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Robust Capacitated Train Rescheduling with Passenger Reassignment under Stochastic Disruptions

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Abstract

During railway operations unexpected events may occur, influencing normal traffic flows. This paper focuses on a train rescheduling problem in a railway system with seat-reserved mechanism during large disruptions, such as a rolling stock breakdown leading to some canceled services, where passenger reassignment strategies have also to be considered. A novel mixed-integer linear programming formulation is established with consideration of train retiming, reordering, and reservicing. Based on a time–space modeling framework, a big-*M* approach is adopted to formulate the track occupancy and extra train stops. The formulation aims to maximize the passenger accessibility measured by the amount of the transported passengers subject to canceled services and to minimize the weighted total train delay for all trains at their destinations. The proposed mathematical formulation also considers planning extra stops for non-canceled trains to transport the disrupted passengers, which were supposed to travel on the canceled services, to their pre-planned destinations. Other constraints deal with seat capacity limitation, track capacity, and some robustness measures under uncertainty of disruption durations. We propose different approaches to compute advanced train dispatching decisions under a dynamic and stochastic optimization environment. A series of numerical experiments based on a part of "Beijing–Shanghai" high-speed railway line is carried out to verify the effectiveness and efficiency of the proposed model and methods.

Railway transportation systems provide an efficient and sustainable service for passengers and have a strong competitiveness compared with the other transport modes. However, trains do not always arrive or depart on time, sometimes are even canceled. In daily railway operations, some external and internal unpredictable disruptions of the railway system, such as severe weather conditions and rolling stock breakdowns, may lead to reduced capacity of tracks and stations. As a result, train dispatchers need to make appropriate train rescheduling decisions, like retiming and reordering, to recover the affected rail operations within a short computation time. During severe disruptions, other dispatching measures such as changing the stopping plans, canceling or inserting train services may also be taken to allow disrupted passengers to travel to their destinations. Consequently, in a railway network of high density and limited capacity, real-time train rescheduling becomes extremely complicated and may strongly affect the quality of passenger services and the performance of the overall rail system (*1*, *2*).

Under severe disruptions, train rescheduling is even more challenging in a railway system with a seat reservation mechanism compared with a mechanism without seat reservations. The seat reservation mechanism means that passengers should buy tickets in advance with a planned departure at the original station, a planned arrival at the destination station, and a seat number. Once a train service is canceled as a result of a disruption, all passengers who were supposed to travel on it will be affected, and they need eventually be assigned to other

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trains, with limited remaining seat capacity, to travel to their destination station. Therefore, in case of serious disruptions, passenger reassignment strategies have to be developed together with train rescheduling strategies. Both strategies have the main objective of ensuring that more passengers arrive at their destinations as early as possible within an acceptable time horizon. With a seat reservation mechanism, the combination of effective train rescheduling with passenger reassignment is critical for railway operators in a disrupted high-density network with limited capacity, because the disrupted passengers may occupy seats in other trains and additional train stops may be required to serve as many as possible disrupted passengers.

This paper deals with the problem of rescheduling trains with passenger reassignment in a railway system with a seat reservation mechanism under a rolling stock breakdown. A mixed-integer programming formulation is proposed for this problem with the following objective function components: the maximization of the passenger accessibility, measured by the amount of disrupted passengers transported to their destination station, and the minimization of a weighted total train delay at their destination stations. The characteristics of uncertainty and randomness are taken into account during the optimization process. We introduce Robust Train Stop Changing (RTSC) constraints into the proposed formulation to ensure that the stopping plans of the rescheduled trains are consistent under disruption scenarios of different durations. This definition of timetable robustness can be viewed as a robust passenger reassignment because all the extra stops added during the rescheduling phase aim to transport passengers affected by disruptions. Various deterministic and stochastic approaches are proposed to solve the studied problem with the aim of investigating the trade-off between the accuracy of managing uncertainty information and the time of computing optimal rescheduling solutions.

The remainder of this paper proceeds as follows. The next section introduces a literature review and the statement of paper contribution. The problem description is then provided. The next section introduces the mixed-integer linear programming formulation and three methods proposed to solve the problem. The computational results on the mathematical formulation and solving methods are then presented. The final section outlines some directions for further research on the studied problem.

Literature Review and Paper Contribution

Literature Review

The train rescheduling problem has been extensively studied in the past few decades. Advances in scheduling theory make it possible to solve a real-time train scheduling problem, in which train departure/arrival times, train orders, and routes will be determined (*3–5*). Recent surveys (*6–9*) summarize recent methods and solution techniques for train timetabling, train dispatching, and train rescheduling.

A recent research stream is focused on train rescheduling during disruptions, and with a focus on the passenger flows affected by them. Sato et al. introduce a timetable rescheduling algorithm when train traffic is disrupted (*10*). A mixed-integer programming formulation is proposed to minimize further inconvenience to passengers. Louwerse and Huisman focus on adjusting the timetable of a passenger railway operator in case of partial or complete track blockage (*11*). The main objective is to maximize the service level offered to passengers by minimizing the number of canceled trains and the delays of the operated trains and by distributing the operated trains evenly over time. Veelenturf et al. propose a railway timetable rescheduling approach for handling large-scale disruptions on a macroscopic modeling level (*12*). An integer linear programming formulation is introduced to minimize the number of canceled and delayed train services. Binder et al. focus on the railway timetable rescheduling problem from a macroscopic level of infrastructure representation in the case of large disruptions (*13*). An integer linear programming formulation is proposed to minimize the passenger dissatisfaction, the operational costs, and the deviation from the disrupted timetable. Corman et al. integrate train rescheduling and delay management by developing microscopic passenger-centric models (*14*). Corman and D'Ariano investigate a set of disruption resolution scenarios involving cancelation of train services, rerouting, and shuttle trains, to manage seriously disturbed traffic conditions in large networks (*15*). Detailed performance indicators about train delay and passengers' discomfort are computed.

Most of the approaches presented in the literature mainly concern the train rescheduling problem in a static environment, where key parameters are set to fixed values. However, railway traffic management is a complex and dynamic system, and the information is always with some degree of uncertainty. Because of this, some researchers have recently introduced dynamic and stochastic approaches with consideration of uncertainty factors related, for example, to the duration of scheduled and unscheduled process times.

Meng and Zhou solve a train dispatching problem focused on a major service disruption on a single-track rail line, with the objective of minimizing the expected additional delay under different forecasted operational conditions (*16*). A robust meet–pass plan is selected and determined for every rolling period. Quaglietta et al. propose several metrics plus a framework to assess the stability of railway dispatching solutions, by adopting a rolling horizon approach under a stochastic and dynamic environment (*17*). Larsen et al. focus on the stochastic evaluation of train schedules computed by a microscopic optimization-based scheduler of railway operations based on deterministic information (*18*). Meng et al. propose a cumulative flow variables-based integer programming model for dispatching trains under a stochastic environment on a general railway network (*19*). Stable train routing constraints are introduced to ensure that trains traverse the same route across different capacity breakdown scenarios. Cavone et al. propose a mixed-integer linear programming for robust real-time train rescheduling in case of disturbances (*20*). A self-learning decision making procedure determines appropriate relevance weights for the distribution of buffer times when disturbances of the same type affect the network. Davydov et al. propose a stochastic model by using specific distributions of operating times, which depend on the actual traffic conditions (*21*). The arrival time distribution is obtained by adjusting the train trajectory and corresponds well with the experimental data derived by Russian Railways. D'Ariano et al. focus on the optimization of train sequencing, routing, and timing decisions related to short-term maintenance works in a railway network subject to disturbed process times (*22*). The bi-objective problem minimizes the deviation from a scheduled plan and maximizes the number of aggregated maintenance works under stochastic disturbances. Cacchiani et al. pay attention to the passenger demand uncertainty by formulating robust optimization models for integrated train stop planning and timetabling with constraints on passenger demand (*23*). Their goal is to determine robust solutions in planning so as to reduce the passenger inconvenience that may occur in real-time as a result of additional passenger demand.

Even though some researchers have done studies about railway operations with consideration of dynamic and stochastic characteristics, all the above research has been developed by assuming a railway system with no seat reservation, which means that the seat capacity for trains is not considered. Some of the studies pay attention to passenger inconvenience, which is measured as a total traveling time consisting of on-board time, waiting time, and transfer time. However, the disrupted train means that disrupted passengers need to be reassigned to other trains. Therefore, a main question in a railway system with seat reservation is whether passengers may be able to arrive at their destinations in a disrupted situation in which several trains operate with a limited available seat capacity and with a pre-defined stopping plan. Furthermore, trains should be rescheduled by considering both the need for changing their stopping plans to accommodate disrupted passengers and the presence of a disrupted train in the railway network. The disrupted passengers should be assigned to specific trains, which still have enough capacity to take new passengers. The assignment optimization of the disrupted passengers is a very complicated task, because of the seat reservation mechanism (similar to airlines [*24*, *25*]). At the same time, having a train performing an extra stop would result in a longer travel time, which would ultimately lead to delays, and possibly even to a domino effect of delay propagation in the overall railway network during operations.

Statement of Contributions

This paper focuses on rescheduling a train timetable facing a disruption, such as a rolling stock failure. The passenger reassignment problem is solved under the assumption of a seat-reserved mechanism. The disruption is modeled under a dynamic and stochastic environment, that is, its duration is not known beforehand. This paper aims to offer the following contributions:

(1) This research focuses on a railway system with a seat-reserved mechanism, like the railway operations in China. There are some differences with previous research. First, not only are tracks of a limited capacity, but also trains are capacitated with a limited number of available seats, and passengers have booked their seats in a specific train. Because of the passenger seat reservation system, passengers in a disrupted train must change their ticket, as their booked service is canceled and they have to travel on other trains with sufficient available free seats. This behavior is substantially different from a railway system with a non-reserved mechanism, such as the usual ticketing systems in most European countries in relation to regional/local trains or urban transit systems. In the latter cases, passengers with a ticket, between an origin and a destination, can choose their favorite trains when they enter the system, leading to crowding and route choice under uncertainty.

