travel by train in a subsequent distribution of passengers with the logit model and compute revenue *R*, cost *C*, and profit *P* based on the modal share *MS*.

In the column *MS*, we report the average modal share based on the estimation with the logit model. We observe that for all tests with model (*LP*), the *preassigned* share of travelers is higher than the modal share *MS estimated* with the logit model.

Whereas the preassigned and estimated modal shares are comparable for a low number of assigned travelers, they significantly differ for high numbers. That means, the line plans designed by model (*LP*) do not attract the passenger numbers that they were designed for. This observation can be explained with the logit model as shown in Figure 1, which we use to estimate the modal share. There, the positive impact of additional routes decreases for an increasing number of available routes, making the achievement of a high modal share unrealistic, even if the line plan was designed for high numbers of passengers. This indicates that line planning models without an integrated estimation of mode choice are not suitable for operators that strive for increasing their modal share. It is striking that the highest modal share is achieved by the solution of model (*LPwMC*).

The monetary observation variables revenue *R*, cost *C*, and profit *P* are based on the modal share *MS* as estimated with the logit model. Both revenue and cost increase with the given passenger share for model (*LP*), but the profit stagnates at around 75% of the profit generated by the solution of model (*LPwMC*). The last column gives the anticipated profit $P^{(LP)}$ based on the *assigned* share of travelers, that is, the objective value of model (*LP*). Especially for high values of the assigned passenger share, the profit $P^{(LP)}$ is significantly higher than the corresponding profit based on estimated passenger numbers. By fixing the modal share before making the line plan, line planning models such as (*LP*) are prone to drastically overestimating the number of passengers and, with this, their revenue and profit. This causes these models to choose for solutions in which many lines are established, without attracting sufficient passengers to be profitable in the end.

The experiments clearly show the importance of appropriate consideration of passenger demand during line planning. By integrating a line plan-dependent mode choice model, we find solutions with our model (*LPwMC*) that significantly increase the operator profit and attract a higher number of travelers compared with a line planning model without an integrated mode choice. We conclude that there is much to gain for operators and their passengers, which makes a mode choice estimation during line plan optimization worthwhile.

Table 2. Results of Models (*LP*) with Different Passenger Shares and (*LPwMC*)

	MS	*R*	*C*	*P*	$P^{(LP)}$
(*LP*)(0.4)	0.36	174.1	115.3	59.2	98.4
(*LP*)(0.5)	0.36	176.0	115.3	60.7	151.7
(*LP*)(0.6)	0.42	205.0	128.7	76.3	191.6
(*LP*)(0.7)	0.42	203.4	129.3	74.1	244.5
(*LP*)(0.8)	0.44	211.2	133.6	77.6	293.8
(*LP*)(0.9)	0.48	229.9	155.5	74.5	325.2
(*LPwMC*)	0.49	240.5	140.5	100.0	—

Note. The modal share is given as decimal, and the monetary values are normalized such that the profit of the solution of (*LPwMC*) equals 100.

