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On Conservation of Renewable Resources with Stock-Dependent Return and Nonconcave Production*, †

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This paper analyzes conservation and extinction of renewable resources when the production function is nonconcave and the return function depends on consumption and the resource stock. The paper focuses on the efficiency of global conservation, a safe standard of conservation, and extinction. Which outcome obtains depends on the marginal rate of substitution between investment and the stock in addition to the discount factor and the marginal productivity of the resource. Conservation may be efficient even if the discounted intrinsic growth rate of the resource is less than 1. *Journal of Economic Literature* Classification Numbers: D90, Q20.

1. INTRODUCTION

There are two important reasons renewable resources can be harvested in a fashion that leads to their extinction. The first arises from the fact that such resources are harvested under open access conditions. The associated common property externalities may cause serious overexploitation [4].[1] The second reason extinction may occur is purely capital theoretic. Even if

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[1] This need not always be true (see, e.g., [10]).

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a resource is managed by a single agent whose objective is intertemporal efficiency, the preferences of the agent and the ecology of the resource may be such that an efficient policy is one that leads to extinction. In this paper, we focus on this second aspect, that is, under what conditions does optimal intertemporal allocation lead to conservation or extinction of a resource.

We consider a discrete time model of a renewable resource whose harvest yields an immediate return (or utility) that depends on both current consumption and the size of the resource stock.[2] Such stock-dependence is important whenever the resource stock influences harvest costs. Growth in the resource stock is governed by an *S*-shaped natural production function that allows for increasing growth rates when the stock is low, but diminishing growth rates as the stock approaches the natural carrying capacity of the environment. The agent in our model can represent a private monopolist, a social welfare maximizer, or a set of atomistic producers in a perfectly competitive industry. In the absence of externalities and in a partial equilibrium setting, the perfectly competitive outcome is identical to that determined by a social planner maximizing the discounted sum of producer and consumer surplus over time [13].

Alternatively, the model is equivalent to a classical optimal growth model, with more general assumptions on both utility and production. The classical growth model is based on a concave production function and a utility function that is independent of the stock. The primary concern of the classical growth literature involves issues other than conservation. In fact, it is common to impose assumptions that are specifically designed to rule out extinction as a possibility.

Clark [5, 6] was the first to examine the problem of extinction with non-concave production under the assumption of a linear, stock-independent utility function. He showed that the efficiency of extinction or conservation depends on the discounted marginal productivity of the resource. He also conjectured the existence of a safe standard of conservation if the discount rate is less than the intrinsic growth rate of the resource, but greater than the maximum average productivity. The existence of such a critical stock was proved by Majumdar and Mitra [15, 16] and characterized more completely by Dechert and Nishimura [9]. An algorithm to calculate the critical stock is given in Anant and Sharma [2] for the linear utility case considered by Clark. Dechert and Nishimura [9] provide a fairly complete characterization of optimal resource allocation policies for the stock-independent model. Their analysis shows that optimal resource investment is an increasing function of the current stock so that optimal resource stocks converge to a steady state over time. Amir *et al.* [1] interpret this

[2] In continuous time the problem of resource extinction has been analyzed by [5–8, 14, 26], among others.

monotonicity property as a second-order condition for local optimality. In these stock-independent models, the significant implications of nonconcave production are: (i) the possible existence of a critical stock or safe standard of conservation; (ii) the optimal policy may not be continuous; and (iii) optimal consumption may be a nonmonotone function of the resource stock. The discount rate and the marginal growth rate of the resource are the primary determinants of the efficiency of conservation or extinction and the return function plays an insignificant role in determining the ultimate fate of the resource.

The optimal growth problem with concave production and stock-dependent return was first considered by Kurz [12]. He showed that stock-dependence introduces the possibility of multiple optimal steady states where in the classical model there is at most one. Levhari *et al.* [13] discuss similar results in the context of renewable resources. Majumdar and Mitra [17] show that stock-dependence can create striking departures from classical behavior by making it possible for optimal programs to exhibit periodic or chaotic behavior over time.

Our analysis of conservation and extinction attempts to account for the joint implications of nonconcave production and the fact that immediate returns depend on the resource stock. When the production function is nonconcave, conservation and extinction are not global properties. When the return function is stock-dependent, optimal programs can exhibit cycles or chaos and a safe standard of conservation may not exist. Together, nonconcave production and stock-dependent returns substantially enlarge the range of possibilities for conservation and extinction when compared with traditional models.

In this paper we show that the efficiency of conservation or extinction depends on more than the relation between the discount rate and the natural growth rate of the species. We show that conservation is efficient even if the discount rate exceeds the natural growth rate everywhere, provided the stock effect on returns is strong enough. Unlike models with a stock-independent return, demand and cost influences can play a crucial role in determining the fate of optimally managed resources and resource conservation is efficient under less stringent restrictions. This suggests that the possibility of extinction for optimally managed resources may be less of a concern than previously presumed. Further, in our framework there is a means by which direct policy intervention can influence the conservation of a species by a single private owner or a perfectly competitive market (through taxes on consumer surplus). This was not possible in earlier models. Finally, our analysis can accommodate the possibility that there is direct social payoff from preservation of species. This allows one to examine what kind of social consciousness is needed for conservation of a species to be efficient, given a production function and discount rate.