(2) We propose a novel mixed-integer linear programming formulation of the problem of determining a robust disposition timetable under disruptions with the consideration of the following key elements: passenger reassignment (how many passengers in which passenger groups travel in which trains), train rescheduling (which trains stop and where, which train times, orders, and routes are adopted in the network), track capacity utilization (the minimum safety separations must be respected between any pair of consecutive trains on rail resources), and various

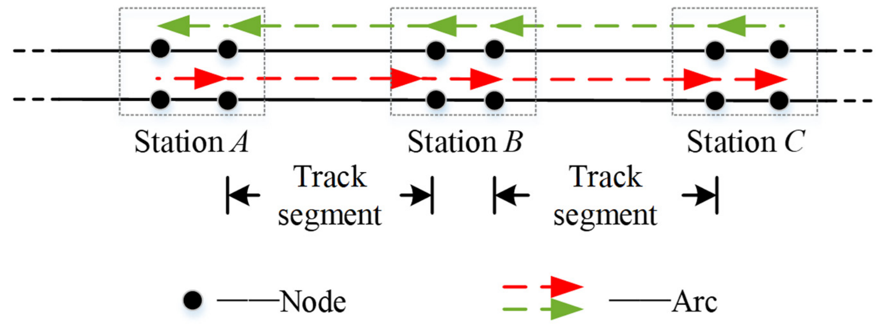


Figure 1. Illustration of railway network.

passenger and train indicators are optimized. The disrupted passengers who were supposed to travel in the broken train, and have the same origin and destination station, form one passenger group.

(3) We deal with an uncertain duration of disruptions. To improve the robustness of our dispatching plan under this uncertainty, we introduce RTSC constraints in the mathematical formulation. Three different solution approaches are proposed to deal with different assumptions on the disruption duration. Numerical experiments are also carried out to demonstrate the potential benefits of the approaches.

Problem Description

In railway operations, punctuality is one of the main performance indicators to be optimized. This indicator can be viewed as the minimization of weighted train delays on a railway network, by considering the importance of each train for train operating companies. From a passenger perspective, each person would like to travel according to what is indicated in his/her ticket. However, if a disruption occurs, the passenger may be satisfied if he/she can arrive at his/her destination as early as possible, even with a revised ticket. RTSC constraints will affect the actual train operation and passenger organization. For example, if train dispatchers continuously change their plans with the updating of disruption information, rescheduling actions will have to be updated multiple times and the organization of affected passengers will be disordered by a changeable response system.

The inputs include the following information:

(1) Railway network

We consider a railway network composed of several stations and track segments with a macroscopic representation of the infrastructure, shown in Figure 1. We view the railway network as a directed graph $G = (N, A)$ with a set of nodes N and a set of directed arcs A. Nodes represent entry and exit points of stations and directed arcs represent track segments and stations.

(2) Passenger travel demands

For each passenger origin–destination (OD) group, we consider a volume (demand) of passengers with the same origin and destination stations, the train assigned to perform their service, and their planned arrival time at the destination station.

(3) Timetable

The timetable is the planned schedule of all train services, with a detailed description of train timing, ordering, and routing, plus arrival/departure times of each train at each station, and the carrying capacity of each train in relation to the available passenger seats.

(4) Disrupted train

We know the location of the train which has a failure and the stochastic characteristic of the disruption duration. We assume the disruption can be resolved by having the affected train pulled to the next station along its running direction. As for the disruption characteristic, we consider several scenarios with different duration times. In our assumption, a probability is given for each scenario, following a given distribution.

A detailed train timetable and passenger reassignment plan can be obtained as the major outputs.

We also make the following assumptions.

(1) We do not consider the reassignment of each individual passenger. Passengers are divided into

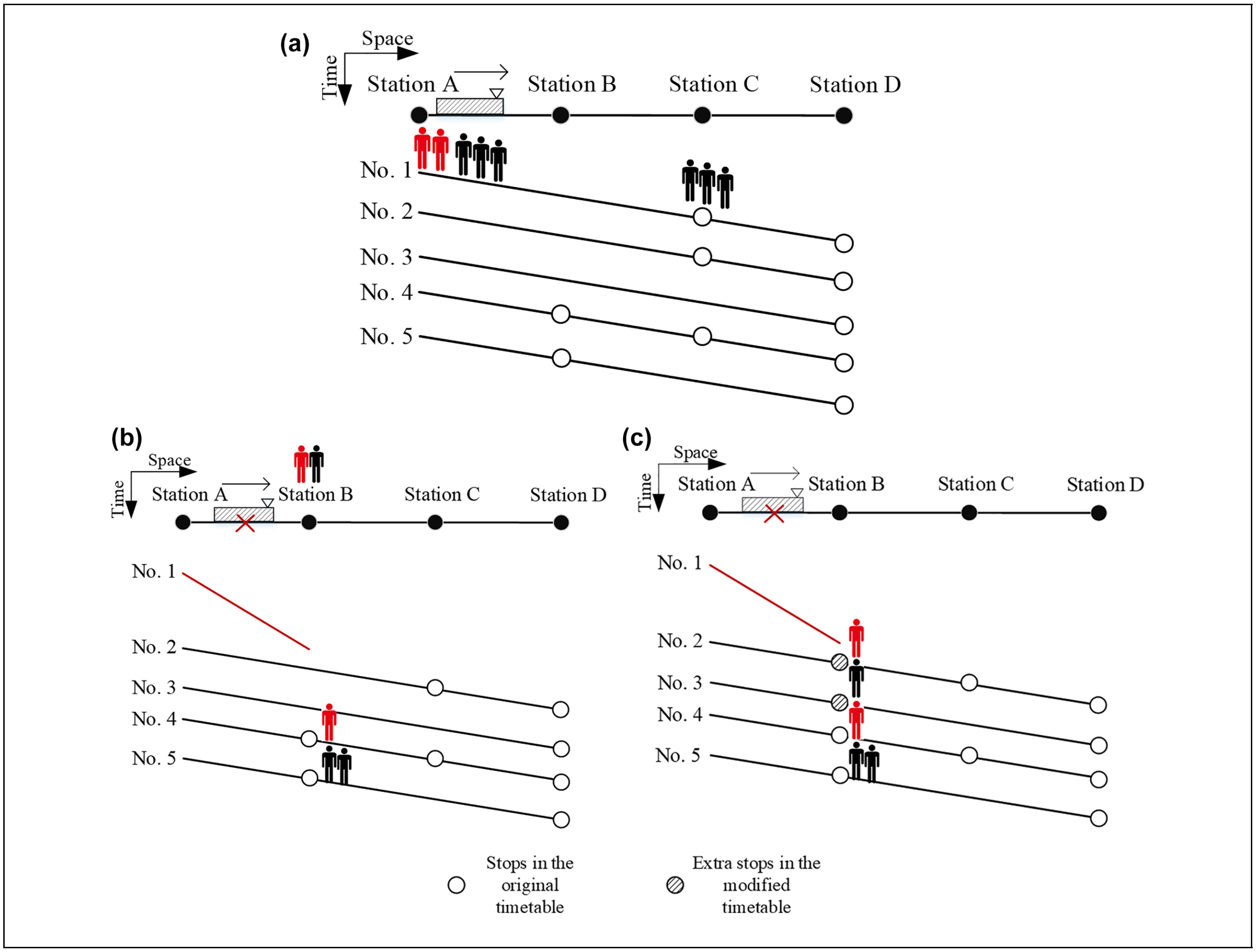


Figure 2. Illustration of example: (*a*) original timetable and passenger flow, (*b*) feasible condition without passenger reassignment, and (*c*) feasible condition with passenger reassignment.

different passenger groups. Passengers in each group have the same demand.

(2) We do not consider the reassignment of passengers who are not in the train facing a disruption. The available seat capacity on the following trains is static. Penalties are set in the objective function for the on-board (undisrupted) passengers. In other words, real-time ticket changes only involve disrupted passengers.

(3) We consider one transfer only for passengers on the disrupted train, from the disrupted train to the following train that can transport them to their destinations.

(4) We assume that stations have a relatively large capacity. The station capacity constraints are thus not included in our mathematical model.

We next describe a fictitious example to illustrate the problem in more detail. We compare a passenger reassignment strategy with the condition without passenger reassignment. In this example, five trains, named No.1 to No.5, travel from Station *A* to Station *D* with some intermediate stops in the original timetable. There are two passenger groups: a red one (two passengers) and a black one (three passengers). Both groups travel on train No.1 but with different destinations (Station *C* for the red group and Station *D* for the black group). Figure 2*a* shows the passenger flows in the original timetable. When train No.1 is at Station C, the black-group passengers are still on board, while the red-group passengers disembark train No.1.

During operations, something goes wrong with train No.1 on the track section from Station *A* to Station *B* and this train cannot move toward Station *B*. A rescue train is required to move train No.1 to Station *B*. All passengers on train No.1 must disembark and wait for the available following trains to (eventually) travel to their destinations. Clearly, the disrupted passengers can only choose trains with a scheduled stop at Station B (so that they can use those trains) and with a scheduled stop at their destinations (so that they can arrive at their destination without any further need of changing train). A ticket change needed for disrupted passengers is only possible if there is available seat capacity.

