Received: 31 May 2022 | Accepted: 10 March 2023

DOI: 10.1111/poms.13995

POMS

ORIGINAL ARTICLE

Robin Hood to the Rescue: Sustainable Revenue-Allocation Schemes for Data Cooperatives

Milind Dawande[1] | **Sameer Mehta**[2] | **Liying Mu**[3]

[1]Naveen Jindal School of Management, The University of Texas at Dallas, Richardson, Texas, USA

[2]Rotterdam School of Management, Erasmus University, Rotterdam, The Netherlands

[3]Alfred Lerner College of Business and Economics, University of Delaware, Newark, Delaware, USA

Correspondence
Sameer Mehta, Rotterdam School of Management, Erasmus University, Rotterdam, The Netherlands.
Email: mehta@rsm.nl

Handling Editor: Subodha Kumar

Abstract

The promise of consumer data along with advances in information technology has spurred innovation not only in the way firms conduct their business operations but also in the manner in which data are collected. A prominent institutional structure that has recently emerged is a *data cooperative*—an organization that collects data from its members, and processes and monetizes the pooled data. A characteristic of consumer data is the externality it generates: Data shared by an individual reveal information about other similar individuals; thus, the marginal value of pooled data increases in both the quantity and quality of the data. A key challenge faced by a data cooperative is the design of a revenue-allocation scheme for sharing revenue with its members. An effective scheme generates a beneficial cycle: It incentivizes members to share high-quality data, which in turn results in high-quality pooled data—this increases the attractiveness of the data for buyers and hence the cooperative's revenue, ultimately resulting in improved compensation for the members. While the cooperative naturally wishes to maximize its total surplus, two other important desirable properties of an allocation scheme are individual rationality and coalitional stability. We first examine a natural *proportional allocation scheme*—which pays members based on their individual contribution—and show that it simultaneously achieves individual rationality, the first-best outcome, and coalitional stability, when members' privacy costs are homogeneous. Under heterogeneity in privacy costs, we analyze a novel *hybrid allocation scheme* and show that it achieves both individual rationality and the first-best outcome, but may not satisfy coalitional stability. Finally, our *RobinHood allocation scheme*—which uses a fraction of the revenue to ensure coalitional stability and allocates the remaining based on the hybrid scheme—achieves all the desirable properties.

KEYWORDS
data cooperatives, data institutions, data monetization, data sharing, revenue-allocation scheme

1 | INTRODUCTION

The last decade has seen an unprecedented collection, storage, and use of consumer data. Technology firms are increasingly collecting massive amounts of individual-level consumer data that include information on various aspects of their lives, for example, demographic, health, location, job, and spending. Firms use consumer data for a variety of purposes—to build accurate machine-learning models (Bertsimas & Kallus, 2020; Jagabathula et al., 2020), target consumers with suitable advertisements (Lee et al., 2018; Marotta et al., 2022), offer personalized search results and product recommendations (Ghose et al., 2019), examine the creative idea-generation process (Aggarwal et al., 2021), and in general, improve their services.

The promise of consumer data along with advances in information technology has spurred innovation not only in the way firms conduct their business operations but also in the manner in which data are collected (Pentland & Hardjono, 2020)—a prominent institutional structure that has recently emerged is a *data cooperative*. A data cooperative is an organization that collects data from its members, processes and monetizes the pooled data, and compensates members for their individual contributions. Data

Accepted by Subodha Kumar, after two revisions.

TABLE 1 An illustrative example of a high-quality data set shared by a user.

Instance ID	User ID	URL	Day (m/d/y)	Time	Has Adblocker	OS	Browser	Gender	Zip
1	axb2	fb.com	06/03/2023	11:34	Yes	Mac	Safari	F	02138
2	axb2	amzn.com	06/03/2023	14:56	Yes	Mac	Safari	F	02138
3	axb2	chase.com	06/03/2023	17:45	Yes	Mac	Chrome	F	02138
4	axb2	uber.com	06/04/2023	20:18	No	Mac	Safari	F	02138
5	axb2	chess.com	06/04/2023	18:09	No	Mac	Safari	F	02138

cooperatives differ in the type of data they aggregate; for instance, Swash (https://swashapp.io/) pools web-surfing data from its members, Driver's Seat (https://www.driversseat.co/) is a driver-owned cooperative that aggregates work-related data from the smartphones of gig-economy workers, and Resonate (https://resonate.is/) is a data cooperative collectively owned by musicians, labels, and fans.

Data cooperatives are fast becoming attractive to potential members for a variety of reasons. Whereas members naturally incur privacy invasion (costs) for sharing their personal data, cooperatives can provide handsome monetary compensation in return. Moreover, members have control over the quality of the data they share with the cooperative. By offering granular privacy controls to its members, data cooperatives not only establish an ecosystem of trust but also complement the recent data-protection laws such as the General Data Protection Regulation (GDPR) in the European Union and the California Consumer Privacy Act (CCPA) in the United States (Hardjono & Pentland, 2019). Further, an individual's personal data by themselves do not have much value. Rather, the value of data for business analytics largely stems from the 3Vs of data—volume, variety, and velocity (McAfee et al., 2012). By aggregating members' data, a cooperative naturally achieves these three properties and, therefore, can command a higher price from data buyers. Finally, the revenue received by the cooperative is distributed among its members. Thus, the larger the size of a cooperative, the better is its bargaining power, which leads to higher compensation for its members (Mehta, Dawande, & Mookerjee, 2021).

At a conceptual level, the operations of a data cooperative are straightforward and easy to understand. Consider, for instance, the Swash data cooperative, which aggregates the web-surfing data of its members. Swash provides a browser extension on popular web browsers such as Google Chrome, Mozilla Firefox, Microsoft Edge, and Brave. The browser extension collects and aggregates the browsing activities of users (who have installed that extension) by running a JavaScript code on the web pages they visit. As mentioned above, a key aspect of data cooperatives is that they provide privacy controls to its members to help them decide the quality of the data they share. Tables 1 and 2 display illustrative examples of how users can control the quality of web-surfing data they share. In both the tables, the rows represent the instances of website visits by the respective users and the columns represent various data types generated during the visits. Depending on their privacy preferences, users can select the data types (columns) that they would like to share with the cooperative. Table 1 shows the data shared by user *axb*2—this user decided to share all the data types with the cooperative; as a result, the data generated in Table 1 are of high quality. On the other hand, in Table 2, user *byn*8 decided to share only the data types *URL, HasAdBlocker, OS*, and *Browser*, and chose to mask all the other data types, resulting in a relatively low-quality data set. In summary, privacy controls allow users to share data of varying quality with the cooperative. The privacy costs members incur, of course, depend on the quality of the data they share. The cooperative collects data from its members and packages the pooled data for monetization.

From the viewpoint of the data cooperative, a key challenge is to allocate (i.e., distribute) the revenue it receives, upon monetization of the pooled data, to its members. In other words, an allocation scheme specifies how the cooperative's revenue is distributed to the individual members. The design of an effective revenue-allocation scheme is important for a data cooperative as it generates a beneficial effect: It incentivizes members to share high-quality data, which in turn results in high-quality pooled data—this increases the attractiveness of the data for buyers and hence the cooperative's revenue, ultimately resulting in improved compensation for members. In a blog post,[1] the Swash data cooperative acknowledges:

> *…the earnings users receive…are currently distributed unfairly. We intend to implement a fairer revenue sharing mechanism where Swash members are rewarded according to the quality and quantity of their contributions. One of our priorities in the Swash roadmap is to make sure that payments are fair, calculated according to the value of the contribution each user puts in. This is complicated…*

Currently, Swash distributes its revenue equally among the members, regardless of their relative individual contributions:

> *Distributing these profits equally is not fair as some users will contribute more to the value of the data set, which can be measured through the quality and quantity of data points provided.*

TABLE 2 An illustrative example of a low-quality data set shared by a user.

Instance ID	User ID	URL	Day (m/d/y)	Time	Has Adblocker	OS	Browser	Gender	Zip
1	byn8	nyt.com	–	–	Yes	Windows	Chrome	–	–
2	byn8	nyt.com	–	–	Yes	Windows	Chrome	–	–
3	byn8	nike.com	–	–	Yes	Windows	Chrome	–	–
4	byn8	dior.com	–	–	Yes	Windows	Chrome	–	–
5	byn8	usps.com	–	–	Yes	Windows	Chrome	–	–

The development of a revenue-allocation scheme that accounts for the individual contributions of members and maximizes the total surplus (i.e., total revenue less the privacy costs incurred by members) of the cooperative is precisely the goal of our analysis in this paper. We now formalize the desirable properties of such a scheme and outline our contribution.

1.1 Summary of contributions

Our setting consists of a data cooperative that designs and announces a revenue-allocation scheme. In response, each member of the cooperative decides the quality of data she shares with the cooperative. The quantity of data shared by a member is a random amount, drawn independently from a known distribution. The cooperative's objective is natural—to maximize the total surplus of the members of the cooperative from monetizing the pooled data, in equilibrium. To discuss the desirable properties of a revenue-allocation scheme, we first introduce the notion of *centralized surplus*: For any set of members, the centralized surplus of that set is the maximum total expected surplus obtained under the following conditions: (i) The cooperative consists only of the members in that set and (ii) the quality decisions of all the members in that set are determined in a centralized manner. We now define three properties associated with an allocation scheme:

(a) An ideal benchmark on the cooperative's objective is the centralized surplus of the set of all the members in the cooperative, since this value is an upper bound on the total expected surplus the cooperative can achieve under *any* allocation scheme. Accordingly, an allocation scheme is said to achieve the *first-best* surplus if the cooperative's objective value under that scheme equals this benchmark.

(b) An allocation scheme is individually rational if the surplus obtained by each member of the cooperative under that scheme is at least the centralized surplus of that member, that is, the centralized surplus of the singleton set consisting of that member. Thus, under such a scheme, no member has an incentive to break away and share data as a separate entity.

(c) An allocation scheme satisfies *coalitional stability* if, for each (sub)set of members of the cooperative, the total expected surplus of the members in that set under the scheme is at least the centralized surplus of that set. Thus, if a scheme satisfies coalitional stability, then no set of members has an incentive to break away and share data as a separate coalition.

Throughout the paper, we will focus on these three properties of a revenue-allocation scheme. We now summarize the road map of our analysis.

- We begin by examining an intuitive allocation scheme—namely, the *proportional allocation scheme* (hereafter, PROP scheme)—where the share of the total revenue that members receive is proportional to their respective individual contributions. When the privacy costs of the members are homogeneous, we show that this scheme achieves the first-best surplus, is individually rational, and satisfies coalitional stability (Section 4). Interestingly, in this case, we also show that the PROP scheme is equivalent (i.e., it implements the same allocation) to a widely known allocation scheme in cooperative games—namely, the Shapley-value allocation scheme (hereafter, SHAP scheme); see, for example, Roth (1988) and Driessen (2013). As we will discuss later, while the SHAP scheme is difficult to implement in our context, the PROP scheme is intuitive and easy to implement. However, when the privacy costs are heterogeneous, the PROP scheme may not achieve the first-best surplus; in Section 5.1, we characterize the conditions under which it achieves the first-best surplus.
- Next, we improve upon the PROP scheme by developing a *hybrid allocation scheme* (hereafter, HYB scheme) that accounts for the heterogeneity in the privacy costs of the members via a carefully chosen adjustment in payment. This scheme can be broadly described as follows: Under the conditions where the PROP scheme does not achieve the first-best surplus, the HYB scheme offers an additional payment (vis-à-vis the PROP scheme) to members with high privacy cost and deducts a payment (vis-à-vis the PROP scheme) for members with low privacy cost. In Sections 5.2 and 5.3, we show that the HYB scheme achieves the first-best surplus; however, it does not

guarantee coalitional stability and individual rationality—this property may be violated when the payment deductions for low–privacy-cost members are taxing to the extent that they prefer to break away from the cooperative to form a separate coalition.

- The analysis of the HYB scheme reveals that simply adjusting members' payoffs on the basis of their privacy costs is not sufficient to develop an allocation scheme that satisfies the three desirable properties defined above. To this end, we develop our *RobinHood* allocation scheme (hereafter, RHOOD scheme) that implements a more sophisticated adjustment in the payoffs of the members to account for their heterogeneity. In situations where the PROP scheme achieves the first-best surplus, the RHOOD scheme is the same as the PROP scheme. Otherwise, the RHOOD scheme functions as follows: A fraction $\beta, \beta \in [0, 1]$, of the revenue received upon monetization is set aside and the remaining $(1 - \beta)$ fraction is allocated to the members by adjusting their payoffs in a manner similar to that in the HYB scheme. The reserved β fraction of the revenue is used to compensate members' costs, including additional compensation to low–privacy-cost members to prevent them from breaking away from the cooperative, for ensuring coalitional stability. For an appropriately chosen value of β, we show that the RHOOD scheme achieves the first-best surplus, is individually rational, and satisfies coalitional stability. Furthermore, we show that the surplus of *each* individual member is higher under the RHOOD scheme as compared to that under the PROP scheme.