The paper is organized as follows. The formal model is presented in Section 2 along with some preliminary lemmas. These focus on the Ramsey–Euler equation, the interiority of optimal allocations, and monotonicity results. Section 3 defines the different types of conservation and extinction considered in this paper and it provides an explicit example that illustrates the complex set of possibilities that may emerge. Section 4 examines the conditions under which there exists a safe standard of conservation, while Section 5 outlines conditions for global conservation. Each of these two sections is divided into two sections. Sections 4.i and 5.i analyze the case where the immediate return function exhibits complementarity between resource investment and the current stock. In this case, optimal resource allocations follow monotonic time paths. The general case, where optimal stocks may exhibit complex nonlinear dynamics, is considered in Sections 4.ii and 5.ii. Finally, Section 6 examines the efficiency of extinction. Proofs of all theorems are given in the text. Proofs of lemmas are omitted and can be found in [25].

2. The Model and Preliminary Results

At each date there is a renewable resource stock, $y_t \in \mathbb{R}_+$, from which an agent harvests and consumes, c_t. Investment in future resource stocks (or escapement) is denoted $x_t = y_t - c_t$. The resource stock in period $t+1$ is determined by a production or stock-recruitment function, $f: \mathbb{R}_+ \to \mathbb{R}_+$, where $y_{t+1} = f(x_t)$. Economic returns in each period are denoted $R(c, y)$. Given an initial resource stock, $y_0 > 0$, the agent chooses resource consumption and investment to maximize the discounted sum of returns over time. The value function for this problem is given by:

$$V(y_0) = \text{Max} \sum_{t=0}^{\infty} \delta^t R(c_t, y_t)$$

subject to: $0 \leqslant c_t$, $0 \leqslant x_t$, $c_t + x_t \leqslant y_t$, and $y_{t+1} = f(x_t)$, where $0 < \delta < 1$ is the discount factor. A *feasible program* is any sequence $\{c_t, x_t, y_t\}_{t=0 \cdots \infty}$ that satisfies the feasibility conditions of the problem. A *stationary program* is a feasible program such that (c_t, x_t, y_t) are constant over time. A feasible program that solves the agent's optimization problem is an *optimal program.* An optimal program such that $0 < c_t < y_t$ for all t is said to be an *interior optimal program.* The set of all optimal resource investments from y_0 defines an *optimal investment policy correspondence* $X(y) = \{x: x = x_0$ for some optimal program $\{c_t, x_t, y_t\}_{t \geqslant 0}$ from $y_0 = y\}$. The *optimal consumption policy correspondence* is defined by $C(y) = \{y - x: x \in X(y)\}$.

Throughout the paper we impose the following assumptions on f:

(T.1) $f(0)=0$, $f(x)$ is strictly increasing on $\mathbb{R}_+$;

(T.2) f is twice continuously differentiable on $\mathbb{R}_+$;

(T.3) $f'(0)>1$;

(T.4) There exists a unique $K>0$ such that $f(K)=K$ and $f(x)<x$ for all $x>K$;

(T.5) There exists a $\gamma\in[0, K)$ such that $f''(x)>0$ for $x<\gamma$ and $f''(x)<0$ for $x>\gamma$;

(T.6) $y_0\in(0, K]$.

Assumption (T.5) allows increasing returns to investment when the resource stock is between 0 and γ.

Define $P=\{(c, y): 0\leqslant c\leqslant y\}$ and $P_0=\{(c, y): 0<c\leqslant y\}$. The return function, R, is assumed to satisfy the following assumptions.

(R.1) $R(c, y)$ is nondecreasing in y, and $R(0, y_1)=R(0, y_2)$ for all $y_1, y_2\in\mathbb{R}_+$.

(R.2) $R(c, y)$ is twice continuously differentiable on P_0, and R_y is bounded above on $\{(c, y): 0<c\leqslant y\leqslant K\}$.

(R.3) $R(c, y)$ is concave in (c, y) on P.

(R.4) For any $y>0$, either $R_c(c, y)>0$ for all $c\in(0, y]$, or there exists a $\xi(y)\in(0, y]$ such that $R_c(c, y)>0$ for all $c\in(0, \xi(y))$ and $R_c(c, y)<0$ for all $c>\xi(y)$.

(R.1)–(R.3) are standard. (R.4) implies that if R is not increasing in c, then for each stock there is a unique strictly positive maximizer of R.

Under (T.1)–(T.6) and (R.1)–(R.4) it is well-known that the following results hold.

(i) An optimal program exists and the value function satisfies the optimality equation

$$V(y)=\max_{0\leqslant c\leqslant y}\left[R(c, y)+\delta V(f(y-c))\right].$$

(ii) V is continuous and nondecreasing.

(iii) $V(y)>-\infty$ for all $y>0$.

(iv) The optimal policy correspondences $X(y)$ and $C(y)$ are upper-hemicontinuous.

(v) If $\{c_t, y_t\}$ is an optimal program, then $c<c_t$ implies $R(c, y_t)\leqslant R(c_t, y_t)$, or $R_c(c_t, y_t)\geqslant 0$; i.e., myopic agents overexploit the resource relative to the optimum.