In Figure 2*b*, dispatchers take some train rescheduling measures (such as train retiming and reordering) without consideration of passenger reassignment, that is, without the addition of unscheduled stops for the following trains to serve disrupted passengers. The red-group passengers can only choose train No.4, and the black-group passengers can only choose trains No.4 and No.5. In relation to the available seat capacity of the following trains, disrupted passengers know about the timetable of the following trains and the possibilities to change their ticket in advance. In this way, they can be ready to board a following train with enough seat capacity and with a feasible stopping plan (i.e., a plan covering their OD). A ticket change mechanism is considered such that not all disrupted passengers can choose the earliest feasible train from Station B in case there is not enough seat capacity. However, not all disrupted passengers can be served.

The passenger reassignment strategy works as follows. In Figure 2*c*, all disrupted passengers can reach their destination. This is made possible by adding an extra stop to trains No.2 and No.3 at Station B. However, the number of disrupted passengers that embark on trains No.2 and No.3 depends on the available seat capacity on those trains. The drawback of serving all passengers is the need to increase the travel time of the following trains as a result of the addition of an unscheduled stop for trains No.2 and No.3. The travel time increase may generate dissatisfaction for the undisrupted passengers if they arrive at their destination with an additional delay. In a high-density timetable, the addition of unscheduled stops for some trains may generate consecutive delays for the following trains. The investigated problem is thus inherently multi-objective when considering the needs of disrupted versus non-disrupted passengers.

To the best of our knowledge, the problem studied in this paper is original and of high practical interest both for train operating companies and infrastructure managers in the presence of disruptions that enforce taking re-ticketing decisions for some passengers.

Methodology

Notations

We first introduce the general subscripts, input parameters and decision variables in Table 1. It should be explained that the exit points from the transfer and destination stations are the transfer and destination nodes for passenger OD pairs, respectively.

Mathematical Formulation

Objective function (1) is proposed in this paper with the following components: the maximization of passenger accessibility, measured by the number of disrupted passengers that are reassigned to following available trains to travel to their destinations; the minimization of the weighted total train delay that considers the positive deviation of all trains with respect the scheduled arrival time at their destinations. It should be explained here that the weight for each train is determined by the amount of undisrupted passengers originally traveling on the following trains, $\omega_f = NP_f/1000$. In this way, the negative impact of passenger reassignment on the undisrupted passengers can be taken into consideration. The two objective function components are optimized via a weighted-sum approach and by considering all the disruption scenarios. When the total available seat capacity is enough, all the disrupted passengers can be saved by setting a relatively large weight for the objective component of maximizing the passenger accessibility.

$$Z = \text{Max } \varepsilon \times \sum_{w \in W} \upsilon_w \times \sum_{p \in P} \sum_{f \in F_{avai}} y_{f,p,w} - (1-\varepsilon) \times \sum_{w \in W} \upsilon_w \times \sum_{f \in F/\{f^*\}} \omega_f \times (a_{f,d_f,w} - AT_f) \quad (1)$$

Subject to:

Capacity loss constraints:

$$a_{f^*,b_w,w} = Dst_w, \forall w \in W \quad (2)$$

$$a_{f^*,e_w,w} \geqslant Det_w, \forall w \in W \quad (3)$$

Start time constraints at the origin node:

$$a_{f,o_f,w} \geqslant EST_f, \ \forall f \in F, w \in W \quad (4)$$

Within cell transition constraints:

$$a_{f,j,w} \geqslant a_{f,i,w}, \ \forall f \in F, (i,j) \in A_f : i,j \in N, w \in W \quad (5)$$

Minimum and maximum running time constraints:

$$TT_{f,i,j,w} = a_{f,j,w} - a_{f,i,w}, \forall f \in F, (i,j) \in A_f : i,j \in N, w \in W \quad (6)$$

$$TT_{f,i,j,w} \leqslant \delta_{f,i,j} + \vartheta^{\max}_{f,i,j}, \forall f \in F, (i,j) \in A_f : i,j \in N, w \in W \quad (7)$$

$$TT_{f,i,j,w} \leqslant \vartheta^{\max}_{f,i,j} \times \sum_{p \in P} \Delta_{f,p,j,w} + \delta_{f,i,j} \times M, \quad \forall f \in F_{avai}, (i,j) \in A_f \cap A_s : i \in N, j \in \{s, d_p\}, w \in W \quad (8)$$

$$TT_{f,i,j,w} \geqslant \delta_{f,i,j}, \forall f \in F, (i,j) \in A_f : i,j \in N, w \in W \quad (9)$$

Table 1. General Subscripts, Input Parameters, and Decision Variables

Symbol	Description
General subscripts	
i, j, k	Node index, $i, j, k \in N$, N is the set of nodes in a railway network
a	Arc index, $a \in A$, A is the set of arcs in a railway network
t	Time index, $t \in \{1mT\}$, T is the rescheduling time horizon
p	Passenger origin–destination (OD) pair index, $p \in P$, P is the set of passenger OD pairs in the broken train f^*. Each passenger OD pair refers to a group of passengers who have the same origin and destination stations
f	Train index, $f \in F$, F is the set of all trains
w	Random scenario index, $w \in W$, W is the set of all scenarios
Input parameters	
f^*	Broken train index, $f^* \in F$
F_{avai}	Set of available following trains, which can be rescheduled to serve the disrupted passengers, $F_{avai}CF$
A_f	Set of arcs that train f may use, A_fCA
A_b	Set of arcs between two consecutive stations, A_bCA
A_s	Set of arcs in stations, A_sCA
$\delta_{f,i,j}$	Scheduled running (dwelling) time of train f to travel through arc (i, j)
$\vartheta^{max}_{f,i,j}$	Maximum extra running (dwelling) time for train f on arc (i, j)
h	Safety time interval (headway) between two consecutive trains
o_f	Origin node of train f
d_f	Destination (sink) node of train f
EST_f	Predetermined earliest start time of train f at its origin node
AT_f	Predetermined arrival time at the destination node of train f in the original timetable
ω_f	Weight for train f, determined by the amount of undisrupted passengers originally traveling on train f
NP_f	The number of undisrupted passengers on board for following train $f, f \in F/\{f^*\}$
ρ_f	Available passenger-carrying capacity (available seats) in train f for the passengers of the broken train f^*, $f \in F_{avai}$
$\sigma_{f,i,j}$	Extra running time for train f deceleration with an extra stop at arc (i, j) for passenger OD pair p getting on or off the train
$\pi_{f,i,j}$	Extra running time for train f acceleration with an extra stop at arc (i, j) for passenger OD pair p getting on or off the train
$\tau_{f,i,j}$	Extra dwelling time for train f stopping at arc (i, j) with an extra stop at arc (i, j) for passenger OD pair p getting on or off the train
s	Transfer node where passengers need to get off their original train and then (eventually) get on a following train
d_p	Destination node for passenger OD pair p
η_p	Volume of passenger OD pair p
υ_w	Occurrence probability of scenario w
Dst_w	Start time of disruption under scenario w
Det_w	End time of disruption under scenario w
b_w	Begin node of disruption under scenario w
e_w	End node of disruption under scenario w
ε	Weight for objective function
M	A relatively large positive number
Decision variables	
$a_{f,i,w}$	Arrival time variable of train f arriving at node i under scenario w
$order_{f_1,f_2,i,j,w}$	0 to 1 binary train order variable: =1, if train f_2 is scheduled after train f_1 on arc (i, j) under scenario w; =0, otherwise
$y_{f,p,w}$	Passenger reassignment variable, the number of passengers with OD pair p reassigned to the following train f under scenario w
$\Delta_{f,p,i,w}$	0 to 1 binary train extra stop variable: =1, if train f needs to perform an unscheduled stop at node i for passenger p, $p \in P$, under scenario w; =0, otherwise
$TT_{f,i,j,w}$	Travel time for train f on arc (i, j) under scenario w

$$TT_{f,i,j,w} \geq \delta_{f,i,j} + \Delta_{f,p,j,w} \times \tau_{f,i,j},$$
$$\forall f \in F_{avai}, p \in P, (i,j) \in A_f : i \in N, j \in \{s, d_p\}, w \in W \quad (10)$$

$$TT_{f,j,k,w} \geq \delta_{f,j,k} + \Delta_{f,p,j,w} \times \pi_{f,j,k},$$
$$\forall f \in F_{avai}, p \in P, (i,j),(j,k) \in A_f : \quad (11)$$
$$i \in N, k \in N, j \in \{s, d_p\}, w \in W$$

$$TT_{f,k,i,w} \geq \delta_{f,k,i} + \Delta_{f,p,j,w} \times \sigma_{f,k,i}, \\ \forall f \in F_{avai}, p \in P, (k,i), (i,j) \in A_f : \\ k \in N, i \in N, j \in \{s, d_p\}, w \in W \quad (12)$$

$$TT_{f,k,i,w} \geq \delta_{f,k,i} + \Delta_{f,p,k,w} \times \pi_{f,k,i} + \Delta_{f,o,j,w} \times \sigma_{f,k,i}, \\ \forall f \in F_{avai}, p \in P, o \in P, (q,k), (k,i), (i,j) \in A_f : \\ q \in N, i \in N, k \in \{s, d_p\}, j \in \{s_o, d_o\}, w \in W \quad (13)$$

Order constraints:

$$order_{f_1,f_2,i,j,w} + order_{f_2,f_1,i,j,w} = 1, \\ \forall f_1, f_2 \in F, (i,j) \in A_f \cap A_b : i,j \in N, w \in W \quad (14)$$