In summary, our analysis shows that while distributing revenue proportional to members' data contributions is appealing, such a distribution may not achieve properties that are deemed desirable by the cooperative. Instead, the cooperative should engage in the act of a Robin Hood—take from the "privacy rich" (i.e., low–privacy-cost members) and compensate the "privacy poor."

In the classical analysis of cooperative games, the values associated with the various plausible coalitions of the members (specified by what is commonly known as the characteristic function) are typically exogenously given and the focus is on the analysis of the allocation schemes that satisfy certain structural properties, for example, coalitional stability. However, in the context of data cooperatives, the value associated with a coalition (i.e., the revenue received upon monetization of the pooled data of that coalition) *endogenously* depends on the decisions of the individual members of that coalition—namely, the quality of data they share; see, for example, Carraro (2003) for a comprehensive discussion on the endogenous formation of economic coalitions. Thus, while the quality decisions of the members are clearly influenced by the allocation scheme, a surplus-maximizing allocation scheme should, in turn, take into account the quality decisions of the members. Our study contributes to the extant literature by developing and analyzing surplus-maximizing allocation schemes in the presence of this endogeneity.

The emerging regulations on data governance across the globe, coupled with modern-day information technologies, has led to the formation of new institutions such as data cooperatives. The analysis of data cooperatives is becoming an important area of study, since they create value from pooling the data of individuals and thereby empowering them by monetizing the data they generate. To the best of our knowledge, ours is the first comprehensive study that develops a tractable framework for data pooling and analyzes attractive revenue-allocation schemes.

2 | RELATED LITERATURE

Our work is related to two streams of literature: (i) data monetization and (ii) applications of cooperative game theory. We review the relevant literature in these streams.

The growth in the value of personal data has led to firms engaging in the buying and selling of data sets. Najjar and Kettinger (2013) analyze the pathways for firms that seek data monetization. Data are an information good and there is a rich stream of literature that studies a variety of issues related to the monetization of information goods such as bundling (Bakos & Brynjolfsson, 1999), pricing (Choudhary, 2010; Sundararajan, 2004), and differentiation (Bhargava & Choudhary, 2001, 2008). More recently, Mehta, Dawande, Janakiraman, et al. (2021) and Mehta et al. (2022) argue that the context of data monetization offers a richer setting compared to traditional information goods. In these papers, the authors model data as structured information that can be arranged in a row–column format and analyze the performance of popular pricing mechanisms.

The rise in data monetization has led to the emergence of data markets; we refer the reader to Liang et al. (2018) for a recent survey on various aspects of these markets. These markets cater to different needs of the buyers and sellers, and face various challenges. For instance, Cai et al. (2019) analyze the market for trading the web-browsing histories of individuals with heterogeneous privacy preferences and Rasouli and Jordan (2021) study the equilibrium outcome for unilateral (trading data in exchange of money) and bilateral (trading data in exchange of data) data exchanges. Bimpikis et al. (2019) study the problem of selling information (e.g., forecasts) to a set of competing buyers in a downstream market. Miklós-Thal et al. (2023) show that as data buyers become progressively better at drawing inferences from consumer data, the data-sharing choices of the consumers polarize—the proportion of consumers who partially share data diminishes whereas as the proportion of consumers who either share no data or full data grows.

There is an extensive literature on applications of cooperative games in the broader domain of management science. Forming coalitions and alliances offers players several advantages, including enabling them to better serve uncertain demand (Nagarajan & Sošić, 2007), improve coordination (Cai & Vairaktarakis, 2012), share common costs and revenue (Hu et al., 2013; Zhang et al., 2022), share data (Ghoshal et al., 2020), and improve bargaining power (Nagarajan & Bassok, 2008). The specific business settings considered in the literature are numerous; we briefly mention a few. Bergantiños and Moreno-Ternero (2020) study the problem of sharing the revenue from broadcasting sports events among the participating teams. Kemahlıoğ and Bartholdi III (2011) consider inventory-pooling coalitions and study ways to allocate the excess profit due to pooling among the players. These studies assume that the value functions associated with the possible coalitions are exogenously given. There are fewer papers with endogenous value functions. Fang and Cho (2020) and Chen et al. (2020) examine the cooperative approach of "joint" auditing for firms to manage social-responsibility violations of their suppliers. Granot and Yin (2008) and Yin (2010) consider coalitions formed by suppliers who produce complementary components, to better coordinate their pricing and production decisions. In these studies, each coalition makes a common decision and allocates the resulting total profit among its members. This is different from our setting where each member makes an individual decision based on the announced allocation scheme. Granot and Sošić (2003) and Fang and Cho (2014) study a problem where retailers first make individual inventory decisions and then form a cooperative and transship excess inventories among themselves. Gopalakrishnan et al. (2021) consider supply chains with joint production of greenhouse gas emissions and develop a scheme to reapportion the total emissions among the firms and induce them to reduce emissions. We contribute to this stream of literature by examining a relatively new organizational structure, namely, a data cooperative. Our main goal here is to design effective revenue-allocation schemes that are individually rational and satisfy coalitional stability, and enable data cooperatives to achieve an attractive surplus under heterogeneous privacy costs incurred by individual members.

Our analysis also examines the relationship of our PROP scheme with the well-known Shapley-value allocation (Roth, 1988) and the traditional "proportional" allocation where only two kinds of coalitional values matter: utilities of individual members and the utility of the grand coalition (Amer et al., 2008). The notion of proportional allocation has a very broad connotation in the theory of cooperative games, in the sense that the unit that needs to be proportionally allocated among the members of the cooperative can either be the revenue earned (Hu et al., 2013), or the cost incurred (Fiestras-Janeiro et al., 2011), or the net surplus generated, and so forth, by the cooperative. Furthermore, the unit that is being allocated among the members can be proportional to their effort (Kamijo, 2009), their valuation (Amer et al., 2008), and so forth.

3 | MODEL PRELIMINARIES

In this section, we introduce the key elements of our model.

Data: The data set generated by each member of the cooperative is assumed to be represented in a tabular form (see Tables 1 and 2 for illustrative examples), which consists of rows and columns (Mehta, Dawande, Janakiraman, et al., 2021). The rows of a data set represent the instances of data generation and the columns represent the various data types associated with each instance. We define the *quantity* of a data set as the total number of instances (rows) in that data set. The *quality* of a data set depends on the set of data types (columns) that the individual member decides to share with the cooperative. Let $\mathcal{C}$ denote the set of all the columns in a data set. For any set $\mathcal{S} \subseteq \mathcal{C}$ of columns, the mapping $q : \mathcal{S} \rightarrow [0, 1]$ specifies the quality of the data set. The quality set function q satisfies the following properties: $q(\phi) = 0$, $q(\mathcal{C}) = 1$, and for any $\mathcal{S} \subset \mathcal{T}, q(\mathcal{S}) < q(\mathcal{T})$, where $\mathcal{S}, \mathcal{T} \subseteq \mathcal{C}$.

Remark 1 (Quality of a data set). The quality of a member's data set is defined above as a function of the set of columns shared by that member. This simplicity is purely for expositional clarity. The quality function q can be generalized to include several other characteristics of the data set, such as the frequency and nature of website visits and the time of data generation. Our analysis only requires that the members know the quality function and can decide the precise quality of data they want to share. Further, the quality set function q need not map subsets of the same cardinality to the same number. Put differently, the set function can map subsets of attributes that are more important to a higher number compared to those attributes that are less important.

Contribution of a data set:
Let $\mathcal{N} = \{1, 2, \ldots, n\}$ denote the set of all the members of the cooperative. Let $\widetilde{Q}_i$ denote[2] the random quantity of data shared by the ith member of the cooperative, $i \in \mathcal{N}$. We assume that the quantity of data shared by each member of the cooperative is drawn independently from an identical distribution, denoted by $\mathcal{F}$, with mean Q and variance σ^2. The quality of the data set shared by the ith member of the cooperative is denoted by q_i, $i \in \mathcal{N}$. We define the *contribution* of a data set with quality q and quantity $\widetilde{Q}$ as $\widetilde{T} := \widetilde{Q} \cdot q$; intuitively, this captures the "effective volume" of the data set. Thus, the contribution of the ith member's data set is $\widetilde{T}_i = \widetilde{Q}_i \cdot q_i$.

Privacy cost:
Members incur a privacy cost for sharing their data; this cost is proportional to the contribution of the data set they share with the data cooperative. Thus, the privacy cost incurred by member i is $c_i \cdot \widetilde{Q}_i \cdot q_i$, where $c_i \geq 0$ is a constant that signifies this member's unit cost of privacy. We assume that the constants $c_i, i \in \mathcal{N}$, are known to the cooperative.[3] Our communication with several data cooperatives revealed that they elicit a significant amount of information—via instruments such as personal interviews, surveys, simulated games, and actual evaluation of members' data-sharing patterns over a probationary period—targeted at precisely estimating a potential member's sensitivity to privacy before that member is compensated for sharing her data. A rigorous approach for estimating a member's privacy cost is the VOPE (Value of

Privacy Estimator) methodology (Hirschprung et al., 2016), which has its foundations in Prospect Theory (Kahneman & Tversky, 1979) and requires the member to participate in simulated iterative games. Another approach is to learn a member's privacy cost over a probationary period, based on that member's data-sharing responses to a set of carefully designed price offers. Other approaches in the literature for estimating members' privacy costs include (a) direct and indirect surveys (Braunstein et al., 2011); (b) states of privacy transition, which requires members to ordinally rank a set of privacy decisions in a variety of real-world scenarios (Preibusch, 2013); and (c) worst-case analysis, which computes the maximal monetary loss incurred by a member due to information disclosure (Longpré & Kreinovich, 2006).

Data monetization:
The cooperative aggregates the individual data sets shared by the members, monetizes the pooled data set, and distributes the revenue among the members. The higher the contribution of the pooled data set, the higher is the cooperative's bargaining power, which leads to a better price upon monetization.

There are two distinguishing characteristics of consumer data pooled by a data cooperative: (i) Consumer data generate externality: Data shared by an individual disclose information about other similar individuals; further, the higher the quantity and quality of the data, the stronger is this externality (Acemoglu et al., 2019); and (ii) members of the cooperative *voluntarily*[4] share their data for monetization. Data buyers usually do not have access to such voluntarily shared consumer data, and therefore such data are considered new or fresh data. The value of data (to data buyers) follows an S-shaped curve (Farboodi & Veldkamp, 2021); that is, there are increasing returns to scale until one collects a sufficiently large amount of fresh data, followed by diminishing returns. Individuals typically generate a small amount of data and most modern-day data cooperatives are of modest size; thus, the pooled data from a data cooperative provide increasing returns upon monetization. Together, the above two features imply that the marginal value of pooled data is increasing in both the quantity and quality of the data shared by the individuals.