We begin our analysis with two lemmas.

LEMMA 2.1. *Let* $\{c_t, x_t, y_t\}_{t \geqslant 0}$ *be an optimal program.*

(a) *If* $c_t > 0$ *then* $R_c(c_t, y_t) \geqslant \delta[R_c(c_{t+1}, y_{t+1}) + R_y(c_{t+1}, y_{t+1})] f'(x_t)$.

(b) *If* $c_{t+1} > 0$ *then* $R_c(c_t, y_t) \leqslant \delta[R_c(c_{t+1}, y_{t+1}) + R_y(c_{t+1}, y_{t+1})] f'(x_t)$.

(c) *If* $c_t > 0$ *and* $c_{t+1} > 0$ *then* $R_c(c_t, y_t) = \delta[R_c(c_{t+1}, y_{t+1}) + R_y(c_{t+1}, y_{t+1})] f'(x_t)$.

Some of the later results require that optimal programs be interior. The following lemma is useful in regard to such cases.

LEMMA 2.2. (a) *If* $\lim_{c \downarrow 0} R_c(c, y) = +\infty$ *then* $C(y) > 0$ *for all* $y > 0$.

(b) *If for any strictly positive* $\{y_n\} \downarrow 0$ *there exists* $\{c_n\} \downarrow 0$ *such that* $0 \leqslant c_n \leqslant y_n$ *and* $\lim_{n \to \infty} [R(c_n, y_n) - R(0, y_n)]/y_n = \infty$, *then* $X(y) > 0$ *for all* $y > 0$.

Lemma 2.1 establishes Ramsey–Euler type inequalities for optimal allocations across successive periods. In Lemma 2.2, if $R(c, y) = U(c)$ then the conditions of (a) and (b) reduce to the standard Inada condition on utility.

Next, we develop results on the monotonicity of optimal policies and programs when the immediate return function exhibits complementarity (resp., strict complementarity) between the current stock and investment in future stocks of the form,

(R.5) $R_{cc} + R_{cy} \leqslant 0$ on P_0, or

(R.6) $R_{cc} + R_{cy} < 0$ on P_0.

Define $\phi(x, y) = R(y - x, y)$. (R.5) implies $\phi_{12} \geqslant 0$, which means that ϕ is a supermodular function on the set $\{(x, y): (y - x, y) \in P_0\}$ [27, Section 3]. Under (R.6) ϕ is strictly supermodular. The economic interpretation of (R.5) or (R.6) is that an increase in the current resource stock raises the marginal value of investment in future stocks. Under this type of complementarity optimal investment satisfies the following monotonicity property.

LEMMA 2.3. *Assume* (R.5) *and let* $y \geqslant y'$. *Then* $x \in X(y)$ *and* $x' \in X(y')$ *implies* $\max[x, x'] \in X(y)$ *and* $\min[x, x'] \in X(y')$, *i.e.,* $X(y)$ *is an ascending correspondence.*

This lemma unifies existing results in [1, 9, 19, and 23]. Define the minimum and maximum selections from $X(y)$ by $\underline{X}(y) = \min\{x: x \in X(y)\}$ and $\overline{X}(y) = \max\{x: x \in X(y)\}$. Lemma 2.3 has the following corollary.

Corollary to Lemma 2.3. *Under* (R.5), $\underline{X}(y)$ *and* $\bar{X}(y)$ *are nondecreasing in* y.

Define $\{\underline{c}_t, \underline{x}_t, \underline{y}_t\}_{t \geq 0}$ by $\underline{x}_t = \underline{X}(\underline{y}_t)$, $\underline{c}_t = \underline{y}_t - \underline{x}_t$, $\underline{y}_{t+1} = f(\underline{x}_t)$, where $\underline{y}_0 > 0$ is given. This is the optimal program obtained by following $\underline{X}(\cdot)$ in each period, i.e., $\underline{x}_t = \underline{X}^t(f(\underline{x}_0))$. Similarly, define $\{\bar{c}_t, \bar{x}_t, \bar{y}_t\}$ to be the optimal program obtained by following $\bar{X}(\cdot)$ in each period. The Corollary to Lemma 2.3 implies that these optimal programs are monotonic over time.

Lemma 2.4. *Assume* (R.5). *Then either* $\underline{x}_t \leqslant \underline{x}_{t+1}$ *and* $\underline{y}_t \leqslant \underline{y}_{t+1}$ *for all* t, *or* $\underline{x}_t \geqslant \underline{x}_{t+1}$ *and* $\underline{y}_t \geqslant \underline{y}_{t+1}$ *for all* t. *Similarly, either* $\bar{x}_t \leqslant \bar{x}_{t+1}$ *for all* t, *or* $\bar{x}_t \geqslant \bar{x}_{t+1}$ *for all* t.

The investment sequences $\{\underline{x}_t\}$ and $\{\bar{x}_t\}$ provide lower and upper bounds on the sequence of optimal resource stocks and investments along any optimal program from y_0.