Track capacity constraints:

$$a_{f_2,i,w} + (1 - order_{f_1,f_2,i,j,w}) \times M >= a_{f_1,i,w} + h, \\ \forall f_1, f_2 \in F, (i,j) \in A_f \cap A_b : i,j \in N, w \in W \quad (15)$$

$$a_{f_2,j,w} + (1 - order_{f_1,f_2,i,j,w}) \times M >= a_{f_1,j,w} + h, \\ \forall f_1, f_2 \in F, (i,j) \in A_f \cap A_b : i,j \in N, w \in W \quad (16)$$

$$a_{f_1,i,w} + (1 - order_{f_2,f_1,i,j,w}) \times M >= a_{f_2,i,w} + h, \\ \forall f_1, f_2 \in F, (i,j) \in A_f \cap A_b : i,j \in N, w \in W \quad (17)$$

$$a_{f_1,j,w} + (1 - order_{f_2,f_1,i,j,w}) \times M >= a_{f_2,j,w} + h, \\ \forall f_1, f_2 \in F, (i,j) \in A_f \cap A_b : i,j \in N, w \in W \quad (18)$$

Passenger flow balance constraints:

$$\sum_{f \in F_{avai}} y_{f,p,w} \leq \eta_p, \forall p \in P, w \in W \quad (19)$$

Passenger-carrying capacity constraints:

$$\sum_{p \in P} y_{f,p,w} \leq \rho_f, \forall f \in F_{avai}, w \in W \quad (20)$$

Mapping constraints between passenger reassignment and train stop plan:

$$\Delta_{f,p,j,w} \times \rho_f \geq y_{f,p,w}, \forall f \in F_{avai}, p \in P, j \in \{s, d_p\}, w \in W \quad (21)$$

$$\Delta_{f,p,j,w} \leq y_{f,p,w}, \forall f \in F_{avai}, p \in P, j \in \{s, d_p\}, w \in W \quad (22)$$

Robust passenger reassignment constraints:

$$y_{f,p,w} = y_{f,p,w-1}, \forall f \in F_{avai}, p \in P, w \in W : w > 1 \quad (23)$$

All the above constraints work for every scenario w. Constraints (2) and (3) are capacity loss constraints to ensure that the broken train occupies the arcs (from the start node to the end node of disruption) during the whole disruption horizon. The start node of the disruption corresponds to a station node. We assume that the disruption cannot start in the middle of the run between stations or during stops. Constraints (4) and (5) are space–time network constraints. Constraints (4) are needed to make sure that every train will depart from its origin station after its earliest departure time. The earliest departure time is consistent with the planned departure time from the origin station. Constraints (5) ensure the time transition within arcs. It means that the arrival time at node i is no less than the arrival time at node j within arc (i, j). Constraints (6) give the calculation method of the travel time on each arc for each train. Constraints (7–13) enforce the required maximum and minimum running and dwelling times. Constraints (7) make sure the running time and dwelling time no more than the maximum value. The available following trains are limited to add the extra stops only if they serve the disrupted passengers, by constraints (8). The latter constraint is an additional limitation on the maximum running time in stations. If one train goes through a station (i.e., the planned dwelling time is 0 min, $\delta_{f,i,j} = 0$), then if no extra stop is added ($\sum_{p \in P} \Delta_{f,p,j,w} = 0$), the rescheduling dwelling time should be 0 min; if the planned dwelling time is more than 0 min, constraints (8) are not restrictive. The maximum dwelling time is then limited by constraints (7). Constraints (9) guarantee the running time and dwelling time no less than the scheduled time. Constraints (10) guarantee the minimum dwelling time for disrupted passengers embarking or disembarking. Constraints (11) ensure the minimum running time with acceleration for arc (j, k) owing to an extra stop at arc (i, j). Constraints (12) make sure the minimum running time with deceleration for arc (k, i) owing to an extra stop at arc (i, j). If one train is added with two stops at two consecutive stations (q, k) and (i, j), the minimum running time for arc (k, i) should be considered with extra running time for acceleration and deceleration at the same time, by constraints (13).

Constraints (14) guarantee that trains cannot take over each other in the section between two consecutive stations. Constraints (15–18) ensure that the number of trains occupying arc (i, j) is less than the capacity of arc (i, j). We note that there can be more than one train traveling on each track segment between two consecutive stations as long as the minimum time interval (headway) between trains is satisfied at any time.

Constraints (19–22) are formulated for passenger reassignment. Constraints (19) require that the total number of passengers in the OD pair p assigned to the following trains is no more than the total passenger demand of OD pair p. Constraints (20) guarantee that the available seat capacity of each train is not violated. Constraints (21) and (22) make sure that once a following available train f is assigned to serve disrupted

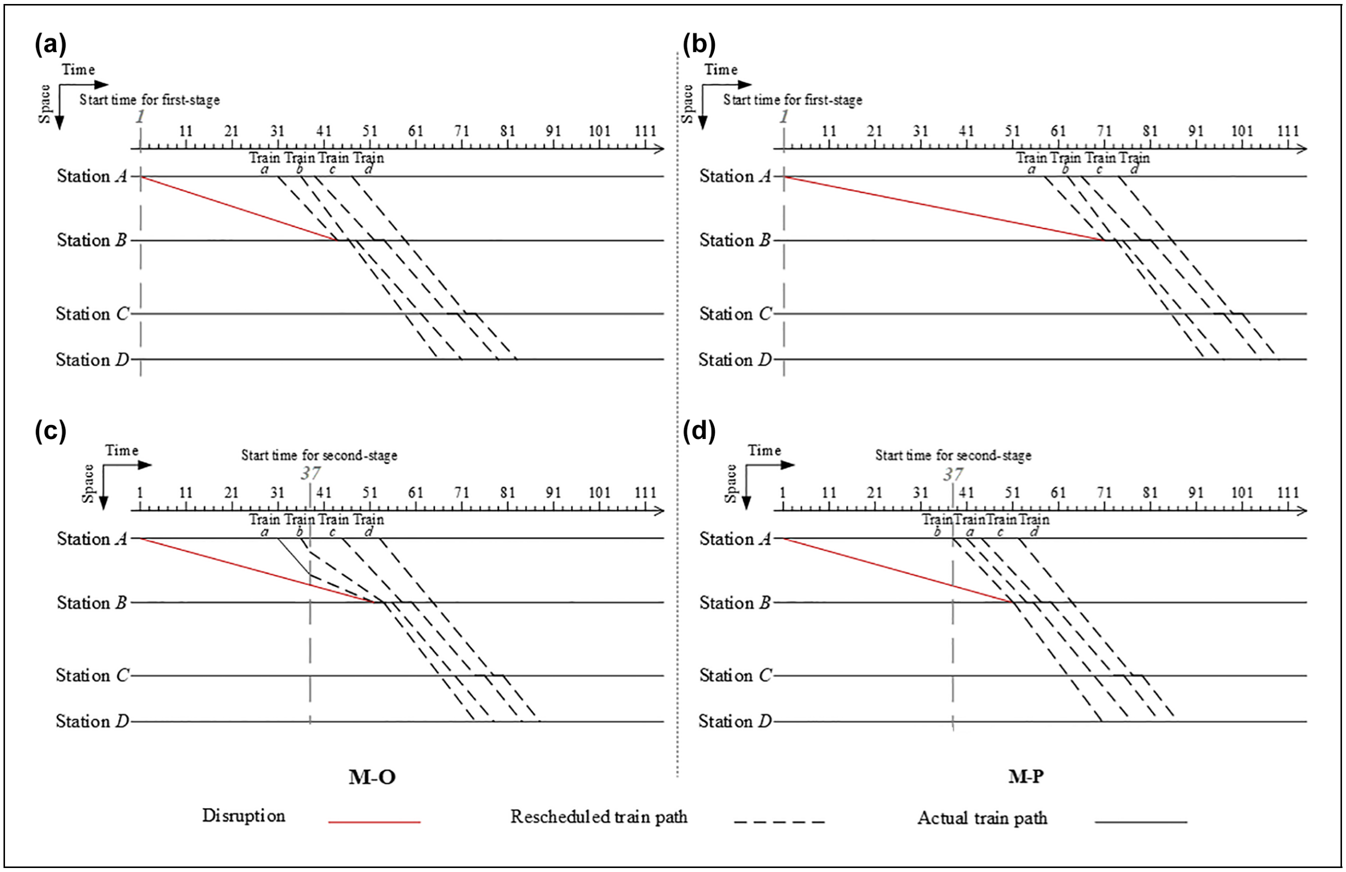


Figure 3. Illustrative example for various settings of approach M: (*a*) first stage for M-O with an optimistic estimation of duration as 40 min, (*b*) first stage for M-P with a pessimistic estimation of duration as 70 min, (*c*) second stage for M-O with an exact duration of 50 min, and (*d*) second stage for M-P with an exact duration of 50 min.

passengers of a passenger OD pair p, this train must stop at the transfer node s and their destination node d_p. In this work, the transfer station is the one just after the disruption location, where the disrupted passengers will wait for the following trains.

Constraints (23) formulate the RTSC constraints, which ensure that the same train stopping plan is enforced for each train under the different disruption duration scenarios. These constraints impose the robustness of dispatching solutions.

Solution Approaches

We next describe the three approaches proposed in this paper: One-stage approach (O), Multi-stage approach (M), and Stochastic approach (S). The two latter approaches can solve the problem in presence of dynamic information and uncertainty.