This assumption has been used in the literature for modeling the revenue received by a cooperative; see, for example, Mu et al. (2019). Accordingly, we assume that the total revenue received by the cooperative by monetizing a pooled data set with contribution T is $P(T) := p_0 \cdot T + p_1 \cdot T^2$, where p_0 and p_1 are nonnegative constants.[5] The contribution of the pooled data set in which the ith member's contribution is $\widetilde{T}_i = \widetilde{Q}_i \cdot q_i$, is $\widetilde{T}_{\mathcal{N}} := \sum_{i\in\mathcal{N}} \widetilde{T}_i = \sum_{i\in\mathcal{N}} \widetilde{Q}_i \cdot q_i$. Thus, the total revenue received by the cooperative for the pooled data set is

$$P(\widetilde{T}_{\mathcal{N}}) = p_0 \cdot \left(\sum_{i\in\mathcal{N}} \widetilde{Q}_i \cdot q_i\right) + p_1 \cdot \left(\sum_{i\in\mathcal{N}} \widetilde{Q}_i \cdot q_i\right)^2.$$

Properties of allocation schemes:
We consider allocation schemes that distribute the *entire revenue* of the cooperative to its members[6]. For an allocation scheme σ, let $\widetilde{x}_i^\sigma$ denote the allocation of the total monetized revenue $P(\widetilde{T}_{\mathcal{N}})$ to the ith member of the cooperative, and let $\widetilde{\pi}_i^\sigma := \widetilde{x}_i^\sigma - c_i\widetilde{T}_i$ denote her corresponding surplus under that scheme. For any set $\mathcal{S}, \mathcal{S} \subseteq \mathcal{N}$, let $\pi_{\mathcal{S}}^{\text{CS}}$ denote the maximum total expected centralized surplus of $\mathcal{S}$; this is obtained by imposing that (a) the cooperative consists only of the members in $\mathcal{S}$ and (b) the quality decisions of these members are determined in a centralized manner. Mathematically,

$$\pi_{\mathcal{S}}^{\text{CS}} = \max_{q_i, i\in\mathcal{S}} \mathbb{E}\left[P(\widetilde{T}_{\mathcal{S}}) - \sum_{i\in\mathcal{S}} c_i\widetilde{T}_i\right], \ \mathcal{S} \subseteq \mathcal{N}. \quad (1)$$

Thus, $\pi_{\mathcal{S}}^{\text{CS}}$ is an upper bound on the total expected surplus that the members of $\mathcal{S}$ can obtain as a separate coalition, under any scheme. For $\mathcal{S} = \mathcal{N}$, we refer to $\pi_{\mathcal{N}}^{\text{CS}}$ as the first-best surplus and the corresponding optimal data-quality decisions as the first-best outcome.

Coalitional stability:
We say that an allocation scheme σ satisfies coalitional stability if

$$\sum_{i\in\mathcal{S}} \mathbb{E}\left[\widetilde{\pi}_i^\sigma\right] \geq \pi_{\mathcal{S}}^{\text{CS}} \ \forall \mathcal{S} \subseteq \mathcal{N}. \quad (2)$$

Thus, a scheme σ satisfies coalitional stability if for *each* set $\mathcal{S} \subseteq \mathcal{N}$, the total expected surplus of the members in $\mathcal{S}$ under that scheme is at least the centralized surplus of that set, that is, $\pi_{\mathcal{S}}^{\text{CS}}$. Note that the centralized surplus of a set is the maximum possible expected surplus that the set of members can achieve. Thus, coalitional stability ensures that no set of the members has an incentive to break away, *regardless* of the allocation scheme. In other words, if an allocation scheme satisfies coalitional stability, then for *any* set of members, no other allocation scheme can offer a strictly higher surplus to that set of members.

Remark 2 (Coalitional stability in our setting). In the literature on cooperative games, the notion of *core* is well established when the grand coalition is assumed and the value function $v(\mathcal{S}), \mathcal{S} \subseteq \mathcal{N}$, is exogenously given; see, for example, Driessen (2013). An allocation scheme σ belongs to the core if $\sum_{i\in\mathcal{S}} z_i \geq v(\mathcal{S})$ for $\forall \mathcal{S} \subseteq \mathcal{N}$, where z_i denotes the allocation to member i under σ. In this case, the grand coalition is said to satisfy coalitional stability under σ. For our analysis, however, we need a modified definition of coalitional stability suitable for our setting where agents make endogenous quality decisions in response to the allocation scheme and the equilibrium coalition may not be the grand coalition.

Individual rationality:
An allocation scheme σ is said to be individually rational if the inequalities in (2) hold for all singleton sets, that is,

$$\mathbb{E}\left[\widetilde{\pi}_i^{\sigma}\right] \geq \pi_i^{\text{CS}} \; \forall i \in \mathcal{N}. \tag{3}$$

An individually rational scheme ensures that the expected surplus of each member $i \in \mathcal{N}$ under that scheme is at least the centralized surplus of the singleton set $\{i\}$. Note that if a scheme σ satisfies coalitional stability, then it is also individually rational.

Equilibrium outcomes:
Throughout the analysis, we focus on symmetric equilibrium outcomes, wherein members with the same privacy-cost parameter decide to share the same quality of data. Further, when there are multiple symmetric equilibria, we focus on the Pareto-dominant equilibrium outcome; that is, the equilibrium under which all members are (weakly) better-off as compared to the other equilibria.

We begin our analysis by examining an intuitive allocation scheme—namely, the proportional (PROP) allocation scheme.

4 | THE PROPORTIONAL (PROP) ALLOCATION SCHEME

Under the PROP scheme, the allocation $\widetilde{x}_i^{\text{PROP}}$ to the ith member of the cooperative, whose contribution is $\widetilde{T}_i$, is defined as:

$$\widetilde{x}_i^{\text{PROP}} := \frac{\widetilde{T}_i}{\widetilde{T}_{\mathcal{N}}} \cdot P(\widetilde{T}_{\mathcal{N}}) = \widetilde{T}_i \cdot (p_0 + p_1 \widetilde{T}_{\mathcal{N}}), i \in \mathcal{N}. \tag{4}$$

Thus, under the PROP scheme, each member receives a share of the total revenue that is proportional to her individual contribution. For better exposition, we first analyze the case where the per-unit privacy costs of the members are homogeneous, that is, $c_i = c$, for all $i \in \mathcal{N}$; the case of heterogeneous privacy costs will be analyzed in Section 5. Let $\widetilde{\pi}_i^{\text{PROP}}$ denote the surplus (revenue less the privacy cost) of the ith member with contribution $\widetilde{T}_i$ under the PROP scheme. Then,

$$\widetilde{\pi}_i^{\text{PROP}} = \widetilde{x}_i^{\text{PROP}} - c \cdot \widetilde{T}_i, i \in \mathcal{N}. \tag{5}$$

Finally, let $\widetilde{\pi}_{\mathcal{N}}^{\text{PROP}} := \sum_{i \in \mathcal{N}} \widetilde{\pi}_i^{\text{PROP}}$ denote the total surplus of the cooperative under the PROP scheme. Theorem 1 states the equilibrium decisions of the members and the total expected surplus of the cooperative under the PROP scheme.

Theorem 1 (Homogeneous privacy cost: Equilibrium under the PROP scheme). *Assume that the per-unit privacy costs of all the members are homogeneous, that is, $c_i = c$ for all $i \in \mathcal{N}$.*

1. *If $c > p_0 + p_1(nQ + \frac{\sigma^2}{Q})$, then none of the members share their data with the cooperative (specifically, $q_i^* = 0$, for all $i \in \mathcal{N}$) in the unique equilibrium under the PROP scheme, and the total expected surplus of the cooperative, $\mathbb{E}[\widetilde{\pi}_{\mathcal{N}}^{PROP}]$, is 0.*
   - 2(*a*). *If $p_0 + p_1(Q + \frac{\sigma^2}{Q}) < c \leq p_0 + p_1(nQ + \frac{\sigma^2}{Q})$, then all the members share the highest quality of data with the cooperative (i.e., $q_i^* = 1$, for all $i \in \mathcal{N}$) in the Pareto-dominant equilibrium under the PROP scheme.*
   - 2(*b*). *If $c \leq p_0 + p_1(Q + \frac{\sigma^2}{Q})$, then all the members share the highest quality of data with the cooperative (i.e., $q_i^* = 1$, for all $i \in \mathcal{N}$) in the unique equilibrium under the PROP scheme.*

   In both cases 2(a) and 2(b), the total expected surplus of the cooperative is $\mathbb{E}[\widetilde{\pi}_{\mathcal{N}}^{PROP}] = nQ(p_0 + p_1(nQ + \frac{\sigma^2}{Q}) - c)$.

The proof of Theorem 1 is in Appendix A. We now elaborate on the main message from Theorem 1: Under the PROP scheme and homogeneous privacy cost, members share data with the cooperative only if their per-unit privacy cost is lower than a threshold. Note that the surplus of the ith member is convex in the quality q_i of the data she shares with the cooperative. Since $q_i \in [0, 1] \; \forall i \in \mathcal{N}$, it follows that a utility-maximizing member either shares no data (i.e., $q_i^* = 0$) or shares the highest quality of data (i.e., $q_i^* = 1$), in equilibrium. Further, since the privacy cost is homogeneous, a symmetric equilibrium achieves one of the following two outcomes: (i) None of the members share their data ($q_i^* = 0 \; \forall i \in \mathcal{N}$) and (ii) all the members share the highest quality of data ($q_i^* = 1 \; \forall i \in \mathcal{N}$). When the per-unit privacy cost is low ($c \leq p_0 + p_1(Q + \frac{\sigma^2}{Q})$), the unique equilibrium is $\{q_i^* = 1, i \in \mathcal{N}\}$, and when this cost is high ($c > p_0 + p_1(nQ + \frac{\sigma^2}{Q})$), the unique equilibrium is $\{q_i^* = 0, i \in \mathcal{N}\}$. On the other hand, when the cost is in between ($p_0 + p_1(Q + \frac{\sigma^2}{Q}) < c \leq p_0 + p_1(nQ + \frac{\sigma^2}{Q})$), both $\{q_i^* = 0, i \in \mathcal{N}\}$ and $\{q_i^* = 1, i \in \mathcal{N}\}$ are possible equilibrium outcomes. The latter equilibrium—where all the members share the highest quality of data—is Pareto-dominant.

Next, we examine key properties of the PROP scheme under homogeneous privacy cost.

4.1 | Properties of the PROP scheme under homogeneous privacy cost

We begin by comparing the equilibrium of the PROP scheme with the first-best outcome. The first-best outcome, that is, the solution to the centralized setting where the cooperative itself decides the quality decisions of the members, provides a natural upper bound on the total surplus of the cooperative (i.e., the total revenue to the cooperative less the sum of privacy costs incurred by all the members). If each individual member shares data of quality q, then the total expected surplus of the cooperative is:

$$\mathbb{E}\left[(p_0 + p_1\widetilde{T}_{\mathcal{N}}) \cdot \widetilde{T}_{\mathcal{N}} - \sum_{i\in\mathcal{N}} cq\widetilde{Q}_i\right] = p_1 n(nQ^2 + \sigma^2)q^2 + n(p_0 - c)Qq. \tag{6}$$

Maximizing the total surplus with respect to quality q gives us the first-best outcome, which we state in Theorem 2.

Theorem 2 (Homogeneous privacy cost: First-best outcome). *Assume that the per-unit privacy costs of all the members are homogeneous, that is, $c_i = c$ for all $i \in \mathcal{N}$.*

- *If $c > p_0 + p_1(nQ + \frac{\sigma^2}{Q})$, then none of the members share their data with the cooperative (i.e., $q_i^* = 0$, for all $i \in \mathcal{N}$), and the total expected surplus of the cooperative is 0.*
- *If $c \le p_0 + p_1(nQ + \frac{\sigma^2}{Q})$, then all the members share the highest quality of data with the cooperative (i.e., $q_i^* = 1$, for all $i \in \mathcal{N}$), and the maximum total expected surplus of the cooperative is $nQ(p_0 + p_1(nQ + \frac{\sigma^2}{Q}) - c)$.*

The proof of Theorem 2 is in Appendix B. It follows immediately from Theorems 1 and 2 that when the privacy cost is homogeneous, the PROP scheme achieves the first-best surplus. The following result notes this and also comments on the individual rationality and coalitional stability of the PROP scheme.

Proposition 1 (Homogeneous privacy cost: Properties of the PROP scheme). *If the per-unit privacy costs of the members are homogeneous, that is, $c_i = c$, for all $i \in \mathcal{N}$, then the PROP scheme achieves the first-best surplus and satisfies coalitional stability (and hence is also individual rational). Further, in the nontrivial case where $c \le p_0 + p_1(nQ + \frac{\sigma^2}{Q})$, all the members of the cooperative share the highest quality of data.*

The proof of Proposition 1 is in Appendix C. In Supporting Information Appendix D and P, we contrast the PROP scheme with the well-known Shapley-value (SHAP) allocation. Thus far, our analysis assumed that the per-unit privacy cost of the members is homogeneous. We now examine the case where these costs are heterogeneous.

5 | HETEROGENEOUS PRIVACY COST: THE HYBRID (HYB) ALLOCATION SCHEME

In this section, we consider heterogeneous privacy costs. Specifically, we assume two types of members[7] in the cooperative: type-L members have a per-unit privacy cost of c_L and type-H members have a per-unit privacy cost of c_H, where $0 \le c_L < c_H$. The type-L (resp., type-H) members comprise a γ (resp., $1-\gamma$) fraction of the $|\mathcal{N}| = n$ members of the cooperative; $0 < \gamma < 1$. Let $\mathcal{N}_L$ (resp., $\mathcal{N}_H$) denote the set of type-L (resp., type-H) members. We begin by analyzing the PROP scheme under these two types of privacy costs. In Section 5.1, we characterize the equilibrium under the PROP scheme and show that, under certain conditions, the PROP scheme does not achieve the first-best outcome. Next, in Section 5.2, we develop a hybrid allocation (HYB) scheme that accounts for the heterogeneity in privacy costs. Finally, in Section 5.3, we examine individual rationality and coalitional stability under the HYB scheme.