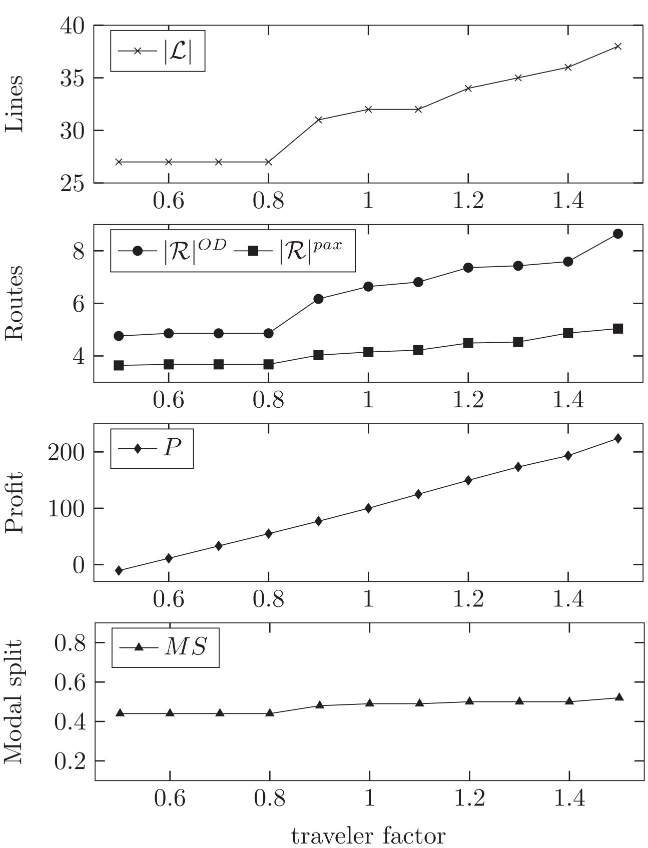
6. Practical Insights

Over years, travel demand is constantly changing, and public transport operators need to react with an adjustment of their service. For operators, it is important to understand the impact of changes in travel demand on the quality of their service and the generated profit. Because of the integrated estimation of route and mode choices, model (*LPwMC*) is capable of computing profit-optimal line plans for different levels of travel demand. In this section, we outline which insights this can provide for operators.

The integrated traveler mode choice allows conducting a sensitivity analysis of the travel demand on passenger service level and operator performance. The different levels of traveler numbers are obtained by multiplying the original traveler demand with a factor ranging from 0.5 to 1.5. Figure 3 shows the results per traveler factor. The corresponding data are given in Table 9 in Online Appendix D. We analyze the number of lines $|\mathcal{L}|$ in the line plan, the number of available routes $|\mathcal{R}|$, the modal share *MS*, and the profit *P* of the line plans found by model (*LPwMC*) for different levels of traveler demand. The profit *P* is normalized such that the profit of the solution for the traveler factor 1.0 equals 100. The number of available routes $|\mathcal{R}|^{OD}$ averaged per OD pair and the number of available routes $|\mathcal{R}|^{pax}$ averaged per passenger are examined separately.

With higher traveler numbers, more lines $|\mathcal{L}|$ can be installed and the supply for passengers improves. Accordingly, the number of available routes for OD pairs $|\mathcal{R}|^{OD}$ and passengers $|\mathcal{R}|^{pax}$ increase. Large OD pairs mostly have direct routes in the route choice sets, whereas for small OD pairs, we often observe transfer routes. Because the number of lines increases, many more line combinations form feasible transfer routes for passengers. Hence, the number of available routes increases proportionally more for small OD pairs than for large OD pairs, which explains the steeper increase of $|\mathcal{R}|^{OD}$ than of $|\mathcal{R}|^{pax}$.

Figure 3. Results of Model (*LPwMC*) for Varying Passenger Load



The higher number of available routes implies an improved service level for passengers and with this a higher modal share for rail. The increase in modal share from 44% to 52% is rather moderate, considering that the overall number of travelers as specified in the input data triples, and the average number of routes per OD pair almost doubles. The low effect on the modal share can be explained with the structure of the choice model that we use to estimate the travelers' mode choice. As indicated in Figure 1, in the logit model, the increase in modal share flattens for a higher number of available routes. This is consistent with observations in the real world where the impact of improving the service level for passengers on the modal share decreases once a certain service level is reached. For example, after increasing the frequency of Intercity trains from four to six trains per hour on the Amsterdam-Utrecht-'s-Hertogenbosch-Eindhoven corridor in 2017, NS registered an increase in ridership of 15% between 's-Hertogenbosch and Eindhoven in the following two years (NS 2020). Using the same method as in de Keizer, Kouwenhoven, and Hofker (2015), it could be shown that this increase was higher than the regular increase on other corridors during that period; however, it is lower than the expected increase of 50% assuming a linear relation between service quality and modal share.