O: One-stage approach with perfect disruption information refers to the ideal case in which the exact disruption duration is known *a priori*, and no further re-planning is needed. This is an extreme (often non-realistic) situation, which helps to benchmark other approaches. In the one-stage approach, the set of all scenarios only includes one scenario, that is, $W = \{w_0\}$, the occurrence probability of this scenario is 1, that is, $\upsilon_{w_0} = 1$.

M: Multi-stage approach with the dynamic and random disruption information refers to the case in which the exact disruption duration is not known at the beginning of the disruption, and the information becomes more precise with time. Optimistic estimation (M-O), Pessimistic estimation (M-P) or Expected value information (M-E) of the disruption duration is adopted at the first stage. Trains are rescheduled according to the estimated duration. When the new information is available, re-planning work will be performed with the updated disruption duration. Also, some of the train paths rescheduled at the previous stages have been achieved and will be the inputs when re-planning.

Figure 3 illustrates a traffic situation to further discuss the approach M. The disruption is assumed to happen at time point 1, and the second stage starts at time point 37. In M-O, we have an optimistic estimation of the

Table 2. Stopping Plans and Delay Caused by Extra Stops for Train *a* and *b*

Stopping plan	Station *B*	Station *C*	Station *D*	Delay (min)
Train *a* ($C=40$)				
I	√	x	√	0
II	√	√	x	7
Train *b* ($C=40$)				
III	√	x	√	7
IV	√	√	x	14

Note: C = available seat capacity.

disruption duration fixed to 40 min at the first stage. The rescheduled plan is shown in Figure 3*a* with the dotted lines. With a consideration of the disruption duration of 50 min at the second stage, trains *a* and *b* have departed from Station *A* before time point 37, so the part of their paths before time point 37 cannot be changed at the second stage, as shown by the full line in Figure 3*c*. In M-P, a pessimistic estimation of the disruption duration, that is, 70 min, is adopted at the first stage. The corresponding rescheduling solution for the first stage is reported in Figure 3*b*, where no trains have departed from Station *A* at time point 37, and they will depart after the start time of the second stage, no matter how long the disruption duration will be. Figure 3*d* shows the second stage in M-P with the disruption duration of 50 min.

S: Stochastic optimization approach with robustness consideration. With this approach, we consider the case in which there is a strict correlation between the solutions obtained for different scenarios. In other words, with a robust approach, we guarantee the same stopping plan for each following train and the same number of passengers assigned to the same following trains, under different scenarios. That is, after the dispatcher determines the revised train stopping plan and passenger reassignment strategy, for any scenario which has been taken into account, trains will have the same stops at the same stations, and the same number of disrupted passengers will be assigned to the same following trains, which still strictly ensures the running (dwell) time, headway intervals, and limited train capacity. Without the robust train stopping plan, the dispatcher has to frequently re-adjust and continuously change stopping plans, which could make the station organization difficult in response. Each scenario is considered with a known disruption duration, and the disruption duration is subject to a known distribution. This means that the scenarios are independent, and each scenario has a probability.

Based on the example shown in Figure 3, we aim to conceptually introduce approach S. Let us consider that for Train *a*, there are two alternative stopping plans, I and II, and stopping plan III and IV can be rescheduled for Train *b*, as shown in Table 2. The delay shown here is caused by extra stops, including extra running time for deceleration and acceleration, and extra dwell time for passengers embarking. The available seat capacity values for Train *a* and *b* are all 40.

We consider that the duration is subject to a normal distribution. The mean of the duration is 50 min. Consequently, the range of the duration is [35, 65], under the principle of "3σ" with standard deviation $\sigma = 5$. For simplicity, here we select two scenarios: (1) scenario w_1 with a duration of 44 min and an occurrence probability of 78%, and (2) scenario w_2 with a duration of 60 min and an occurrence probability of 22%. We know that in scenario w_1, Train *a* and *b* departing from Station *A* are delayed by 20 min and 0 min, respectively. In scenario w_2, Train *a* and *b* departing from Station *A* are delayed by 36 min and 10 min, respectively. Table 3 reports the solutions under robust train stopping plan and normal train stopping plan, which corresponds to approach S and O, respectively. If the normal train stopping plan strategy is considered, Train *b* uses stopping plan IV under scenario w_1, and a different stopping plan III under scenario w_2. If the robust train stopping plan strategy is enforced, Train *b* selects stopping plan IV under both scenarios. If we consider an equal weight for the two objectives, the robust train stopping plan strategy causes less objective value by $20.91 - 20.14 = 0.77$, compared with the normal train stopping plan strategy, which can be viewed as the cost of the robust train stopping plan.

Numerical Experiments

Instance Description

The test bed is based on a part of the Beijing–Shanghai high-speed railway: the line from Nanjing South Station to Shanghai Hongqiao Station, including eight stations. The timetable used in our experiments is a real one for the year 2018. The resulting mathematical formulation is solved by IBM CPLEX Optimization Studio 12.8.0.0 on a computer with Intel(R) Xeon (TM) CPU E7-4850 v4 @ 2.10 GHz and 256 GB memory.

Table 3. Statistics for Solutions under Approach S and O

	Robust train stopping plan (approach S)						Normal train stopping plan (approach O)					
	Scenario $\boldsymbol{w}_1$ (78%)			Scenario $\boldsymbol{w}_2$ (22%)			Scenario $\boldsymbol{w}_1$ (78%)			Scenario $\boldsymbol{w}_2$ (22%)		
	Plan number	#P	Delay (min)	Plan number	#P	Delay (min)	Plan number	#P	Delay (min)	Plan number	#P	Delay (min)
Train *a*	I	40	20	I	40	36	I	40	20	I	40	36
Train *b*	IV	40	14	IV	40	24	IV	40	14	III	40	17
Total	NA	80	34	NA	80	60	NA	80	34	NA	80	53
Z	$(80 \times 0.5 - 34 \times 0.5) \times 0.78 + (80 \times 0.5 - 60 \times 0.5) \times 0.22 = 20.14$						$(80 \times 0.5 - 34 \times 0.5) \times 0.78 + (80 \times 0.5 - 53 \times 0.5) \times 0.22 = 20.91$					

Note: #*P* = number of saved passengers; NA = not available.

Disruption information: We assume that one train named G1 fails on the track segment between Nanjing South Station and Zhenjiang South Station at 19:19, and the next service of Train G1 will be canceled. The duration of this failure is subject to a normal distribution. The mean of the shorter duration and the longer duration is 50 and 65 min, respectively, that is, $\mu_1 = 50$ and $\mu_2 = 65$. Consequently, the range of the shorter duration and longer duration is [35, 65] and [50, 80], respectively, under the principle of "3σ" with standard deviation $\sigma = 5$.

Passenger information: There are four passenger OD pairs on Train G1 with the following destinations: Changzhou North Station, Suzhou North Station, Kunshan South Station, and Shanghai Hongqiao Station. The passenger volume of these four pairs is 80, 160, 80, and 580, respectively. The total passenger demand is 900.

Traffic information: We know the real original timetable for the following trains of Train G1, named G2 to G20, which have different pre-planned stops. Train G20 is the last one operated during the day. The safety time interval (headway) between two consecutive trains is set to 3 min, $h = 3$. The extra running time for train deceleration and acceleration with an extra stop are set to 3 min and 2 min, respectively, $\sigma_{f,i,j} = 3$ and $\pi_{f,i,j} = 2$. The minimum dwelling time for disrupted passengers embarking and disembarking is set to 3 min, $\tau_{f,i,j} = 3$.

The trade-off between these two objective components will be an interesting problem. Dispatchers can set the weights for these two objectives according to the actual situation. If they would like to transport as many disrupted passengers as possible, a relatively larger weight can be set for the maximization of the saved passengers. Conversely, if they consider the train operation a priority, a relatively larger weight can be set for the minimization of the total train delay. Pareto-optimal solutions can be identified by means of the weighted-sum method with the normalization (*21*). In this paper, we will not occupy too much space to discuss the trade-off problem. An equal weight $\varepsilon = 0.5$ for the two objective functions is considered, to illustrate the methodological contribution. The value of big-M cannot be less than the time for the rescheduling period. Here, we set M equal to 400.

Computation Time Versus Instance Size

In this section, all experiments are performed by approach S, which considers the strict correlation among different scenarios at one stage. Compared with other approaches, it will be more difficult to obtain the optimal solution with approach S. Therefore, we choose approach S to test the efficiency of our approach. The efficiency of our method is assessed by varying the number of trains, that is, from two to 20 trains, to be rescheduled in the network. Trains are included to be rescheduled according to their departure sequence from Nanjing South Station in the original timetable. The time horizon for rescheduling will get longer when the number of trains varies from two to 20. When 20 trains are included, the rescheduling time horizon is 4 h, which is comparable to the travel time required if passengers take another transportation mode from Zhenjiang to Shanghai. We consider that if the disrupted passengers wait for an available seat for a longer time than this acceptable time, they will claim a refund on their train tickets and choose another transportation mode. Scenarios are selected symmetrically. Here we test different disruption durations, including 35, 40, 44, 47, 49, 51, 53, 56, 60, and 65 min, with corresponding probabilities of 0.002, 0.028, 0.1, 0.17, 0.2, 0.2, 0.17, 0.1, 0.028, and 0.002.