Lemma 2.5. *Assume* (R.5). *If* $\{c_t, x_t, y_t\}$ *is an optimal program then* $\bar{x}_t \geqslant x_t \geqslant \underline{x}_t$ *and* $\bar{y}_t \geqslant y_t \geqslant \underline{y}_t$ *for all* t.

Lemma 2.4 can be sharpened when the immediate return function is strictly supermodular and optimal programs are interior.

Lemma 2.6. *Let* $\{c_t, x_t, y_t\}$ *and* $\{c'_t, x'_t, y'_t\}$ *be any two optimal programs from* $y_0 > y'_0 > 0$, *respectively. If* (R.6) *holds then,* $x_0 \geqslant x'_0$. *In addition, if* $c_0 > 0$, $c_1 > 0$, $c'_0 > 0$, *and* $c'_1 > 0$, *then* $x_0 > x'_0$.[3]

An immediate consequence of Lemma 2.6 is that when the immediate return is strictly supermodular, interior optimal programs of resource stocks and investment are either stationary or strictly monotonic over time.

Corollary to Lemma 2.6. *Assume* (R.6). *If* $\{c_t, x_t, y_t\}$ *is an interior optimal program from some initial stock* $y_0 > 0$ *then only one of the following is true*: (a) $x_t < x_{t+1}$ *and* $y_t < y_{t+1}$ *for all* $t \geqslant 0$, (b) $x_t > x_{t+1}$ *and* $y_t > y_{t+1}$ *for all* $t \geqslant 0$, *or* (c) $x_t = x_{t+1}$, $c_t = c_{t+1}$, $y_t = y_{t+1}$ *for all* $t \geqslant 0$.

If there is no consumption then the resource is biologically sustainable on $(0, K)$, i.e., $f(x) > x$ for all $x \in (0, K)$. From an economic point of view we are interested in stocks that are both sustainable and satisfy the necessary conditions for an optimum. With this in mind we define a *steady state* stock.

[3] In the case of convex technology (with a unique optimum) one can replace strict supermodularity by its weak form and obtain the result that $y_0 > y'_0$ implies $x_0 \geqslant x'_0$ and $x_t \geqslant x'_t$, as in Nyarko and Olson [23].

DEFINITION. A stock $\hat{y} \in [0, K)$ is said to be a steady state if either $\hat{y} = 0$, or

$$R_c(\hat{y} - \hat{x}, \hat{y}) = \delta f'(\hat{x})[R_c(\hat{y} - \hat{x}, \hat{y}) + R_y(\hat{y} - \hat{x}, \hat{y})],$$

where $\hat{x}$ is given by $f(\hat{x}) = \hat{y}$.[4]

The monotonicity of optimal programs then has the following implication.

LEMMA 2.7. (a) *If* (R.5) *holds then* $\{\underline{x}_t\}$ *and* $\{\bar{x}_t\}$ *converge monotonically to optimal steady states.* (b) *If* (R.6) *holds then every interior program converges monotonically to an optimal steady state.*

It is readily apparent that both the existence and stability of nonzero optimal steady states can be important for the conservation or extinction of the resource. The next section highlights the different possibilities.

3. CONSERVATION CONSIDERED

There are significant possibilities for conservation and extinction that only arise under the combination of stock-dependent return and nonconcave production. These can best be illustrated by considering a concrete example.

EXAMPLE 3.1. Let the resource growth function be

$$f(x) = \begin{cases} x - x(x + 0.1)(x - 1) & \text{if } x \leqslant 0.8 \\ 4x^3 - 12.1x^2 + 12.3x - 3.2 & \text{if } 0.8 < x \leqslant 1 \\ x^{0.1} & \text{if } x > 1. \end{cases}$$

This growth function is continuously differentiable and nonconcave with an inflection point at $x = 0.3$. The stock is normalized so that $x = 1$ represents the natural carrying capacity, and the intrinsic growth rate is $f'(0) - 1 = 0.1$.

For convenience, the discount factor is expressed as $\delta = 1/(1 + \rho)$, where ρ is the discount rate. The return function is assumed to be given by

$$R(c, y) = pc - e^{-\alpha(y - c) - \beta y},$$

[4] Steady states need not be optimal, but an optimal stationary program is equivalent to an optimal steady state.

with p, α, and β positive parameters. By appropriate choice of p and α, R_c can be made positive over all or part of the domain $\{(c, y)\colon 0 \leqslant y \leqslant 1, 0 \leqslant c \leqslant y\}$. R then satisfies (R.1)–(R.4). More importantly, $R_{cc} + R_{cy} > 0$ so that $R(y - x, y)$ is submodular in (x, y). Intuitively, this implies substitutability between the current stock and investment in future stocks. Under this condition Benhabib and Nishimura [3] have shown that optimal resource investment is decreasing in the stock on the interior of P and interior optimal programs exhibit cycles. More recently it has been demonstrated that submodularity is inconsistent with the Inada condition so, in general, one cannot guarantee that optimal programs are always interior. When noninterior programs are allowed, optimal investment may be increasing for some intervals of the resource stock and optimally managed stocks may exhibit complex or chaotic dynamics [17, 22].