Nevertheless, the profit increases approximately linearly from approximately negative 10% to 225% of the reference profit for an increasing number of travelers. The negative profit values for low passenger numbers are a result of the minimal service level required by governmental regulations, which enforces to operate two lines on each track, even if this cannot be done profitably. This increase in profit can be explained by the high costs for operating a basic line plan on the whole network. For up to 80% of the travelers, the capacity of such a basic line plan is sufficient on the considered instance. Until then, the number of lines, the number of routes, and the modal share stay almost constant. Only for more travelers, and thus for more passengers, it pays off to install more lines. It is interesting to see that the slightly higher modal share and thus the higher revenues offset the costs for additional lines and, overall, lead to an approximately linear increase in profit.

The evaluation shows that the operator's profit is very sensitive to changes in the traveler demand. In case of declining traveler demand, operators cannot prevent losses even if they react to demand changes in an optimal way. This is in line with recent observations where operators incur tangible losses because of considerably lower traveler demand caused by the

COVID-19 pandemic despite efforts to reduce the service level. Conversely, operators can profit greatly from growing traveler demand when exploiting the full potential of passenger demand estimation during line planning.

7. Conclusion and Outlook

The line plan significantly determines the service level of public transport for passengers. It has an impact on how many travelers decide to use public transport and which routes passengers use. In this paper, we present a line planning model with integrated mode and route choice estimation. In contrast to most existing approaches, both choices are modeled from a passenger's perspective and are not driven by a system optimum. This allows an accurate estimation of passenger demand during optimization, resulting in line plans that are tailored for the demand they generate.

Considering passenger choice models during line plan optimization is a complex and hard-to-solve problem. To obtain a tractable model, we assume (1) that passengers distribute uniformly on the best available routes and (2) that the utilities of alternative modes are independent of the designed line plan. By considering only shortest routes for passengers and making these two assumptions, the presented model can be linearized and solved with existing branch and bound methods.

The linear model presented in this paper may be combined with any aggregate choice model to estimate the mode choice of travelers. In particular, the choice model does not need to be linear. Because of the assumptions made, the traveler mode choice can be preprocessed using the preferred choice model without affecting the solving of the line planning model. In the paper, a logit model is used to estimate the mode choice.

We discuss additional constraints and branching priorities for improving the computational performance and show their effectiveness in experiments. The advantages of integrated passenger choice estimation are outlined in a comparison with a standard line planning model that relies on a simple preestimation of passenger loads. In fact, our experiments show that line plans based on simple preestimates of demand do not achieve to capture the preestimated demand, whereas line planning with integrated demand estimation by design yields line plans that are well suited for the demand they generate. As a consequence, these line plans are more profitable for operators and feature a higher level of service for passengers compared with line plans found based on fixed passenger demand. Based on the results of this comparison, considering demand estimation during line planning is strongly recommended.

Furthermore, we analyze the sensitivity of the public transport service level and operator profit on fluctuations in travel demand. This gives valuable insights into the business models of operators and suggests that operators should react to changes in travel demand regularly.

An interesting direction for future research could be the development of further approaches to estimate passenger choices during line planning. For example, instead of modeling passenger demand based on choice models, passengers could iteratively be assigned to their available paths with highest utility until the trains' capacity is reached. Such an all-or-nothing assignment with capacity constraints would be a structurally different approach to the challenge of "line planning with mode choice" and could serve as an interesting method for comparison.

Furthermore, it is to be expected that a good preestimation of demand would be able to mitigate the benefit of demand integration to some extent. This could, in particular, be interesting for line planning instances that are significantly bigger than the considered ones, so that the integer programming model developed in this model might not be tractable anymore. The development of a good heuristic for demand preestimation in absence of a line plan could therefore constitute a valuable next step toward finding good line plans for large line planning instances.

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