Under fixed orders, the train order in the original timetable is kept; no reordering is optimized. Under flexible order, the train order can be changed, namely optimized, to serve the purpose of the problem. We set 1 h of computation time to the solver. The computational assessment is reported in Figure 4. There are two indicators: one related to the time required to find the best-known solution (gray curves), and the other one related

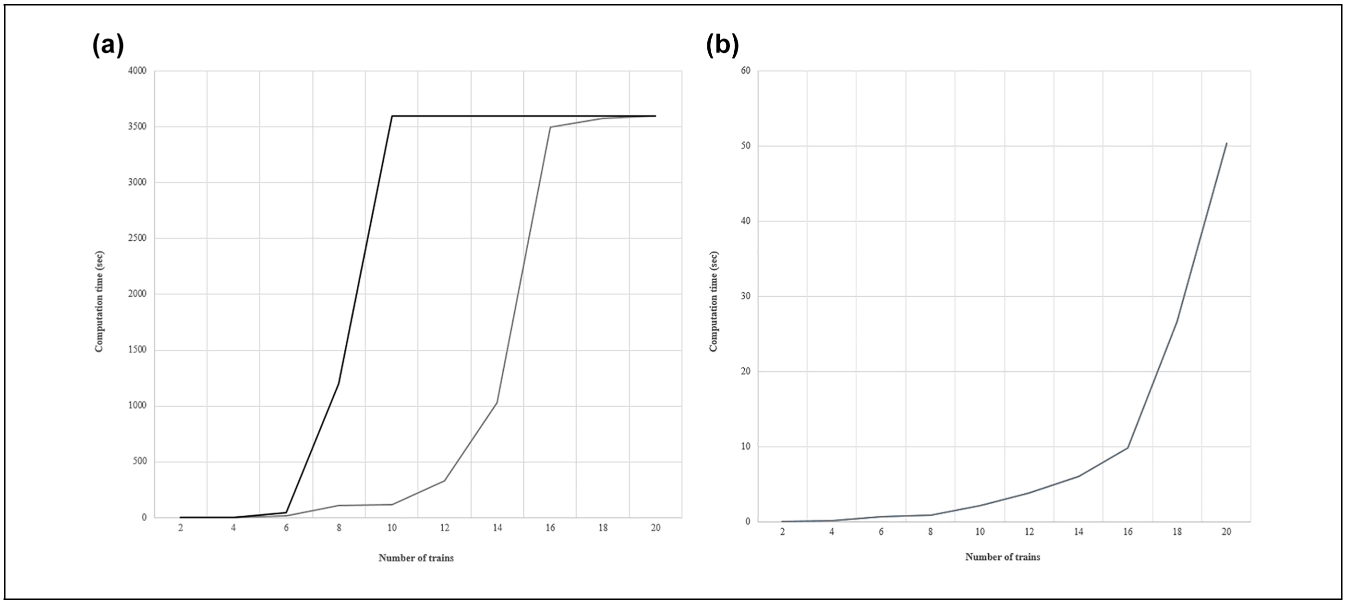


Figure 4. Computation time for instances of different sizes: case (*a*) flexible orders and case (*b*) fixed orders.

Table 4. Number of Decision Variables and Objectives for Investigated Instances

#*w* = 10, *D* = 900		Decision variables					Objectives		
#***f***	*C*	#***a***	#***order***	#***y***	#Δ	#***TT***	***Z*** for case (b)	***Z*** for case (a)	Improve-ment
2	60	280	280	40	80	150	14.746	14.746	0
4	180	560	1120	120	240	410	63.392	67.897	4.505
6	280	840	2520	200	400	670	105.130	109.859	4.729
8	420	1120	4480	280	560	930	168.724	173.624	4.900
10	510	1400	7000	360	720	1190	207.860	212.752	4.892
12	630	1680	10080	440	880	1450	261.925	266.818	4.893
14	730	1960	13720	520	1040	1710	308.145	313.040	4.895
16	860	2240	17920	600	1200	1970	362.985	367.190	4.205
18	1000	2520	22680	680	1360	2230	380.130	384.305	4.175
20	1110	2800	28000	760	1520	2490	380.060	381.375	1.315

to the total computation time (black curves). If we obtain the global optimal solution within 1 h of computation time, the difference between these two curves is the time required to prove optimality, for example the instances of six or eight trains. Table 4 presents the following quantitative information on the investigated instances: including the number of scenarios (#*w*), number of trains (#*f*), passenger demand (*D*), total available seat capacity for the disrupted passengers (*C*), the number of decision variables including train arrival time (#*a*), train order (#*order*), passenger reassignment (#*y*), train extra stop (#Δ) and travel time (#*TT*), and information of objectives (*Z*) (the optimal solution can be obtained within the computation time of 1 h).

From Figure 4 and Table 4, we conclude that the model can easily solve case (b) with fixed orders, as the train order decision variables are much more than others, and it is thus simple searching for an optimal solution when we do not consider train reordering. Practitioners like fixed order because they can better understand the consequences, and it will have less impact in planning the resource occupation downstream. Having a fixed order is sometimes needed, for instance, if a route has been already set, and to achieve a flexible order the signal and route locking must be reset. It should be explained that a feasible solution can be obtained easily even for the instances which involve more trains with the flexible order. For example, we can get a feasible solution within a few seconds for the instance with 16 trains shown in Figure 4*a*. The first feasible solution is 146.888, with a gap of 59.996% compared with the best know solution within the computation time of 1 h. Moreover, we can get a feasible solution of 354.214 within the computation time of 180 s. The gap is 3.534%, accordingly. The computation time can be set according to the actual operation situation, and the best-known solution within this limited computation time will be obtained. The optimal objective value of case (a) with flexible orders increases 0 to 4.900 compared with case (c) with fixed orders. In the investigated instances with different train numbers, the reordering between G2 and G3, and between G2 and G4 departing from Nanjing South Station always happens in most scenarios, whereas the reordering between any other two trains is only decided in a few scenarios. We can conclude that most of the improvement of flexible orders results from the reordering decisions between G2 and G3, and between G2 and G4. This can also explain the almost constant improvement of the objective as the number of trains increases. It will be significant to find out the key reordering decisions in the process of rescheduling to obtain a better solution within a short computation time.

From Figure 5, we can see that the number of reordering is different under different scenarios in one instance. When the disruption duration gets longer, more trains will be disturbed and more reordering will happen to reduce the total train delay. It should be noted here because of the maximum extra dwell time set to 6 min, and the headway set to 3 min, it will not happen that one train dwells at one station for a long time to give chances to other trains to overtake.

For the instance with $\#f = 4$, the reordering happens between trains G2 and G3, and between trains G2 and G4, departing from Nanjing South Station in the scenario with the disruption duration of 49 min. For the instances with $\#f = 16$ and 18, the reordering only happens between trains G2 and G3 in the same scenario. Train G4 is rescheduled with more extra dwell time and stops in these instances. This is why the reordering does not happen between trains G2 and G4 in the instances with $\#f = 16$ and 18. The number of train reordering actions in a solution does not necessarily relate to the number of trains.

For the instance with $\#f = 6$ and 8, the reordering happens between trains G2 and G3, and between trains G2 and G4 departing from Nanjing South Station, in the scenario with the disruption duration of 56 min. Besides, the reordering happens between trains G4 and G5, and between trains G4 and G6 departing from Nanjing South Station, in the scenarios with the disruption duration of 60 and 65 min. As for the instance with $\#f = 8$, two more reorderings happen between G4 and G7, and between G4 and G8, departing from Nanjing South Station, in the scenarios with the disruption duration of 60 and 65 min. For the instances with $\#f = 10$ and 14, the reordering happens between trains G2 and G3, and

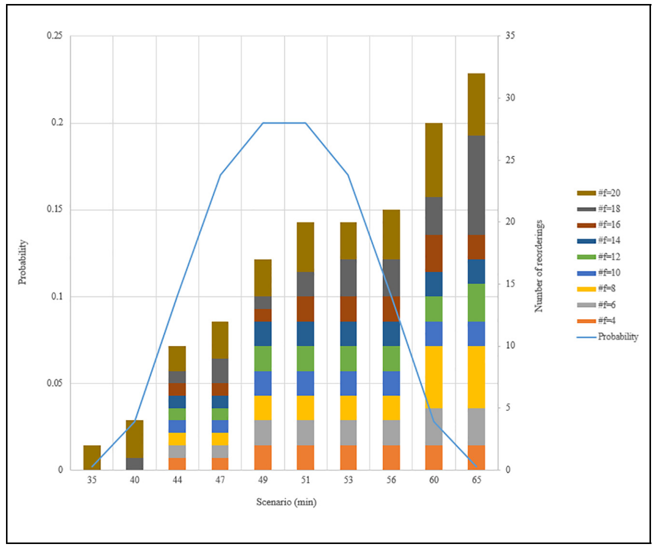


Figure 5. Number of reordering under different scenarios in investigated instances.

between trains G2 and G4, departing from Nanjing South Station in the scenarios with the disruption duration of 56, 60 and 65 min. From the experiments, increasing the disruption duration results in an increasing number of train reorderings when the amount of trains is small. The number of train reordering actions in a solution does not necessarily relate to the disruption duration.