5.1 | The PROP scheme versus the first-best outcome

Let q_{Li}, $\widetilde{T}_{Li}$, and $\widetilde{\pi}_{Li}$ (resp., q_{Hj}, $\widetilde{T}_{Hj}$, and $\widetilde{\pi}_{Hj}$) denote the quality of data, the contribution of the data set, and the equilibrium surplus, of the ith type-L (resp., jth type-H) member, respectively.

For the heterogeneous-privacy-cost case, the PROP scheme is defined as follows:

$$\tilde{x}_{Li}^{\text{PROP}} := \frac{\widetilde{T}_{Li}}{\widetilde{T}_{\mathcal{N}}} \cdot P(\widetilde{T}_{\mathcal{N}})\ \forall i \in \mathcal{N}_L, \text{and } \tilde{x}_{Hj}^{\text{PROP}} := \frac{\widetilde{T}_{Hj}}{\widetilde{T}_{\mathcal{N}}} \cdot P(\widetilde{T}_{\mathcal{N}})\ \forall j \in \mathcal{N}_H.$$

To characterize the PROP scheme under heterogeneous privacy costs, it is convenient to first define a few thresholds. Let $\tau_1 := p_0 + p_1(\gamma nQ + \frac{\sigma^2}{Q})$, $\tau_2 := p_0 + p_1(nQ + \frac{\sigma^2}{Q})$, and $\tau_3 := p_0 + p_1((1+\gamma)nQ + \frac{\sigma^2}{Q})$. The following result characterizes the equilibrium outcome under the PROP scheme.

Theorem 3 (Heterogeneous privacy cost: Equilibrium under the PROP scheme). *Under heterogeneous per-unit privacy costs of the members, the equilibrium under the PROP scheme is characterized in Table 3; a visual representation is shown in the left panel of Figure 1.*

The proof of Theorem 3 is in Appendix E. Theorem 3 specifies a threshold structure of the equilibrium outcome under the PROP scheme: Type-L members share the highest quality of data if either (i) their per-unit privacy cost c_L is less than the threshold τ_1 or (ii) if the per-unit privacy cost of type-H members is less than the threshold τ_2. On the other hand, type-H members share the highest quality of data only when their per-unit privacy cost c_H is less than the threshold τ_2. Note that the thresholds τ_1 and τ_2 are both increasing in the size (n) of the cooperative. Thus, as more members join the cooperative, the benefit from monetizing the pooled data helps offset members' privacy costs, which in turn incentivizes them to share data. In the left panel of Figure 1, this effect corresponds to the expansion of the green region (where both types

TABLE 3 Equilibrium outcome under the PROP scheme.

Parametric condition	Equilibrium outcome	Equilibrium surplus of type-L members	Equilibrium surplus of type-H members
$(c_L > \tau_1) \cap (c_H > \tau_2)$	No one shares $q_{Li} = q_{Hj} = 0, \forall i \in \mathcal{N}_L, j \in \mathcal{N}_H$	0	0
$(c_L \le \tau_1) \cap (c_H > \tau_2)$	Only type-L shares $q_{Li} = 1, q_{Hj} = 0, \forall i \in \mathcal{N}_L, j \in \mathcal{N}_H$	$n\gamma \cdot Q(\tau_1 - c_L)$	0
$c_H \le \tau_2$	Both types share $q_{Li} = q_{Hj} = 1, \forall i \in \mathcal{N}_L, j \in \mathcal{N}_H$	$n\gamma \cdot Q(\tau_2 - c_L)$	$n(1-\gamma) \cdot Q(\tau_2 - c_H)$

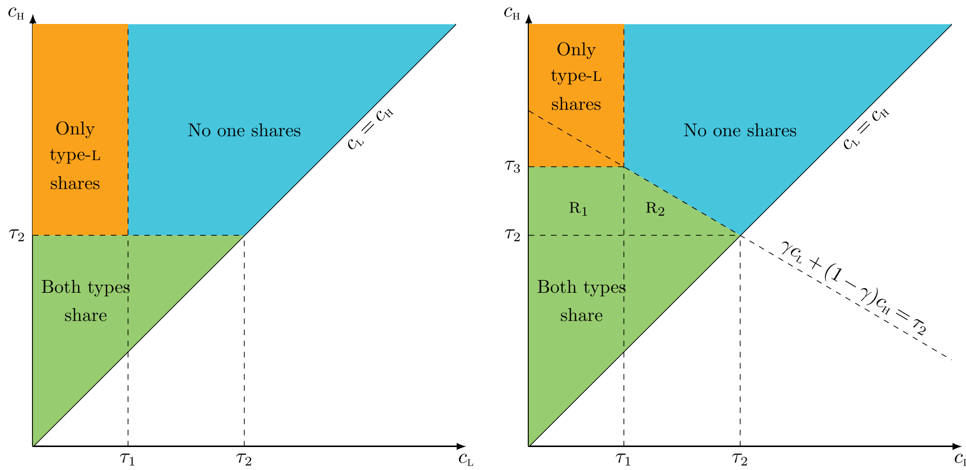

FIGURE 1 Left panel: Equilibrium under the PROP scheme with heterogeneous privacy costs of the members. Right panel: The first-best outcome under heterogeneous privacy costs. [Color figure can be viewed at wileyonlinelibrary.com]

share their data) and the orange region (where only type-L members share their data).

Next, we characterize the first-best outcome, where the cooperative itself decides the quality decisions of the members; let q_L and q_H denote the quality of data shared by type-L and type-H members, respectively. The total expected surplus of the cooperative is:

$$\mathbb{E}\left[(p_0 + p_1\widetilde{T}_{\mathcal{N}})\widetilde{T}_{\mathcal{N}} - \sum_{i\in\mathcal{N}_L} \widetilde{Q}_i c_L q_L - \sum_{j\in\mathcal{N}_H} \widetilde{Q}_j c_H q_H\right]$$

$$= p_0 n Q(\gamma q_L + (1-\gamma)q_H) + p_1 n^2 Q^2 (\gamma q_L + (1-\gamma)q_H)^2$$
$$+ p_1 n\sigma^2\left(\gamma q_L^2 + (1-\gamma)q_H^2\right) - \gamma n Q c_L q_L$$
$$- (1-\gamma) n Q c_H q_H.$$

Theorem 4 specifies the first-best outcome obtained by maximizing the total expected surplus over the quality decisions q_L and q_H.

Theorem 4 (Heterogeneous privacy cost: First-best outcome). *Under heterogeneous per-unit privacy costs of the members, the first-best outcome is characterized in Table 4; a visual representation is shown in the right panel of Figure 1.*

The proof of Theorem 4 is in Appendix F. From Theorem 3 and Theorem 4, we see that there are two regions of the (c_L, c_H) parametric space where the PROP scheme does not achieve the first-best outcome:

- Region 1 (R_1): $(c_L \le \tau_1) \cap (\tau_2 < c_H \le \tau_3)$.
- Region 2 (R_2): $(c_L > \tau_1) \cap (c_H > \tau_2) \cap (\gamma c_L + (1-\gamma)c_H \le \tau_2)$.

First, we note that apart from Regions R_1 and R_2, the PROP scheme achieves the first-best revenue in the entire (c_L, c_H) parametric space. The regions R_1 and R_2 are characterized by low values of c_L and intermediate values of c_H. Under the PROP scheme, only type-L members share their data in region R_1 and no member shares data in region R_2. However, both type-L and type-H members share their data in regions R_1 and R_2 in the first-best outcome. Collectively, in both these regions, the PROP scheme fails to provide sufficient incentive to the members for sharing their data. The main reason behind the PROP scheme not being able to achieve the first-best outcome is that the scheme only considers the contributions of the members and fails

TABLE 4 The first-best outcome under heterogeneous privacy costs.

Parametric condition	Equilibrium outcome	Equilibrium surplus of type-L members	Equilibrium surplus of type-H members
$(c_L > \tau_1) \cap (\gamma c_L + (1-\gamma)c_H > \tau_2)$	No one shares $q_L^* = q_H^* = 0$	0	0
$(c_L \le \tau_1) \cap (c_H > \tau_3)$	Only type-L shares $q_L^* = 1, q_H^* = 0$	$n\gamma \cdot Q(\tau_1 - c_L)$	0
$(\gamma c_L + (1-\gamma)c_H \le \tau_2) \cap (c_H \le \tau_3)$	Both types share $q_L^* = q_H^* = 1$	$n\gamma \cdot Q(\tau_2 - c_L)$	$n(1-\gamma) \cdot Q(\tau_2 - c_H)$

to account for the privacy costs they incur when they share their data. Further, the PROP scheme also fails to satisfy coalitional stability in regions R_1 and R_2. The instability in these two regions stems from the insufficient compensation to the L-type members, which incentivizes them to break away from the cooperative. Proposition 2 below summarizes the properties of the PROP scheme under heterogeneous privacy costs.

Proposition 2. *Under heterogeneous per-unit privacy costs of the members, the PROP scheme is individually rational but does not achieve the first-best outcome and does not satisfy coalitional stability in regions R_1 and R_2.*

The proof of Proposition 2 is in Appendix G. The gap between the equilibrium outcome under the PROP scheme and the first-best outcome in regions R_1 and R_2 motivates our HYB scheme, which we develop and analyze in Section 5.2. In Supporting Information Appendix L, we analyze the status quo uniform allocation scheme that is being used at the Swash data cooperative (introduced in Section 1).

5.2 | The HYB scheme

Recall that, in both regions R_1 and R_2, the PROP scheme fails to incentivize type-H members to share their data. In the former region, only type-L members share data while, in the latter region, both the types do not share their data. Given the high privacy cost of type-H members, a natural idea to incentivize them to share data is to offer them additional compensation (vis-à-vis the PROP scheme) by reducing the compensation of type-L members. If we can carefully determine this additional compensation to ensure that (i) in region R_1, type-L members continue to share data despite their reduced compensation, and (ii) in region R_2, the sharing of data by type-H members, in turn, incentivizes type-L members to also share data, then one can hope to achieve the first-best outcome in both regions R_1 and R_2. Such an adjustment is precisely the goal of our next allocation scheme, which we refer to as the hybrid (HYB) scheme.

Consider the following adjustment to the PROP scheme in regions R_1 and R_2:

reduced compensation

$$\tilde{x}_{Li} = \tilde{x}_{Li}^{PROP} - k_L \widetilde{T}_{Li} \ \forall i \in \mathcal{N}_L,$$
$$\tilde{x}_{Hj} = \tilde{x}_{Hj}^{PROP} + k_H \widetilde{T}_{Hj} \ \forall j \in \mathcal{N}_H,$$

additional compensation

where $k_L, k_H \ge 0$ are constants. In regions R_1 and R_2, the surplus of the type-L and type-H members under this allocation scheme can then be written as:

$$\tilde{\pi}_{Li} = \tilde{x}_{Li}^{PROP} - k_L \widetilde{T}_{Li} - c_L \widetilde{T}_{Li} = \tilde{x}_{Li}^{PROP} - (c_L + k_L)\widetilde{T}_{Li} \ \forall i \in \mathcal{N}_L,$$
$$\tilde{\pi}_{Hj} = \tilde{x}_{Hj}^{PROP} + k_H \widetilde{T}_{Hj} - c_H \widetilde{T}_{Hj} = \tilde{x}_{Hj}^{PROP} - (c_H - k_H)\widetilde{T}_{Hj} \ \forall j \in \mathcal{N}_H.$$

This resembles the PROP scheme, with the "adjusted" per-unit privacy cost of type-L members as $c_L + k_L$ and that of type-H members as $c_H - k_H$. From Proposition 1, we know that the PROP scheme achieves the first-best outcome when the per-unit privacy cost is homogeneous. In the adjustment above, this is achieved when $c_L + k_L = c_H - k_H$. Further, balancing, in equilibrium, the inflow of the total expected adjusted payment to type-H members with the outflow of the total adjusted payment from type-L members, we get

$$\mathbb{E}\left[\sum_{i \in \mathcal{N}_L} k_L \widetilde{T}_{Li}\right] = \mathbb{E}\left[\sum_{j \in \mathcal{N}_H} k_H \widetilde{T}_{Hj}\right], \text{ which implies } n\gamma k_L Q = n(1-\gamma) k_H Q.$$

Solving these two equations gives us $k_L = (1-\gamma)(c_H - c_L)$ and $k_H = \gamma(c_H - c_L)$. Thus, we have the following adjustment to the PROP scheme in regions R_1 and R_2:

$$\tilde{x}_{Li}^{PROP\text{-}ADJ} = \tilde{x}_{Li}^{PROP} - (1-\gamma)(c_H - c_L)\widetilde{T}_{Li} \ \forall i \in \mathcal{N}_L, \qquad (7)$$

$$\tilde{x}_{Hj}^{PROP\text{-}ADJ} = \tilde{x}_{Hj}^{PROP} + \gamma(c_H - c_L)\widetilde{T}_{Hj} \ \forall j \in \mathcal{N}_H. \qquad (8)$$

TABLE 5 Equilibrium outcome under the HYB scheme.