The implications for resource conservation can be seen in Figs. 1–3. These show the optimal stock policies for low ($\rho = 0.1$), intermediate ($\rho = 0.45$), and higher ($\rho = 0.47$) discounting, where $p = 10.0$, $\alpha = 1.0$, and

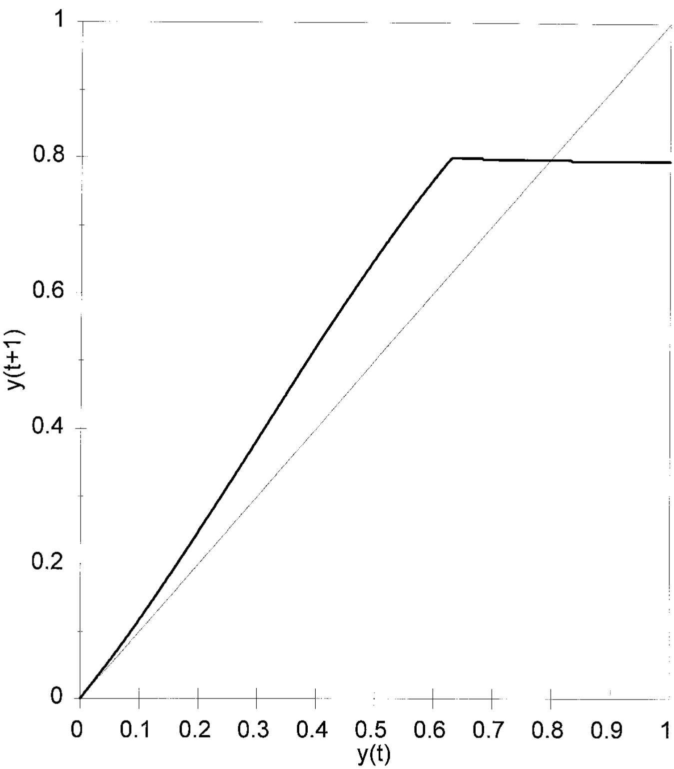

FIG. 1. Optimal stock transition, low discount rate, $\rho = 0.1$.

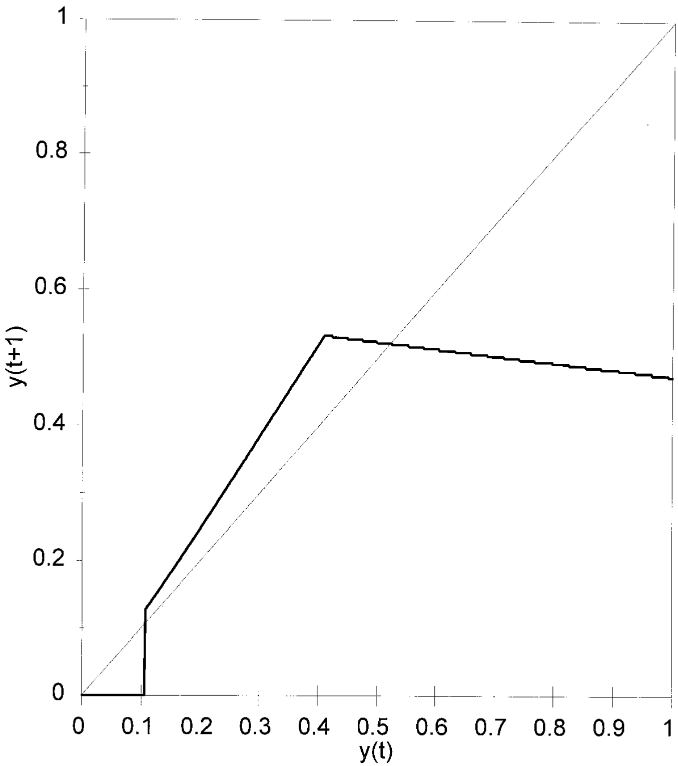

FIG. 2. Optimal stock transition, intermediate discount rate, $\rho = 0.45$.

$\beta = 1.0$.[5] With low discounting, an optimal program from low stocks involves a moratorium on harvests until stocks are sufficiently large. Beyond this point the optimal program converges along strongly damped cycles to a positive optimal steady state. Extinction never occurs from any initial stock. With intermediate discounting the optimal policy from low stocks is to immediately harvest the resource to extinction. There is a critical stock or safe standard of conservation such that extinction is efficient for small stocks, but conservation is efficient from larger stocks. Figure 3 illustrates one important feature of the problem that is specific to a model that allows for both stock-dependent return and nonconcave production. Extinction is efficient from both low and high stocks, while conservation is efficient from intermediate stocks. There is no critical stock such that from larger stocks conservation is always guaranteed. In fact, the set of stocks from which conservation is efficient may become arbitrarily small as the

[5] The solutions are obtained via numerical methods using the method of successive approximations.