Table 5 presents detailed results for investigated instances, including the number of saved passengers, total weighted train delay, and the number of extra stops for transporting disrupted passengers. It shows that most extra stops are rescheduled at the transfer station for disrupted passengers to embark, and mostly trains are rescheduled to serve the disrupted passengers with destinations consistent with the pre-planned train stops, to avoid the extra running time caused by the extra stops. Moreover, when the number of trains is less than 18, the total available seat capacity is less than the passenger demand. The number of saved passengers depends on the total available seat capacity. When the number of trains increases to 18, the total available seat capacity is more than the passenger demand. The number of saved passengers equals the passenger demand. All disrupted passengers can be saved for a timetable with enough available seat capacity (*22*). Not all following trains require the addition of extra stops or dwell times to transport the disrupted passengers, while the available seat capacity of some following trains is enough for satisfying the passenger demand. Therefore, the following trains, with pre-planned stops at the transfer stations or destination stations of disrupted passengers, will be more likely to be rescheduled to serve the disrupted passengers,

Table 5. Detailed Results for Investigated Instances

Instance		Case (a) with flexible orders			Case (c) with fixed orders		
#w = 10, *D* = 900							
#f	*C*	#Saved passengers	Weighted train delay	#Extra stops	#Saved passengers	Weighted train delay	#Extra stops
2	60	60	30.508	0	60	30.508	0
4	180	180	44.206	0	180	53.217	0
6	280	280	60.282	2 (2 + 0)	280	69.740	2 (2 + 0)
8	420	420	72.752	3 (3 + 0)	420	82.552	3 (3 + 0)
10	510	510	84.496	5 (5 + 0)	510	94.280	5 (5 + 0)
12	630	630	96.364	6 (6 + 0)	630	106.150	6 (6 + 0)
14	730	730	103.920	7 (7 + 0)	730	113.710	7 (7 + 0)
16	860	860	125.620	9 (9 + 0)	860	134.030	9 (9 + 0)
18	1000	900	131.390	10 (8 + 2)	900	139.740	9 (8 + 1)
20	1110	900	137.250	8 (7 + 1)	900	139.880	8 (7 + 1)

Note: In brackets presented in the column of the number of extra stops, the number before and after the symbol of " + " reports the number of extra stops at the transfer station and destinations for transporting disrupted passengers, respectively.

with the aim to decrease the train delay resulting from extra stops or dwell times. In the considered instance with 20 trains (G1–G20), Train G19 (which has a pre-planned stop at the transfer station) is rescheduled to transport the disrupted passengers, while in another instance with 18 trains (G1–G18), Train G19 is not involved. The reduction of the number of extra stops in the above-mentioned instance with 20 trains is compensated by an additional travel time for the disrupted passengers.

Impact of Passenger Reassignment on Traffic Flows

In this section, a set of experiments is focused on analyzing the properties of the studied problem. The impact of passenger reassignment on traffic flows for each scenario will be discussed. The duration of disruption is known in each scenario. All experiments are performed by approach O, which considers one scenario with the full information of disruption duration at one stage. Approach S and M keep the key point on solving the problem of the dynamic and randomness of the disruption. Approach S limits the same passenger reassignment strategy under different scenarios with different durations. With this consideration of robustness, the passenger reassignment maybe not optimal for each scenario. It may conclude a worse impact of passenger reassignment on traffic flows than approach O. As for approach M, some decisions are made at the first stage, which may limit the optimality of the solution at the second stage. It may also conclude a worse impact of passenger reassignment on traffic flows than approach O. The worse impact of passenger reassignment on traffic flows owing to the consideration of the dynamic and randomness of the disruption is not what we would like to discuss, and

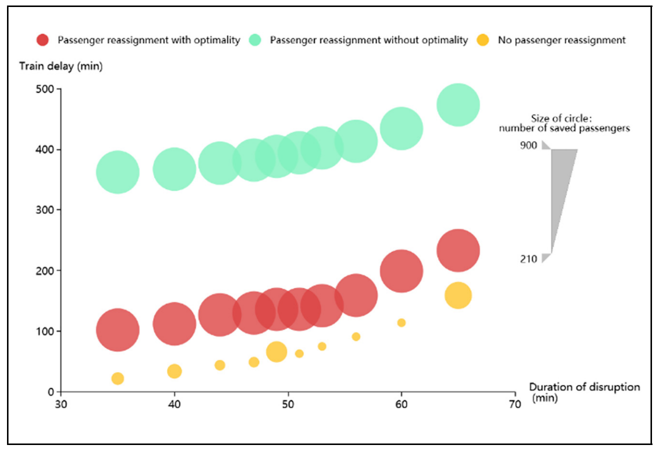


Figure 6. Illustration for the two objectives as duration of disruption increases.

is obvious to understand. We would like to explore how the passenger reassignment affects the traffic flow under different durations. Therefore, we choose approach O in this section.

The instance of 20 trains is considered here and the duration of disruption varies from 35 min to 80 min. Figure 6 presents the total train delay and the number of saved passengers (y-axis, and size of the circles, respectively) for varying duration of the disruption (x-axis) under three conditions: the first condition (shown in red circles) means train rescheduling optimized with passenger reassignment, which is solved by considering formulation (1–22). The second condition considers non-optimized passenger reassignment (presented in green circles), which means that the following trains (G2–G17) with enough available seat capacity (C = 920) will stop at the transfer station (Zhenjiang) and

Table 6. Values of Two Objectives for Investigated Conditions

Instance	Con1		Con2		Con3		Comparison		
Dur (min)	f_1	f_2	f_1	f_2	f_1	f_2	$-f_1$ (*Con1/Con2*) (%)	f_1 (*Con1/Con3*) (%)	f_2 (*Con1/Con3*) (%)
35	102	900	362	900	22	290	71.82	78.43	67.78
40	112	900	367	900	34	330	69.48	69.64	63.33
45	127	900	379	900	44	250	66.49	65.35	72.22
50	133	900	391	900	57	210	65.98	57.14	76.67
55	153	900	411	900	86	210	62.77	43.79	76.67
60	199	900	434	900	114	210	54.15	42.71	76.67
65	233	900	473	900	159	580	50.74	31.76	35.56
70	254	900	506	900	194	370	49.80	23.62	58.89
75	295	900	549	900	248	530	46.27	15.93	41.11
80	354	900	597	900	291	410	40.70	17.80	54.44

Table 7. Detailed Results for First Group of Experiments

	O		M-O		M-E		M-P		S	
Dur (min)	*#ES*	*#RR*	*#ES*	*#RR*	*#ES*	*#RR*	*#ES*	*#RR*	*#ES*	*#RR*
35	8	1	8	1	8	3	8	3	8	2
40	8	1	8	1	8	3	8	3	8	3
44	8	3	8	1	8	3	8	3	8	2
47	8	3	8	3	8	3	8	3	8	3
49	8	3	8	3	8	3	8	3	8	3
51	8	4	8	2	8	4	8	4	8	4
53	8	4	8	2	8	4	8	4	8	3
56	8	4	8	2	8	4	8	4	8	4
60	8	7	8	6	8	7	8	7	8	6
65	8	8	8	6	8	8	8	8	8	5

Note: #ES = number of extra stops; #RR = number of reorderings.

destination stations of every passenger OD pair (Changzhou, Suzhou, Kunshan, and Shanghai) for transporting all disrupted passengers ($D = 900$). We obtain the objective values by solving formulations (1–22) and (24).

$$\Delta_{f,p,i,w} = 1, \forall f \in \{G2, \ldots, G17\}, p \in P, i \in \{s, d_p\}, w \in W \tag{24}$$

The third condition does not consider passenger reassignment (as for the example shown in Figure 2*b*), which means that we optimize the disturbed train timetable by formulations (2–18), and (25), as shown in yellow circles. We obtain the number of saved passengers under the optimized train timetable by solving formulations (2–22), (26), and (27); here Z_1^* indicates the minimum total train delay.

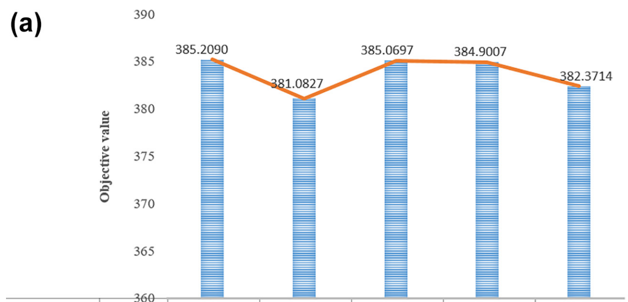
$$Z_1 = Min \sum_{w \in W} \upsilon_w \times \sum_{f \in F} \omega_f \times (a_{f,d_f,w} - AT_f) \tag{25}$$

$$Z_2 = \text{Max} \sum_{w \in W} \upsilon_w \times \sum_{p \in P} \sum_{f \in F_{\text{avai}}} y_{f,p,w} \tag{26}$$

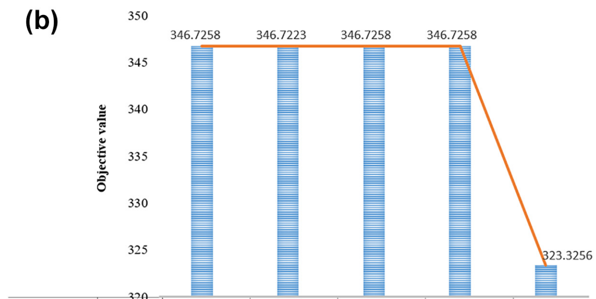
$$Z_1 = Z_1^* \tag{27}$$

Table 6 shows total train delay (f_1) and the number of saved passengers (f_2) under three conditions. We compare train delay (for the same number of saved passengers) of the first condition (*Con1*) with optimized passenger reassignment and the second condition (*Con2*) with non-optimized passenger reassignment, and use the symbol, $-f_1$ (*Con1*/*Con2*), to describe the decreased train delay of *Con1* compared with *Con2*. Moreover, we compare the number of saved passengers of the first condition with optimized passenger reassignment and the third condition (*Con3*) without passenger reassignment, and the symbol, f_2 (*Con1*/*Con3*), is used to describe the increased number of saved passengers of *Con1* compared to *Con3*.