Parametric condition	Equilibrium outcome	Equilibrium surplus of type-L members	Equilibrium surplus of type-H members
$(c_{\rm L} > \tau_1) \cap (\gamma c_{\rm L} + (1-\gamma)c_{\rm H} > \tau_2)$	No one shares $q^*_{{\rm L}i} = q^*_{{\rm H}j} = 0\ \forall i \in \mathcal{N}_{\rm L}, j \in \mathcal{N}_{\rm H}$	0	0
$(c_{\rm L} \leq \tau_1) \cap (c_{\rm H} > \tau_3)$	Only type-L shares $q^*_{{\rm L}i} = 1, q^*_{{\rm H}j} = 0\ \forall i \in \mathcal{N}_{\rm L}, j \in \mathcal{N}_{\rm H}$	$n\gamma \cdot Q(\tau_1 - c_{\rm L})$	0
$R_1 \cup R_2$	Both types share $q^*_{{\rm L}i} = q^*_{{\rm H}j} = 1\ \forall i \in \mathcal{N}_{\rm L}, j \in \mathcal{N}_{\rm H}$	$n\gamma \cdot Q(\tau_2 - \gamma c_{\rm L} - (1-\gamma)c_{\rm H})$	$n(1-\gamma)\cdot Q(\tau_2 - \gamma c_{\rm L} - (1-\gamma)c_{\rm H})$
$c_{\rm H} \leq \tau_2$	Both types share $q^*_{{\rm L}i} = q^*_{{\rm H}j} = 1\ \forall i \in \mathcal{N}_{\rm L}, j \in \mathcal{N}_{\rm H}$	$n\gamma \cdot Q(\tau_2 - c_{\rm L})$	$n(1-\gamma) \cdot Q(\tau_2 - c_{\rm H})$

We now define our HYB allocation scheme as follows:

$$\tilde{x}^{\rm HYB}_{{\rm L}i} = \begin{cases} \tilde{x}^{\rm PROP\text{-}ADJ}_{{\rm L}i} & \text{if } (c_{\rm L}, c_{\rm H}) \in R_1 \cup R_2 \\ \tilde{x}^{\rm PROP}_{{\rm L}i} & \text{otherwise} \end{cases} \quad \forall i \in \mathcal{N}_{\rm L}, \quad (9)$$

$$\tilde{x}^{\rm HYB}_{{\rm H}j} = \begin{cases} \tilde{x}^{\rm PROP\text{-}ADJ}_{{\rm H}j} & \text{if } (c_{\rm L}, c_{\rm H}) \in R_1 \cup R_2 \\ \tilde{x}^{\rm PROP}_{{\rm H}j} & \text{otherwise} \end{cases} \quad \forall j \in \mathcal{N}_{\rm H}. \quad (10)$$

We note that the adjustment in the PROP scheme is designed specifically for regions R_1 and R_2. Recall from Proposition 2 that except in regions R_1 and R_2, the PROP scheme achieves the first-best outcome everywhere else in the $(c_{\rm L}, c_{\rm H})$ parametric space and therefore, no other adjustment in compensation is needed.

The following result states the equilibrium outcome under the HYB scheme.

Theorem 5 (Equilibrium outcome under the HYB scheme). *The HYB scheme—defined via Equations (9) and (10)—achieves the first-best outcome. The equilibrium outcome under this scheme is characterized in Table 5.*

The proof of Theorem 5 is in Appendix H. While the adjustment in compensation discussed above enables the HYB scheme to achieve the first-best outcome, an undesirable consequence of this adjustment is that the scheme does not satisfy individual rationality and coalitional stability. We discuss this next.

5.3 | Individual rationality and coalitional stability of the HYB scheme

The HYB scheme is identical to the PROP everywhere in the $(c_{\rm L}, c_{\rm H})$ parametric space other than the region $R_1 \cup R_2$, and hence from Proposition 2, it satisfies coalitional stability in that domain.

In an effort to sufficiently incentivize the high–privacy-cost (i.e, type-H) members to share data in the regions R_1 and R_2, the HYB scheme awards them additional compensation at the expense of the low–privacy-cost (i.e, type-L) members. Specifically, from (7) and (9) above, the ith type-L member receives a reduced compensation of $(1-\gamma)(c_{\rm H} - c_{\rm L})\widetilde{T}_{{\rm L}i}$ under the HYB scheme (vis-à-vis the PROP scheme) in these two regions. When this reduction in compensation is sufficiently severe—specifically, in the region $R_1 \cap (c_{\rm H} - c_{\rm L} > np_1Q)$ of the $(c_{\rm L}, c_{\rm H})$ space—the type-L members have an incentive to break away, and hence the HYB scheme fails to satisfy coalitional stability in that region. Furthermore, in a subregion $R_1 \cap (c_{\rm H} - c_{\rm L} > np_1Q)$—specifically, in the region defined by

$$R_1 \cap \left(c_{\rm H} - c_{\rm L} > \left(\frac{n-1}{1-\gamma}\right)p_1Q\right),$$

the distortion in the compensation is severe to the extent that the HYB scheme also fails to satisfy individual rationality; see Figure 2. We establish these two results in Theorem 6.

Theorem 6 (Individual rationality and coalitional stability under HYB scheme). *The HYB scheme does not satisfy coalitional stability in the region $R_1 \cap (c_H - c_L > np_1Q)$ of the (c_L, c_H) parametric space. Further, the scheme does not satisfy individual rationality in the region $R_1 \cap (c_H - c_L > (\frac{n-1}{1-\gamma})p_1Q)$. In all the other regions, the HYB scheme satisfies coalitional stability, and consequently, individual rationality.*

The proof of Theorem 6 is in Appendix I.

6 | THE ROBIN HOOD (RHOOD) ALLOCATION SCHEME

While the HYB scheme achieves the first-best outcome, in some regions of the $(c_{\rm L}, c_{\rm H})$ parametric space, the action of this scheme can be destabilizing. Specifically, in these regions, the HYB scheme rewards type-H members by penalizing type-L members to the extent that the latter group finds it preferable to break away as a separate entity. Consequently, the HYB scheme is unable to guarantee coalitional stability (and even individual rationality). Thus, while the core idea behind the HYB scheme—namely, that of preferentially treating the type-H members—is appealing, the advantage it bestows on these members can get excessive

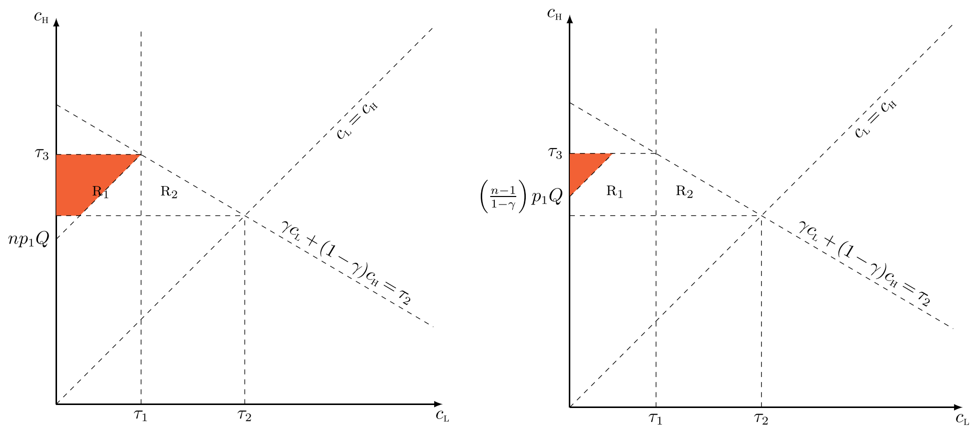

FIGURE 2 The HYB scheme fails to satisfy coalitional stability (left) in the region $\text{R}_1 \cap (c_H - c_L > np_1Q)$ and individual rationality (right) in the region $\text{R}_1 \cap (c_H - c_L > (\frac{n-1}{1-\gamma})p_1Q)$. [Color figure can be viewed at wileyonlinelibrary.com]

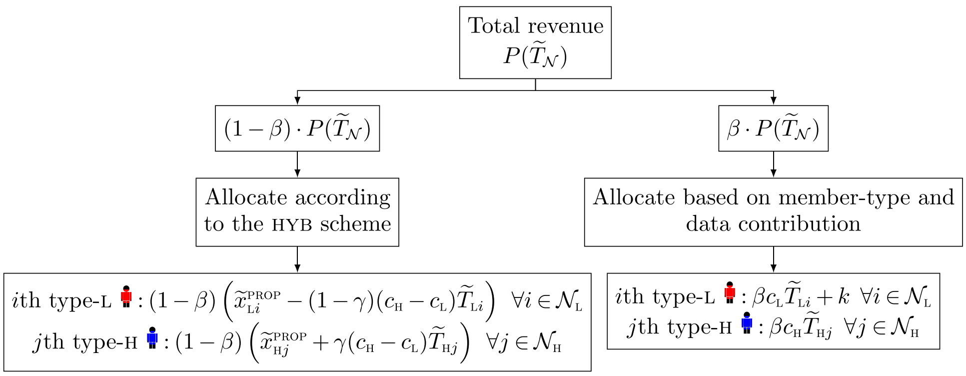

FIGURE 3 A schematic representation of the RHOOD scheme. [Color figure can be viewed at wileyonlinelibrary.com]

in some regions. One possible solution is an allocation that offers only a modest advantage to type-H members and uses a portion of the revenue to ensure coalitional stability. This more sophisticated rebalancing of compensation from the "privacy rich" (i.e., low–privacy-cost members) to the "privacy poor" results in a novel allocation scheme, which we refer to as the RobinHood (RHOOD) scheme. Importantly, we show that the RHOOD scheme achieves all our desirable properties—namely, achieves the first-best outcome and satisfies coalitional stability and individual rationality.

We now describe the main idea behind the development of the RHOOD scheme (see Figure 3 for a schematic representation). In regions R_1 and R_2, we consider the following allocation: For a fixed β, $\beta \in [0,1]$, the cooperative distributes $(1-\beta)$ fraction of the total revenue to the type-L and type-H members according to the HYB scheme (first terms in (11) and (12)). The remaining β fraction of the total revenue is distributed among the two types of members, based both on their type and respective contributions of their data sets, to ensure that the scheme satisfies coalitional stability and, thereby, individual rationality. Precisely, the remaining β fraction of the total revenue is distributed as follows:

- The jth type-H member receives a compensation of $\beta c_H \widetilde{T}_{\text{H}j}$, $\forall j \in \mathcal{N}_H$ (second term in (12)).
- The ith type-L member receives a compensation of $\beta c_L \widetilde{T}_{\text{L}i}$, $\forall i \in \mathcal{N}_L$ (second term in (11)) plus a fixed compensation $k \geq 0$ (third term in (11)) that is aimed at incentivizing that member to not break away from the cooperative.