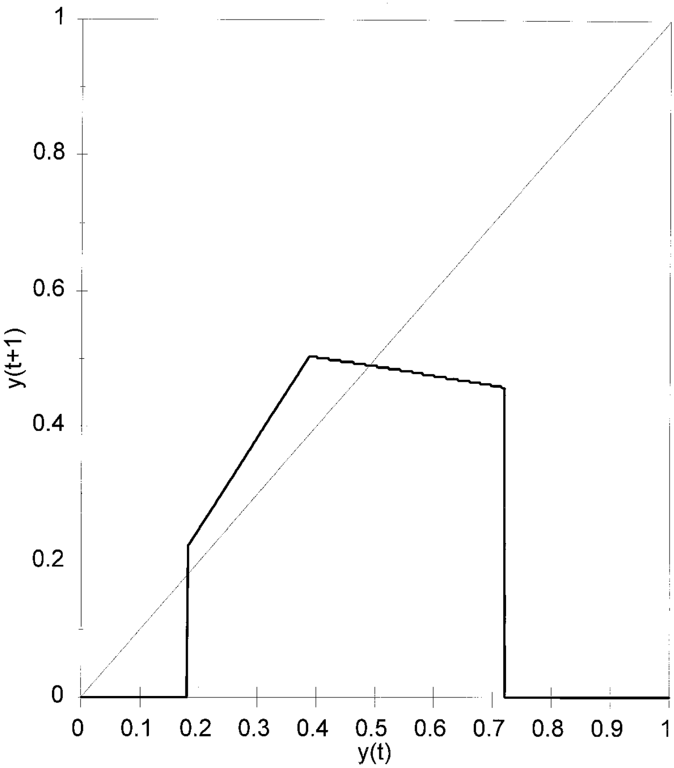

FIG. 3. Optimal stock transition, higher discount rate, $\rho = 0.47$.

discount rate increases. The behavior in this example stems from two basic facts. The first is that with nonconcave production there can be extinction from low stocks. The second is that when the return function is submodular an increase in the current stock lowers the marginal value of investing in future stocks. Thus, the optimal response to a larger stock is to invest less (or a corner solution). When the rate of time preference increases, the incentives to lower investment become even smaller. As the example shows, incentives for investment from large stocks may become so low that the future stock is in the region of extinction from small stocks, in which case it is optimal to harvest the entire stock immediately.

In this paper we focus on the existence of a safe standard of conservation, global conservation, and the possibility of extinction.

DEFINITION. A *safe standard of conservation* exists if there is a $\beta > 0$ such that if y_t is any optimal program from $y_0 \geqslant \beta$, then $\inf\{y_t\} > 0$.

The term safe standard of conservation is adapted from Clark [5]. It refers to a stock, β, such that if the initial stock exceeds β, then conservation of the resource is efficient, while survival of the resource cannot be guaranteed if the stock is less than β. If the resource stock falls below the safe standard of conservation then the economic survival of the resource could be promoted by modifying the structure of the resource management problem through policies directed toward the immediate reward function (e.g., through taxation) or the biological production function (through artificial enhancement), or by a restriction or ban on harvests if the stock falls too low.

Global conservation occurs if the most consumptive optimal policy from any strictly positive stock fails to drive the stock to extinction.

DEFINITION. *Global conservation* is efficient if $\inf\{y_t\} > 0$ for every optimal program $\{y_t\}$ from any $y_0 > 0$.

If there is no safe standard of conservation then it must be the case that for any $Y > 0$, there exists a $y > Y$ such that extinction is efficient from y. This does not mean that conservation is never efficient. As seen in Fig. 3, there may be alternating intervals of conservation and extinction. It may be economically efficient to harvest the resource to extinction from both low stocks and high stocks, with conservation being efficient only from some intermediate stock levels. On the other hand, if conservation is inefficient from all stocks then one has global extinction.

DEFINITION. *Global extinction* is efficient if $\limsup_{t \to \infty} y_t = 0$ along every optimal program.

In this case the economic considerations of the agent exploiting the resource do not justify preservation. Conservation must be defended on other grounds, or policy instruments described above could be used to promote survival of the resource.

4. EXISTENCE OF A SAFE STANDARD OF CONSERVATION

In this section, we establish conditions that ensure the existence of a safe standard of conservation. With stock-dependence, the conditions for a safe standard of conservation ought to involve the return function in an essential way. This is evident from the fact that a strictly positive steady state satisfies

$$\delta f'(x)[1 + (R_y(c, y)/R_c(c, y))] = 1. \tag{4.1}$$

Equation (4.1) is the familiar rule stating that the discounted marginal sustainable rent from an increment to the stock normalized by the marginal value of consumption equals unity. In this expression there is both a productivity effect and a welfare effect from investment in the resource, with the size of the welfare effect determined by the marginal rate of substitution between c and y. This welfare effect will be important in the results that follow.

4.i. *Existence of a Safe Standard of Conservation with Complementarity Between Current and Future Stocks*

In this section we examine the existence of a safe standard of conservation when the immediate return function exhibits complementarity between current stocks and investment in future stocks. Under (R.5), Lemmas 2.4–2.7 imply that a safe standard of conservation exists if and only if there is a stock $\underline{y} > 0$ such that $\{\underline{y}_t\}$ is an optimal stationary program from $\underline{y}$. In models where the return function is stock-independent it is well known that such a program exists if the production function is δ-productive, i.e., if there is some stock $x > 0$ for which $\delta f(x) > x$ (where a weak inequality suffices if R is strictly concave, see [9, 16, 20]). In the stock-independent case the δ-productivity condition arises naturally from the decentralized version of the resource allocation problem, where its primary role is to satisfy Slater's constraint qualification.[6] In problems where the immediate return function is important decentralized methods involve undesirable restrictions since neither Slater's condition nor most of the classical constraint qualifications allow a role for the objective function. For example, in our framework it is easy to construct examples where there are nontrivial optimal steady states but δ-productivity is violated. Thus, we take a direct approach that allows the return function to play an essential role in proving the existence of an optimal stationary program.