From the results in Figure 6 and Table 6, we can find out that in the first and second conditions with the consideration of passenger reassignment, shown in red and green circles, all 900 disrupted passengers can be saved. The optimized condition (red circles) can decrease the total train delay of 40.70% to 71.82% compared with the other one (green circles). The shorter the disruption duration, the larger is the decrease of the total train delay.



Dur (min)	v_w	O	M-O	M-E	M-P	S
35	0.002	1	1	0.9763	0.9730	0.9858
40	0.028	1	1	0.9891	0.9858	0.9928
44	0.1	1	0.9976	1	0.9967	0.9876
47	0.17	1	0.9900	1	1	0.9949
49	0.2	1	0.9899	1	1	0.9949
51	0.2	1	0.9874	1	1	0.9942
53	0.17	1	0.9847	1	1	0.9894
56	0.1	1	0.9864	1	1	0.9940
60	0.028	1	0.9887	1	1	0.9873
65	0.002	1	0.9882	1	1	0.9497
Computation time (sec)		823.12	672.53	618.93	1178.35	3584.32



Dur (min)	v_w	O	M-O	M-E	M-P	S
50	0.002	1	1	1	1	0.9440
55	0.028	1	1	1	1	0.9280
59	0.1	1	1	1	1	0.9206
62	0.17	1	1	1	1	0.9493
64	0.2	1	1	1	1	0.9296
66	0.2	1	1	1	1	0.9202
68	0.17	1	1	1	1	0.9398
71	0.1	1	1	1	1	0.9501
75	0.028	1	1	1	1	0.8863
80	0.002	1	0.9940	1	1	0.7902
Computation time (sec)		3600	3525.42	3600	3600	3600

Figure 7. Relative objective value and computation time for different approaches: (*a*) first group of experiments and (*b*) second group of experiments.

Comparing the optimized condition with passenger reassignment (red circles) and the condition without the consideration of passenger reassignment (yellow circles), the total train delay increases 17.80% to 78.43% as a result of the passenger reassignment; meanwhile, the number of saved passengers increases 35.56% to 76.67%. It can be concluded that the increased train delay for the passenger reassignment decreases with the increase of the duration of the disruption. In other words, the negative impact of the passenger reassignment on the total train delay decreases when the duration of disruption increases. Meanwhile, the total number of disrupted passengers who get to their destination is significantly different between these two conditions.

The gap between the train delays of the first and second conditions is more or less constant as the duration of the disruption increases. This is caused by the similar extra stops in instances with different durations of the disruption in the first condition. All extra stops are the same in instances in the second condition. The rescheduling targets to maximize the number of saved passengers. In each instance in the first condition, most trains are rescheduled to stop at the transfer station for disrupted passengers to embark. Extra stops at corresponding destinations are needed, which is not influenced too much by the duration of the disruption. There is also evidence about this explanation in Table 7.

Performance of Approach O, Approach M, and Approach S

In this section, two groups of experiments are designed to investigate the performance of approaches proposed in this paper. The instance of 20 trains is considered here. In the first group, shorter disruption durations varying from 35 to 65 min are considered, and the optimistic, expected, and pessimistic estimations of the duration are 35, 50, and 65 min, respectively. In the second group, we pay attention to a longer disruption durations varying from 50 to 80 min, and the optimistic, expected, and pessimistic estimations of the duration are 50, 65, and 80 min, respectively. The second stage for approach M starts at 19:49, 30 min after the beginning of the disruption. Ten scenarios with the probability of 0.002, 0.028, 0.1, 0.17, 0.2, 0.2, 0.17, 0.1, 0.028, and 0.002 are considered in these two groups of experiments. Figure 7 shows the objective function value and computation time for all approaches, that is, O, M-O, M-P, M-E, and S. The objective function value for approaches O and M is calculated according to the probability occurrence of 10 scenarios. For each scenario, we consider the objective function value of approach O as normalized to 1, whereas the other approaches' objective function value takes a relative value to approach O's value. It should be noted that here we present the optimal solution obtained within the computation time of 1 h, and the computation time required to find the best-known solution.

From the results of Figure 7, approach O outperforms approach S in every scenario in relation to the objective. The reason for this is that approach O has perfect information on the disruption. As for approach S, the RTSC plan forces the same extra stops for the same disrupted passengers under different scenarios. However, a different passenger reassignment scheme is computed for approach O compared with approach S. Approaches M-E and M-P perform better than approach M-O, because more decisions are fixed in the first stage for approach M-O, while these are determined in a later stage for approaches M-E and M-P. Combined with Table 7, the difference of the number of extra stops rescheduled for disrupted passengers among these three approaches is not too much. The same number of disrupted passengers are served by the following trains in approach O, M, and S. The difference of the objective among these three approaches depends on the total weighted train delay time. As for the computation time, for the shorter duration, approach O is similar to approach M, because even two stages are needed in approach M, while some of the decisions have been limited in the second stage resulting in a smaller number of decision variables. A significantly longer computation time for approach S is the robustness cost. For the longer duration, three approaches have a similar computation time and the robustness cost is less. The duration of the disruption is also key to the computation time.

For the first group of experiments considering the shorter duration, shown in Figure 6, approach O also outperforms approach M, because in approach O the timetable is rescheduled in a single stage with full information, without the need to consider the effects of the rescheduling actions performed in other stages. In approach M, we determine the input data for the second-stage rescheduling according to the results obtained from the first-stage rescheduling. In relation to approach M-O, before the start time of the second stage, trains G2 and G3 have departed from Nanjing and the departure order for these two trains has been determined (in other words, the train departure times and sequencing decisions cannot be changed in the second stage). Similarly, in approach O, when the disruption duration time is 35 and 40 min, Train G2 departs before Train G3, which is the same as the order of these two trains in the original timetable. Approaches O and M-O thus compute the same solution for these two scenarios, while approach O outperforms approach M-O in the other scenarios, where the reordering decision of G2 and G3 departing from Nanjing is made. In the first-stage solutions of approach M-E, Train G3 has departed from Zhenjiang. Other trains should depart from Zhenjiang after 19:49 (start time of the second stage) in the second stage. Compared with the solution of approach O stated above, approach O outperforms approach M-E under the disruption duration of 35 and 40 min, while for other scenarios, these two approaches have the same solution. In the first stage of M-P, no train departs before 19:49. The train departure time and sequencing decisions will thus be taken at the second stage, and all trains will depart later than 19:49. However, in approach O, some trains depart from Zhenjiang earlier than 19:49 when the disruption duration time is 35, 40, and 44 min. Therefore, approach O computes a better solution than approach M-P when the disruption duration time is 35, 40, and 44 min, while the same solution is computed for the other scenarios.

Comparing Figure 7, *a* and *b*, approach M (M-O, M-E, and M-P) has a better performance in the second group of experiments than the first group of experiments. For most scenarios, approach M can obtain the same solution as the approach O. The reason for this performance regards the start time of the second-stage rescheduling that is relatively earlier than the end time of disruption for the second group of experiments. This means that a few decisions have been fixed at the end of the first stage. Only in M-O is G3 departing from Zhenjiang before 19:49 determined at the end of the first stage. We can conclude that the solution quality significantly depends on when we get accurate information on the disruption duration.

Conclusions and Future Research

This paper proposes a mixed-integer linear programming formulation for the train rescheduling problem with passenger reassignment under one rolling stock breakdown in a railway system with a seat-reserved mechanism. All affected passengers should be reassigned to the following available trains, which have to be rescheduled with the addition of extra stops for offering a new service to the disrupted passengers. However, not all disrupted passengers may be served by the following trains, thus causing a high cost for the train operating company. In this paper, we also consider a dynamic and stochastic environment, in which RTSC constraints are set up to guarantee the same extra stops for the same disrupted passengers under different disruption duration scenarios. The resulting stochastic problem is solved by various approaches with different types of information.

Computational experiments on the "Beijing–Shanghai" high-speed railway example show that the proposed model and solution methods can be used to identify the robust optimized train timetable and passenger reassignment scheme effectively and efficiently. The train-ordering decision is a key factor for the computation time. Conclusions made from the experimental results also describe how the passenger reassignment strategy influences the traffic. The longer is the disruption duration, the relatively less is the negative impact on train delays. If disruptions last for a long time, adding extra stops is not worse than keeping the timetable stopping plan, while more passengers can be saved by this train rescheduling measure. As for the different approaches, approach O always outperforms the other approaches. However, approach S can be more effective under a dynamic and stochastic environment, as the same stopping plan and passenger reassignment is performed, thus limiting the complexity of reorganizing the passenger management at busy stations. However, the computation time for approach S is significantly long, and it seems to be not applicable to manage large case studies. Although approach M performs worse than approach O, it deals with the dynamic and random information and passenger reassignment. As these features are managed with much less computation time than approach S for a relative short duration of the disruption, approach M is significant more scalable than approach S. For a longer duration, the robustness cost of the computation time is not significant for approach S.

In future research, the station capacity constraints can be combined into the proposed model. This can be modeled in several ways; see for example Zhan et al. (*26*) and Gao et al. (*27*). As a result of some broken trains, the optimization of rolling stock circulation could be reorganized to ensure a better utilization of rolling stock in the whole railway network. Further research should also be dedicated to the bi-objective characteristic of the studied problem, where the weight setting for the objectives should be discussed.

Author Contributions

The authors confirm contribution to the paper as follows: study conception and design: Xin Hong, Lingyun Meng, Francesco Corman, Andrea D'Ariano, Lucas P. Veelenturf, Sihui Long; data collection: Xin Hong; analysis and interpretation of results: Xin Hong; draft manuscript preparation: Xin Hong, Sihui Long. All authors reviewed the results and approved the final version of the manuscript.

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