Mathematically, these adjustments in regions R_1 and R_2 are defined as follows:

$$\tilde{x}^{\text{STAB-ADJ}}_{\text{L}i} := \underbrace{(1-\beta)\left(\tilde{x}^{\text{PROP}}_{\text{L}i} - (1-\gamma)(c_H - c_L)\widetilde{T}_{\text{L}i}\right)}_{(1-\beta)\text{ of total revenue compensated as per HYB scheme}} + \underbrace{\beta c_L \widetilde{T}_{\text{L}i}}_{\text{privacy-cost–basedcompensation}} + \underbrace{k}_{\text{fixed compensation}} \quad \forall i \in \mathcal{N}_L, \tag{11}$$

$$\tilde{x}^{\text{STAB-ADJ}}_{\text{H}j} := \underbrace{(1-\beta)\left(\tilde{x}^{\text{PROP}}_{\text{H}j} + \gamma(c_H - c_L)\widetilde{T}_{\text{H}j}\right)}_{(1-\beta)\text{ of total revenue compensated as per HYB scheme}} + \underbrace{\beta c_H \widetilde{T}_{\text{H}j}}_{\text{privacy-cost–basedcompensation}} \quad \forall j \in \mathcal{N}_H. \tag{12}$$

TABLE 6 Equilibrium outcome under the RHOOD scheme.

Parametric condition	Equilibrium outcome	Equilibrium surplus of type-L members	Equilibrium surplus of type-H members
$(c_L > \tau_1) \cap (\gamma c_L + (1-\gamma)c_H > \tau_2)$	No one shares $q^*_{Li} = q^*_{Hj} = 0\ \forall i \in \mathcal{N}_L, j \in \mathcal{N}_H$	0	0
$(c_L \le \tau_1) \cap (c_H > \tau_3)$	Only type-L shares $q^*_{Li} = 1, q^*_{Hj} = 0\ \forall i \in \mathcal{N}_L, j \in \mathcal{N}_H$	$n\gamma \cdot Q(\tau_1 - c_L)$	0
$R_1 \cup R_2$	Both types share $q^*_{Li} = q^*_{Hj} = 1\ \forall i \in \mathcal{N}_L, j \in \mathcal{N}_H$	$n\gamma \cdot (Q(1-\beta)(\tau_2 - \gamma c_L - (1-\gamma)c_H) + k)$	$n(1-\gamma) \cdot (Q(1-\beta)(\tau_2 - \gamma c_L - (1-\gamma)c_H))$
$c_H \le \tau_2$	Both types share $q^*_{Li} = q^*_{Hj} = 1\ \forall i \in \mathcal{N}_L, j \in \mathcal{N}_H$	$n\gamma \cdot Q(\tau_2 - c_L)$	$n(1-\gamma) \cdot Q(\tau_2 - c_H)$

The value of k can now be obtained by balancing the expected privacy-cost–based compensation (second terms in (11) and (12)) and fixed compensation (third term in (11)) with the β fraction of the total revenue, in equilibrium. Specifically,

$$\mathbb{E}\left[\sum_{i \in \mathcal{N}_L} (\beta c_L \widetilde{T}_{Li} + k)\right] + \mathbb{E}\left[\sum_{j \in \mathcal{N}_H} (\beta c_H \widetilde{T}_{Hj})\right] = \mathbb{E}\left[\beta \cdot P(\widetilde{T}_{\mathcal{N}})\right].$$

This gives us:

$$k = \frac{\beta Q(\tau_2 - \gamma c_L - (1-\gamma)c_H)}{\gamma}. \tag{13}$$

We now formally define the RHOOD scheme:

$$\widetilde{x}^{\text{RHOOD}}_{Li} = \begin{cases} (1-\beta)\left(\widetilde{x}^{\text{PROP}}_{Li} - (1-\gamma)(c_H - c_L)\widetilde{T}_{Li}\right) + \beta c_L \widetilde{T}_{Li} + k & \text{if } (c_L, c_H) \in R_1 \cup R_2 \\ \widetilde{x}^{\text{PROP}}_{Li} & \text{otherwise} \end{cases}$$
$$\forall i \in \mathcal{N}_L, \tag{14}$$

$$\widetilde{x}^{\text{RHOOD}}_{Hj} = \begin{cases} (1-\beta)\left(\widetilde{x}^{\text{PROP}}_{Hj} + \gamma(c_H - c_L)\widetilde{T}_{Hj}\right) + \beta c_H \widetilde{T}_{Hj} & \text{if } (c_L, c_H) \in R_1 \cup R_2 \\ \widetilde{x}^{\text{PROP}}_{Hj} & \text{otherwise} \end{cases}$$
$$\forall j \in \mathcal{N}_H. \tag{15}$$

Thus, the RHOOD scheme applies the stability-related adjustment in region $R_1 \cup R_2$ and adopts the same allocation as the PROP scheme everywhere else in the (c_L, c_H) parametric space. Note that when $\beta = 0$, the RHOOD scheme reduces to the HYB scheme.

Theorem 7 (Equilibrium outcome under the RHOOD scheme). *The RHOOD allocation scheme—defined via* (14) *and* (15)*—achieves the first-best outcome. The equilibrium outcome under this scheme is characterized in Table 6.*

The proof of Theorem 7 is in Appendix J. From Theorem 5 and Theorem 7 (third row in Tables 5 and 6), it follows that in the region $R_1 \cup R_2$, the expected equilibrium surplus of each type-L member under the RHOOD scheme is $(1-\beta)$-fraction of her surplus under the HYB scheme plus the fixed compensation k, which depends on β. On the other hand, the expected equilibrium surplus of each type-H member under the RHOOD scheme is simply $(1-\beta)$-fraction of her surplus under the HYB scheme. Further, in region $R_1 \cup R_2$, we have:

$$\frac{\partial \mathbb{E}\left[\widetilde{\pi}^{\text{RHOOD}}_{Li}\right]}{\partial \beta} = \frac{(1-\gamma)Q(\tau_2 - \gamma c_L - (1-\gamma)c_H)}{\gamma} \ge 0\ \forall i \in \mathcal{N}_L,$$

$$\frac{\partial \mathbb{E}\left[\widetilde{\pi}^{\text{RHOOD}}_{Hj}\right]}{\partial \beta} = -Q(\tau_2 - \gamma c_L - (1-\gamma)c_H) \le 0\ \forall j \in \mathcal{N}_H.$$

Thus, the expected equilibrium surplus of type-L (resp., type-H) members increases (resp., decreases) with β. Consequently, the difference between the equilibrium surplus of type-L and type-H members increases with β. At the extreme value of $\beta = 1$, the surplus of type-H members reduces to 0 and the entire surplus is distributed among the type-L members.

We now examine coalitional stability and individual rationality of the RHOOD scheme.

6.1 | Individual rationality and coalitional stability of the RHOOD scheme

Since the RHOOD scheme is identical to the PROP scheme everywhere in the (c_L, c_H) parametric space other than $R_1 \cup R_2$, it follows from Proposition 2 that the RHOOD scheme satisfies coalitional stability, and consequently, individual rationality in that domain. From Theorem 7, we know that the total equilibrium surplus of type-L members in the region $R_1 \cup R_2$ is:

$$\sum_{i \in \mathcal{N}_L} \mathbb{E}\left[\widetilde{\pi}^{\text{RHOOD}}_{Li}\right]$$
$$= n\gamma \cdot ((1-\beta)(\tau_2 - \gamma c_L - (1-\gamma)c_H)Q + k).$$

In regions R_1 and R_2, the centralized surplus of the set of type-L members is:

$$\sum_{i\in\mathcal{N}_\text{L}} \pi^{\text{CS}}_{\text{L}i} = \begin{cases} n\gamma \cdot Q(\tau_1 - c_\text{L}) & \text{in region R}_1, \\ 0 & \text{in region R}_2. \end{cases}$$

Thus, in region R_1, type-L members have an incentive to break away, if

$$n\gamma \cdot Q(\tau_1 - c_\text{L}) > n\gamma \cdot ((1-\beta)(\tau_2 - \gamma c_\text{L} - (1-\gamma)c_\text{H})Q + k),$$

or, equivalently,

$$(\gamma + \beta(1-\gamma)) \cdot c_\text{H} - \gamma(1-\beta) \cdot c_\text{L} > n\gamma p_1 Q + \beta\tau_2. \quad (16)$$

In the (c_L, c_H) parametric space, let R-NS denote the intersection of the region R_1 with the region where the type-L members have an incentive to break away. That is,

$$\text{R-NS} := \text{R}_1 \cap ((\gamma + \beta(1-\gamma)) \cdot c_\text{H} - \gamma(1-\beta) \cdot c_\text{L} > n\gamma p_1 Q + \beta\tau_2).$$

The inequality (16) is equivalent to: $\beta < \frac{\gamma(c_\text{H} - c_\text{L} - np_1 Q)}{\tau_2 - \gamma c_\text{L} - (1-\gamma)c_\text{H}}$. Let $\beta_0 := \frac{\gamma(c_\text{H} - c_\text{L} - np_1 Q)}{\tau_2 - \gamma c_\text{L} - (1-\gamma)c_\text{H}}$. In region $\text{R}_1 \cup \text{R}_2$, since $c_\text{H} \le \tau_3$, we have:

$$\beta_0 = \frac{\gamma(c_\text{H} - c_\text{L} - np_1 Q)}{\tau_2 - \gamma c_\text{L} - (1-\gamma)c_\text{H}} \le \frac{\gamma(\tau_3 - c_\text{L} - np_1 Q)}{\tau_2 - \gamma c_\text{L} - (1-\gamma)\tau_3} = 1.$$

Theorem 8 states the coalitional stability and individual rationality of the RHOOD scheme.

Theorem 8 (Individual rationality and coalitional stability of RHOOD scheme).

- *If $\beta \in [0, \beta_0)$, then*
  - *the RHOOD scheme does not satisfy coalitional stability in the region R-NS and*
  - *the RHOOD scheme does not satisfy individual rationality in the region*

$$R_1 \cap \left((\gamma + \beta(1-\gamma))c_H - \gamma(1-\beta)c_L > \frac{n-1}{1-\gamma}\gamma p_1 Q + \beta\tau_2 \right).$$

- *If $\beta \in [\beta_0, 1]$, then the region R-NS is empty and the RHOOD scheme satisfies coalitional stability and consequently, individual rationality in the entire (c_L, c_H) parametric space.*

The proof of Theorem 8 is in Appendix K. From Theorem 7 and Theorem 8, we have the following:

Proposition 3. *For any $\beta \in [\beta_0, 1]$, the RHOOD scheme is individually rational, achieves the first-best outcome, and satisfies coalitional stability.*

6.2 | Insights from numerical study

Recall that the RHOOD scheme is identical to the PROP (and the HYB) scheme in all the regions of the (c_L, c_H) parametric space, except $\text{R}_1 \cup \text{R}_2$. It is therefore natural to examine how the area under the regions R_1, R_2, and $\text{R}_1 \cup \text{R}_2$ (and thereby, the dominance of the RHOOD scheme over the PROP and HYB scheme) varies with respect to problem parameters. From Figure 1 (right panel), it is straightforward to see that the areas under these three regions are as follows:

$$\begin{aligned} \text{Area under R}_1 &= \tau_1 \cdot (\tau_3 - \tau_2) \\ &= np_1\gamma\big(Q(p_0 + np_1 Q\gamma) + p_1\sigma^2\big), \\ \text{Area under R}_2 &= \frac{1}{2} \cdot (\tau_2 - \tau_1) \cdot (\tau_3 - \tau_2) \\ &= \frac{n^2 Q^2 p_1^2 \gamma(1-\gamma)}{2}, \text{ and} \end{aligned}$$

$$\text{Area under R}_1 \cup \text{R}_2 = \frac{np_1\gamma(2p_0 Q + np_1 Q^2(1+\gamma) + 2p_1\sigma^2)}{2}.$$

We now examine these areas with respect to the fraction of L-type members (γ) in the cooperative. We set the parameters as follows: $p_0 = 0.5, p_1 = 0.01$ (constants in the revenue function), $n = 50$ (members in the cooperative), $Q = 1, \sigma = 0.1$ (mean and standard deviation, respectively, of the quantity of data shared by a member).

Figure 4 (left panel) shows that as the proportion of L-type members in the cooperative (γ) increases, the area under region R_2 first increases and then decreases, whereas the area under region R_1 and the area under the combined region $\text{R}_1 \cup \text{R}_2$ increases. Thus, as γ increases, the RHOOD scheme dominates the PROP and the HYB schemes over a progressively larger region of the (c_L, c_H) parametric space.

Next, we assess the impact of variability in the quantity of data shared on the areas under these three regions. For a better illustration, we set the parameters as follows: $p_0 = 0.5, p_1 = 10, n = 10, Q = 1$, and $\gamma = 0.1$. Figure 4 (right panel) shows that as the standard deviation σ in the quantity of data changes, the area under region R_2 remains unchanged, whereas the area under region R_1 and the area under the combined region $\text{R}_1 \cup \text{R}_2$ increases. Further, the increase in the area of the combined region is convex in σ (specifically, $n\gamma p_1^2\sigma^2$). Thus, the rate of increase in the area under the combined region $\text{R}_1 \cup \text{R}_2$ increases as σ increases. Thus, again the RHOOD scheme dominates the PROP and the HYB scheme over a progressively larger region of the (c_L, c_H) parametric space, as the variability in the quantity of data shared by the members increases.