Define the investment that maximizes average product by $x^* = \text{argmax}\{f(x)/x \mid 0 \leqslant x \leqslant K\}$, if $\gamma > 0$ and $x^* = 0$ if $\gamma = 0$.

THEOREM 4.1. *Assume* (R.5). (a) *Suppose* $R_c(f(x) - x, f(x)) > 0$ *for all* $x \in (x^*, K)$ *and that there exists some* $0 < x' \in (x^*, K)$ *such that*

$$\delta f'(x')[1 + (R_y(f(x') - x', f(x'))/R_c(f(x') - x', f(x')))] \geqslant 1.$$

[6] For this reason methods based on δ-productivity are predominant even in the literature on the existence of optimal steady states in multi-sector growth models (e.g., [18]). An exception is Khan and Mitra [11] who note that there is a direct payoff to a primal approach that dispenses with the need for Slater's constraint qualification. Instead, they require that the technology be δ-normal, but this reduces to δ-productivity in our framework.

Then there exists a strictly positive optimal steady state and $y' \geqslant f(x')$ *and* y'' *is a safe standard of conservation.* (b) *Suppose there exists some* $x \in (0, K)$ *such that* $R_c(f(x)-x, f(x)) \leqslant 0$. *Then* $y = f(x)$ *is a safe standard of conservation.*

Proof of Theorem 4.1. (a) If $\gamma > 0$, then $x^* > \gamma$, $f''(x^*) < 0$ and $f'(x^*) = [f(x^*)/x^*]$. Define a modified production function F by

$$F(x) = \begin{cases} [f(x^*)/x^*]\, x, & 0 \leqslant x < x^*, \\ f(x), & x \geqslant x^*. \end{cases}$$

F is concave and satisfies (T.1)–(T.4). Consider the (modified) dynamic optimization problem where F replaces f and everything else stays the same. This is a convex problem. Let W be the value function for this modified problem and let $\underline{H}(y) = \min\{x\colon x \text{ is optimal from } y \text{ in the modified problem}\}$. $\underline{H}(y)$ is nondecreasing under (R.5).

By definition $F(x) \geqslant f(x)$ for all $x \geqslant 0$, so that a feasible program in the original problem is also feasible in the modified problem (from any y_0). Let $\{y_t, x_t, c_t\}$ be the optimal program in the modified problem generated by investing $\underline{H}(y)$ in each period. If this program is such that $y_t \geqslant f(x^*)$ for all $t \geqslant 0$, then it is feasible in the original problem and it must be equivalent to the optimal program generated by investing $\underline{X}(y)$ in every period. To see this note that if there is some optimal program in the original problem where $x_t < \underline{H}(y_t)$ then this program must have the same discounted sum of returns as the program generated by $\underline{H}(\cdot)$ and so must be optimal in the modified problem; however, this violates the definition of $\underline{H}(\cdot)$.

From $R_c(f(x)-x, f(x)) > 0$, $R_y \geqslant 0$, and $F(K-\varepsilon) > K-\varepsilon$ it follows that $R(F(K-\varepsilon) - K - \varepsilon, F(K-\varepsilon)) > R(K-\varepsilon-[K-\varepsilon], K-\varepsilon) = R(0, K)$. Under R.3 and R.4, $R_c(0, K) > 0$. Letting $\varepsilon \to 0$ in the first inequality, multiplying by $\delta/(1-\delta)$, and combining with the second inequality yields $R_c(0, K) > [R_c(0, K) + R_y(0, K)]\, \delta F'(K)$. Then, since $\delta F'(x')[1 + (R_y(F(x')-x', F(x'))/R_c(F(x')-x', F(x')))] \geqslant 1$, the intermediate value theorem implies there exists $x'' \in (x', K)$ such that $R_c(F(x'')-x'', F(x'')) = \delta[R_c(F(x'')-x'', F(x'')) + R_y(F(x'')-x'', F(x''))]\, F'(x'')$ and $F(x'') = f(x'')$ is an optimal steady state in both the modified and the original problem. As $x'' \geqslant x' > x^*$, F is strictly concave in a neighborhood of x''. Using the fact that $W'(F(x'')) > 0$ (as $R_c(F(x'')-x'', F(x'')) > 0$) and the optimality equation for the modified problem, one can show that $\underline{H}(F(x'')) = x''$. By the monotonicity of $\underline{H}(\cdot)$, optimal stocks from $F(x'')$ are all bounded below by $F(x'')$. This is then a lower bound on all optimal programs from $y_0 > F(x'')$ in the original problem.