Recall from Theorem 8 and Proposition 3 that when the parameter β is in the interval $[\beta_0, 1]$, the RHOOD scheme achieves the first-best outcome, and satisfies individual rationality and coalitional stability. Thus, a higher value of

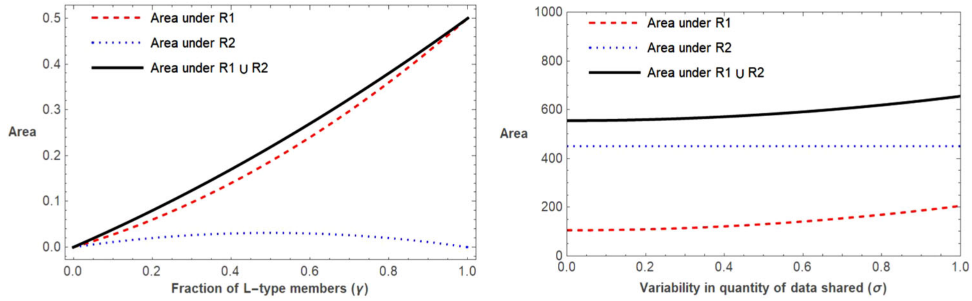

FIGURE 4 Area under regions R_1, R_2, and $R_1 \cup R_2$, with respect to the fraction of L-type members (left panel) and the variability in the quantity of data shared (right panel). [Color figure can be viewed at wileyonlinelibrary.com]

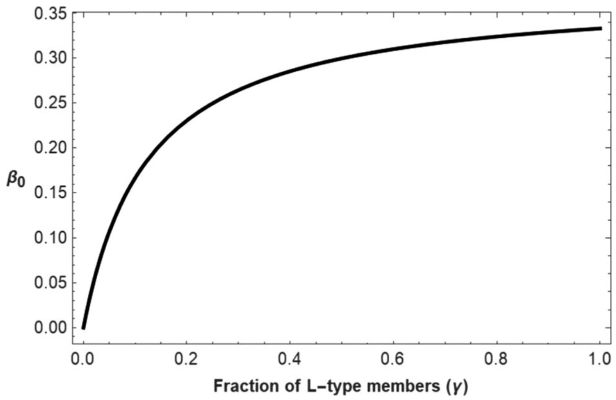

FIGURE 5 The threshold β_0 in the RHOOD scheme with respect to the fraction of L-type members.

β_0 indicates that to implement the RHOOD scheme, the cooperative needs to select a relatively high value of β from a narrower interval. In addition to the parameter values described above for the left panel of Figure 4, we set $c_L = 0.1$ and $c_H = 0.9$. Figure 5 shows that as the proportion of L-type members in the cooperative increases, the value of the threshold β_0 also increases. The reason is as follows: Recall from Section 5.3 that, in certain parametric regions of the (c_L, c_H) space, the L-type members have an incentive to break away from the cooperative under the HYB scheme. Also recall that the RHOOD scheme allocates a $(1-\beta)$ fraction of the total expected revenue to the members based on the HYB scheme (see Figure 3). Thus, as the proportion of L-type members in the cooperative increases, the RHOOD scheme needs to allocate a lower fraction of the total revenue (which corresponds to a higher value of β) to the members according to the HYB scheme to disincentivize the L-type members from breaking away from the cooperative; this, in turn, translates to a higher value of β_0.

7 | MODEL EXTENSIONS

In this section, we discuss some advanced aspects of our model.

7.1 | S-shaped revenue function

In our analysis thus far, we assumed that the revenue function is convex in the total contribution of the cooperative. As mentioned earlier in Section 3, given the modest size of the present-day data cooperatives, this is a reasonable assumption. However, as a cooperative becomes popular and its size increases, it may no longer receive increasing marginal returns from monetizing the pooled data shared by its members. In particular, the revenue function may follow an S-shaped trajectory (Farboodi & Veldkamp, 2021); that is, there are increasing returns to scale until one collects a sufficiently large amount of data, followed by diminishing returns.

To keep the analysis tractable, we focus on the case where the per-unit privacy costs of the members are homogeneous ($c_i = c\ \forall i \in \mathcal{N}$) and set $c = np_1/3$. Further, we assume that the quantity of data shared by the participating members is deterministic ($\sigma = 0$) and normalized to one ($Q = 1$). Since $q_i \in [0,1]$, the total contribution of the cooperative with n members ($T_{\mathcal{N}} = \sum_{i\in\mathcal{N}} q_i \cdot Q$) can at most be equal to n. We model the S-shaped revenue function as follows:

$$P(T_{\mathcal{N}}) = \begin{cases} p_1 T_{\mathcal{N}}^2 & \text{if } T_{\mathcal{N}} \le \frac{n}{2} \\ \frac{2}{3}p_1 n T_{\mathcal{N}} - \frac{p_1}{3}T_{\mathcal{N}}^2 & \text{if } \frac{n}{2} < T_{\mathcal{N}} \le n. \end{cases}$$

Note that the revenue function $P(T_{\mathcal{N}})$ defined above is *convex increasing* in the contribution $T_{\mathcal{N}}$ from 0 to $n/2$ and *concave increasing* from $n/2$ to n. We now state the main results from our analysis. For the S-shaped revenue function, Theorem 9 obtains the equilibrium under the PROP scheme and Theorem 10 obtains the first-best outcome.

Theorem 9 (S-shaped revenue function: Equilibrium under the PROP scheme). *Under the S-shaped revenue function, the Pareto-dominant equilibrium quality is* $q_i^* = \frac{n}{n+1}$*, for all* $i \in \mathcal{N}$*. Each member's equilibrium surplus is* $\pi_i^{PROP} = \frac{p_1 n^2}{3(n+1)^2}$*, and the total surplus of the data cooperative is* $\pi_{\mathcal{N}}^{PROP} = \frac{p_1 n^3}{3(n+1)^2}$.

Theorem 10 (S-shaped revenue function: First-best outcome). *Under the S-shaped revenue function, the first-best equilibrium quality is* $q_i^* = \frac{1}{2}$, *for all* $i \in \mathcal{N}$ *and the total surplus of the data cooperative is* $\pi_{\mathcal{N}}^{CS} = \frac{p_1 n^2}{12}$.

The proof of the results from Section 7.1 are in Appendix M. From Theorems 9 and 10, it is clear that the PROP scheme does not achieve the first-best outcome. This is because the members of the cooperative end up "oversharing" data under the PROP scheme. Based on this observation, we now present an adjustment to the PROP scheme. This adjustment disincentivizes the members to share their data once the total contribution of the cooperative crosses the critical threshold of $n/2$, since any additional contribution leads to diminishing returns under the S-shaped revenue function. This adjustment results in a new scheme, which we refer to as PROP-ADJ, and is defined as follows:

$$x_i^{\text{PROP-ADJ}} := \begin{cases} p_1 T_{\mathcal{N}} T_i & \text{if } T_{\mathcal{N}} \le \frac{n}{2} \\ \frac{n}{4} p_1 & \text{if } T_{\mathcal{N}} > \frac{n}{2} \end{cases} \quad i \in \mathcal{N}.$$

Thus, once the total contribution $T_{\mathcal{N}}$ exceeds the threshold $\frac{n}{2}$, the PROP-ADJ scheme allocates a constant compensation to the members (even if they decide to share high-quality data with the cooperative).

Theorem 11 (Equilibrium under the PROP-ADJ scheme). *Under the S-shaped revenue function, the Pareto-dominant equilibrium quality of data shared by the ith member is* $q_i^* = \frac{1}{2}$, *for all* $i \in \mathcal{N}$. *Each member's equilibrium surplus is* $\pi_i^{PROP\text{-}ADJ} = \frac{p_1 n}{12}$, *and the total surplus of the cooperative is* $\pi_{\mathcal{N}}^{PROP\text{-}ADJ} = \frac{p_1 n^2}{12}$.

From Theorems 10 and 11, it follows that the PROP-ADJ scheme achieves the first-best outcome. Our next result analyzes the coalitional stability and individual rationality of the PROP-ADJ scheme.

Theorem 12 (Coalitional stability and individual rationality of PROP-ADJ scheme). *Under the S-shaped revenue function, the PROP-ADJ scheme satisfies coalitional stability, and consequently, individual rationality.*

From Theorems 10, 11, and 12, it follows that the PROP-ADJ scheme satisfies each of our three desirable properties.

7.2 | Information asymmetry

In Supporting Information Appendix N, we generalize our model by imposing that (i) there are $K \ge 2$ types of members in the cooperative (as opposed to the two member types L and H that we analyzed thus far) and (ii) the per-unit privacy cost of the members is private information; that is, each member knows only her own privacy cost and has only distributional information on the privacy cost of the other members.

TABLE 7 Properties of the three allocation schemes we analyzed under heterogeneous privacy costs.

Allocation scheme	Individual rationality	First-best outcome	Coalitional stability
PROP	√		
HYB		√	
RHOOD	√	√	√

7.3 | Three member types

In Supporting Information Appendix O, we extend our base model to incorporate three types of members with respect to their privacy costs. The main message from this analysis is that our key insights from the analysis of the base model continue to hold—the cooperative should engage in Robin Hood–style distribution of rewards by taking from the "privacy rich" members and giving to the "privacy poor" members.

7.4 | The uniform scheme

A simple allocation scheme is one where the cooperative distributes the total revenue obtained from monetizing the pooled data equally to its members (irrespective of their individual contributions). We refer to this scheme as the uniform scheme. We analyze this scheme in Appendix L.

8 | CONCLUDING REMARKS

Our broad goal in this paper is to develop and analyze attractive revenue-allocation schemes for data cooperatives. We examine three desirable properties in an allocation scheme—individual rationality, first-best outcome, and coalitional stability. Table 7 summarizes these properties for the three allocation schemes we analyzed.

When the privacy costs are homogeneous, the PROP scheme satisfies all the three desirable properties. Under heterogeneous privacy costs however, we show that while the PROP scheme is individually rational, it fails to achieve the first-best outcome and guarantee coalitional stability. To this end, we construct the more sophisticated HYB scheme, and show that it achieves the first-best outcome. However, the HYB scheme is not individually rational and, hence, does not satisfy coalitional stability either. Finally, we construct the RHOOD scheme, which is parameterized by $\beta \in [0, 1]$, and show that for sufficiently high values of β, it is individually rational, achieves the first-best outcome, and satisfies coalitional stability.

TABLE 8 Ordering of type-L and type-H members' equilibrium surplus under the three schemes we analyzed.

Region	Expected equilibrium surplus to the *i*th type-L member, $\forall i \in \mathcal{N}_L$	Expected equilibrium surplus to the *j* the type-H member, $\forall j \in \mathcal{N}_H$
$R_1 \cap (c_H - c_L) \leq np_1Q$	$\mathbb{E}[\tilde{\pi}_{Li}^{PROP}] \leq \mathbb{E}[\tilde{\pi}_{Li}^{HYB}] \leq \mathbb{E}[\tilde{\pi}_{Li}^{RHOOD}]$	$\mathbb{E}[\tilde{\pi}_{Hj}^{PROP}] \leq \mathbb{E}[\tilde{\pi}_{Hj}^{RHOOD}] \leq \mathbb{E}[\tilde{\pi}_{Hj}^{HYB}]$
$R_2 \cup (R_1 \cap (c_H - c_L) > np_1Q)$	$\mathbb{E}[\tilde{\pi}_{Li}^{HYB}] \leq \mathbb{E}[\tilde{\pi}_{Li}^{PROP}] \leq \mathbb{E}[\tilde{\pi}_{Li}^{RHOOD}]$	$\mathbb{E}[\tilde{\pi}_{Hj}^{PROP}] \leq \mathbb{E}[\tilde{\pi}_{Hj}^{RHOOD}] \leq \mathbb{E}[\tilde{\pi}_{Hj}^{HYB}]$
$(R_1 \cup R_2)^c$	$\mathbb{E}[\tilde{\pi}_{Li}^{HYB}] = \mathbb{E}[\tilde{\pi}_{Li}^{PROP}] = \mathbb{E}[\tilde{\pi}_{Li}^{RHOOD}]$	$\mathbb{E}[\tilde{\pi}_{Hj}^{HYB}] = \mathbb{E}[\tilde{\pi}_{Hj}^{PROP}] = \mathbb{E}[\tilde{\pi}_{Hj}^{RHOOD}]$

Table 8 shows how the expected equilibrium surplus of type-L and type-H members fare under the three schemes. Throughout the (c_L, c_H) parameter space, both type-L and type-H members obtain a (weakly) higher surplus under the RHOOD scheme compared to the PROP scheme. In region $R_1 \cup R_2$, which we defined in Section 5, type-H members obtain a higher surplus under the HYB scheme compared to the PROP and RHOOD schemes.

To our knowledge, this is the first study to analyze revenue-allocation schemes for data cooperatives, which are fast becoming attractive among marketers as they naturally serve as a source of fresh and voluntarily shared consumer data. It would be interesting to examine how the emergence of data cooperatives shapes the data markets of the future. For instance, would data cooperatives be able to offer a substantive challenge to existing IT-enabled data brokers? Furthermore, how would data brokers change their data collection and processing strategies in response to the rise of data cooperatives? Although data cooperatives are still in their nascent phase, we believe that the study of how these self-governing institutions scale up is a fertile avenue for future research. While the manner in which we operationalize data quality in our analysis meaningfully captures the broad economic reality faced by a data cooperative, future research can consider a more refined treatment of data quality. For example, modeling quality to capture the correlation among the qualities of the data shared by the members, and the subsequent strategic implications, is also a fruitful direction in which future research can proceed.

ACKNOWLEDGMENTS

The authors would like to thank the Open Data Institute (https://theodi.org/) for facilitating access to data cooperatives, practitioners, and policy makers.

ORCID

Milind Dawande https://orcid.org/0000-0001-6956-0856
Sameer Mehta https://orcid.org/0000-0002-0410-3248

ENDNOTES

[1] https://medium.com/swashapp/swash-integrating-with-chainlink-to-make-sponsored-withdrawals-and-proportional-revenue-sharing-a-dbfb1646e753

[2] Throughout the paper, we use a tilde (∼) on symbols that denote random variables.

[3] In Section 7 and Supporting Information Appendix N, we discuss the information asymmetry setting where each member's privacy cost is his private information.

[4] The type of data that consumers voluntarily share about themselves is now popularly known as zero-party data; see, for example, https://www.salesforce.com/resources/articles/what-is-zero-party-data/.

[5] Later, in Section 7 and Supporting Information Appendix M, we also analyze a specific setting in which the data cooperative's revenue function is an S-shaped function of the total contribution.

[6] Alternatively, suppose the cooperative keeps a fraction $\alpha \in [0, 1]$ of the total revenue to cover its operational expenses and distributes the remainder to its members. Then, we simply scale the total revenue $P(\tilde{T}_{\mathcal{N}})$ by $(1 - \alpha)$. It is easy to see that modified revenue function remains structurally unchanged and all our results continue to hold.

[7] Later, in Section 7 and Supporting Information Appendix O, we extend our analysis to three types of members and show that our insights remain qualitatively the same.

REFERENCES

Acemoglu, D., Makhdoumi, A., Malekian, A., & Ozdaglar, A. (2019). *Too much data: Prices and inefficiencies in data markets*. Tech. rep., National Bureau of Economic Research.

Aggarwal, V., Hwang, E. H., & Tan, Y. (2021). Learning to be creative: A mutually exciting spatiotemporal point process model for idea generation in open innovation. *Information System Research*, *32*(4), 1214–1235.

Amer, R., Carreras, F., & Magaña, A. (2008). Shapley value vs. proportional rule in cooperative affairs. In Luis L. Bonilla, Miguel Moscoso, Gloria Platero, & Jose M. Vega (Eds.), *Progress in industrial mathematics at ECMI 2006* (pp. 877–881). Springer.

Bakos, Y., & Brynjolfsson, E. (1999). Bundling information goods: Pricing, profits, and efficiency. *Management Science*, *45*(12), 1613–1630.

Bergantiños, G., & Moreno-Ternero, J. D. (2020). Sharing the revenues from broadcasting sport events. *Management Science*, *66*(6), 2417–2431.

Bertsimas, D., & Kallus, N. (2020). From predictive to prescriptive analytics. *Management Science*, *66*(3), 1025–1044.

Bhargava, H. K., & Choudhary, V. (2001). Information goods and vertical differentiation. *Journal of Management Information Systems*, *18*(2), 89–106.

Bhargava, H. K., & Choudhary, V. (2008). Research note—When is versioning optimal for information goods? *Management Science*, *54*(5), 1029–1035.

Bimpikis, K., Crapis, D., & Tahbaz-Salehi, A. (2019). Information sale and competition. *Management Science*, *65*(6), 2646–2664.

Braunstein, A., Granka, L., & Staddon, J. (2011). Indirect content privacy surveys: Measuring privacy without asking about it. In *Proceedings of the seventh symposium on usable privacy and security*. Association for Computing Machinery, New York, NY, United States (pp. 1–14).

Cai, H., Ye, F., Yang, Y., Zhu, Y., & Li, J. (2019). Towards privacy-preserving data trading for web browsing history. In *2019 IEEE/ACM 27th international symposium on quality of service* (pp. 1–10). IEEE.

Cai, X., & Vairaktarakis, G. L. (2012). Coordination of outsourced operations at a third-party facility subject to booking, overtime, and tardiness costs. *Operations Research*, *60*(6), 1436–1450.

Carraro, C. (2003). *The endogenous formation of economic coalitions*. Edward Elgar Publishing.

Chen, J., Qi, A., & Dawande, M. (2020). Supplier centrality and auditing priority in socially responsible supply chains. *Manufacturing and Service Operation Management*, *22*(6), 1199–1214.

Choudhary, V. (2010). Use of pricing schemes for differentiating information goods. *Information System Research*, *21*(1), 78–92.

Driessen, T. S. (2013). *Cooperative games, solutions and applications* (Vol. 3). Springer Science & Business Media.

Fang, X., & Cho, S.-H. (2014). Stability and endogenous formation of inventory transshipment networks. *Operations Research*, *62*(6), 1316–1334.

Fang, X., & Cho, S.-H. (2020). Cooperative approaches to managing social responsibility in a market with externalities. *Manufacturing and Service Operation Management*, *22*(6), 1215–1233.

Farboodi, M., & Veldkamp, L. (2021). *A growth model of the data economy*. Tech. rep., National Bureau of Economic Research.

Fiestras-Janeiro, M. G., García-Jurado, I., & Mosquera, M. A. (2011). Cooperative games and cost allocation problems. *Top*, *19*(1), 1–22.

Ghose, A., Ipeirotis, P. G., & Li, B. (2019). Modeling consumer footprints on search engines: An interplay with social media. *Management Science*, *65*(3), 1363–1385.

Ghoshal, A., Kumar, S., & Mookerjee, V. (2020). Dilemma of data sharing alliance: When do competing personalizing and non-personalizing firms share data. *Production and Operations Management*, *29*(8), 1918–1936.

Gopalakrishnan, S., Granot, D., Granot, F., Sošić, G., & Cui, H. (2021). Incentives and emission responsibility allocation in supply chains. *Management Science*, *67*(7), 4172–4190.

Granot, D., & Sošić, G. (2003). A three-stage model for a decentralized distribution system of retailers. *Operations Research*, *51*(5), 771–784.

Granot, D., & Yin, S. (2008). Competition and cooperation in decentralized push and pull assembly systems. *Management Science*, *54*(4), 733–747.

Hardjono, T., & Pentland, A. (2019). Data cooperatives: Towards a foundation for decentralized personal data management. https://doi.org/10.48550/arXiv.1905.08819

Hirschprung, R., Toch, E., Bolton, F., & Maimon, O. (2016). A methodology for estimating the value of privacy in information disclosure systems. *Computers in Human Behavior*, *61*, 443–453.

Hu, X., Caldentey, R., & Vulcano, G. (2013). Revenue sharing in airline alliances. *Management Science*, *59*(5), 1177–1195.

Jagabathula, S., Subramanian, L., & Venkataraman, A. (2020). A conditional gradient approach for nonparametric estimation of mixing distributions. *Management Science*, *66*(8), 3635–3656.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decisions under risk. *Econometrica*, *47*, 278.

Kamijo, Y. (2009). A linear proportional effort allocation rule. *Mathematical Social Sciences*, *58*(3), 341–353.

Kemahlıoğlu-Ziya, E., & Bartholdi III, J. J. (2011). Centralizing inventory in supply chains by using Shapley value to allocate the profits. *Manufacturing and Service Operation Management*, *13*(2), 146–162.

Lee, D., Hosanagar, K., & Nair, H. S. (2018). Advertising content and consumer engagement on social media: Evidence from Facebook. *Management Science*, *64*(11), 5105–5131.

Liang, F., Yu, W., An, D., Yang, Q., Fu, X., & Zhao, W. (2018). A survey on big data market: Pricing, trading and protection. *IEEE Access*, *6*, 15132–15154.

Longpré, L., & Kreinovich, V. (2006). *How to measure loss of privacy*. Tech. rep., University of Texas at El Paso. https://scholarworks.utep.edu/cs_techrep/182/

Marotta, V., Wu, Y., Zhang, K., & Acquisti, A. (2022). The welfare impact of targeted advertising technologies. *Information System Research*, *33*(1), 131–151.

McAfee, A., Brynjolfsson, E., Davenport, T. H., Patil, D., & Barton, D. (2012). Big data: The management revolution. *Harvard Business Review*, *90*(10), 60–68.

Mehta, S., Dawande, M., Janakiraman, G., & Mookerjee, V. (2021). How to sell a dataset? Pricing policies for data monetization. *Information System Research*, *32*(4), 1281–1297.

Mehta, S., Dawande, M., Janakiraman, G., & Mookerjee, V. (2022). An approximation scheme for data monetization. *Production and Operations Management*, *31*(6), 2412–2428.

Mehta, S., Dawande, M., & Mookerjee, V. (2021). Can data cooperatives sustain themselves? LSE Business Review. http://eprints.lse.ac.uk/112008/

Miklós-Thal, J., Goldfarb, A., Haviv, A. M., & Tucker, C. (2023). *Digital hermits*. Tech. rep., National Bureau of Economic Research.

Mu, L., Dawande, M., & Mookerjee, V. (2019). Shaping the values of a milk cooperative: Theoretical and practical considerations. *Production and Operations Management*, *28*(9), 2259–2278.

Nagarajan, M., & Bassok, Y. (2008). A bargaining framework in supply chains: The assembly problem. *Management Science*, *54*(8), 1482–1496.

Nagarajan, M., & Sošić, G. (2007). Stable farsighted coalitions in competitive markets. *Management Science*, *53*(1), 29–45.

Najjar, M. S., & Kettinger, W. J. (2013). Data monetization: Lessons from a retailer's journey. *MIS Quarterly Executive*, *12*(4), 213–225.

Pentland, A., & Hardjono, T. (2020). Data cooperatives. In Alex Pentland, Alexander Lipton, & Thomas Hardjono (Eds.), *Building the new economy*. (0 ed.). https://wip.mitpress.mit.edu/pub/pnxgvubq https://doi.org/10.21428/ba67f642.0499afe0

Preibusch, S. (2013). Guide to measuring privacy concern: Review of survey and observational instruments. *International Journal of Human-Computer Studies*, *71*(12), 1133–1143.

Rasouli, M., & Jordan, M. I. (2021). Data sharing markets. arXiv preprint arXiv:2107.08630. https://doi.org/10.48550/arXiv.2107.08630

Roth, A. E. (1988). *The Shapley value: Essays in honor of Lloyd S. Shapley*. Cambridge University Press.

Sundararajan, A. (2004). Nonlinear pricing of information goods. *Management Science*, *50*(12), 1660–1673.

Yin, S. (2010). Alliance formation among perfectly complementary suppliers in a price-sensitive assembly system. *Manufacturing and Service Operation Management*, *12*(3), 527–544.

Zhang, Z., Liu, G., Wu, J., & Tan, Y. (2022, March 20). Data and algorithm pricing: Incentive mechanisms design for federated learning. Available at SSRN https://ssrn.com/abstract=4061980 or http://doi.org/10.2139/ssrn.4061980

SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

How to cite this article: Dawande, M., Mehta, S., & Mu, L. (2023). Robin Hood to the Rescue: Sustainable Revenue-Allocation Schemes for Data Cooperatives. *Production and Operations Management*, *32*, 2560–2577. https://doi.org/10.1111/poms.13995