(b) Suppose there exists some $x \in (0, K)$ such that $R_c(f(x)-x, f(x)) < 0$. It is clear that starting from $f(x)$ the agent will never consume more than $f(x)-x$, so that next period's stock is at least as large as $f(x)$. The proof follows from the monotonicity of $\underline{X}(y)$. ∎

Note that this theorem does not require the interiority of optimal policies. Further, if the immediate return function is independent of the stock level, then Theorem 4.1.a reduces to the familiar "δ-productivity" result in the existing literature.[7]

4.ii. *Existence of a Safe Standard of Conservation: The General Case*

We now consider the possibilities for conservation of the resource when (R.5) is relaxed and the return function $\phi(x, y)$ is not necessarily supermodular in x and y. As noted earlier, this allows for the possibility of optimal stock and investment programs that are nonmonotonic and nonconvergent over time. The loss of monotonicity renders the task of establishing results on a safe standard of conservation more difficult. As illustrated in Section 3, the fact that extinction does not occur from some stock level does not necessarily imply the existence of a safe standard of conservation. One must show specifically that extinction does not occur from any higher stock. Not surprisingly, therefore, stronger conditions are required to obtain conservation as an efficient outcome.

The following notation is useful. For any (x, y) such that $0<x<y$ define

$$s(y, x)=\inf_c\{[R_y(c, y)/R_c(c, y)]: y-x\leqslant c<\xi(y)\},$$

where $\xi(y)$ is defined in (R.4). $s(y, x)$ represents the smallest welfare effect as consumption varies between $y-x$ and the static utility maximizing amount. As in the previous section, x^* is given by $\operatorname{argmax}\{f(x)/x: x\geqslant 0\}$ if $\gamma>0$ and $x^*=0$ if $\gamma=0$.

THEOREM 4.2. *Assume optimal programs are interior and suppose there exists some* $0<x'\in(x^*, K)$ *such that for all* $y\geqslant y'=f(x')$, *either* (a) $R_c(y-x', y)>0$ *and* $\delta f'(x')(1+s(y, x'))\geqslant 1$ *or* (b) $R_c(y-x', y)\leqslant 0$; *then* y' *is a safe standard of conservation.*

Proof of Theorem 4.2. Define the function $F(x)$ as in the proof of Theorem 4.1 and consider the (modified) dynamic optimization problem where F replaces f and everything else stays the same. Let W be the (concave and differentiable) value function and $\underline{H}(y)$ be the lower bound on optimal investment in the modified problem. Let $\{c_t, x_t, y_t\}$ be a

[7] If f is S-shaped (i.e., $\gamma>0$, $x^*>0$) δ-productivity is equivalent to the requirement that $\delta f'(x^*)>1$; if f is strictly concave (i.e., $\gamma=x^*=0$) it is equivalent to the requirement that $\delta f'(0)>1$.

(modified) optimal program from y_0. Under the assumption that such a program is interior $W'(y_0) = R_c(c_0, y_0) + R_y(c_0, y_0)$ and by Lemma 2.1.c

$$R_c(c_t, y_t) = \delta F'(x_t)[R_c(c_{t+1}, y_{t+1}) + R_y(c_{t+1}, y_{t+1})]. \tag{4.2}$$

Choose any $y_0 \geqslant f(x') = F(x')$. We will show that $x_0 = \underline{H}(y_0) \geqslant x'$. If $R_c(y_0 - x', y_0) \leqslant 0$ it cannot be optimal to invest less than x', even for a myopic agent. If $R_c(y_0 - x', y_0) > 0$ and $\delta f'(x')(1 + s(y_0, x')) \geqslant 1$, then $\delta F'(x')(1 + s(y_0, x')) \geqslant 1$. Suppose $x_0 < x'$. Then $F'(x_0) > F'(x')$ and $y_0 - x' < y_0 - x_0$ with the latter implying $R_c(y_0 - x', y_0) > 0$. Let $y_1 = F(x_0)$. Using (4.2) one obtains

$$\begin{aligned} W'(y_0) &= R_c(c_0, y_0) + R_y(c_0, y_0) \\ &= [\{R_c(c_0, y_0) + R_y(c_0, y_0)\}/R_c(c_0, y_0)] \\ &\quad \times [\delta F'(x_0)][R_c(c_1, y_1) + R_y(c_1, y_1)] \\ &> [1 + \{R_y(c_0, y_0)/R_c(c_0, y_0)\}]\, \delta F'(x')\, W'(y_1) \\ &\geqslant [1 + s(y_0, x')]\, \delta F'(x')\, W'(y_1). \end{aligned} \tag{4.3}$$

Since $[1 + s(y_0, x')]\, \delta F'(x') \leqslant 1$, it follows that $W'(y_0) > W'(y_1)$. This implies $y_1 \geqslant y_0$ so that $F(x_0) \geqslant y_0 \geqslant F(x')$ which contradicts $x_0 < x'$. By induction, if $y_0 \geqslant f(x') = F(x')$, the optimal stocks y_t generated by $\underline{H}$ are bounded below by $y' = F(x')$ for all t. From here, identical arguments to those used in the Proof of Theorem 4.1 establish that the optimal program generated by $\underline{X}(\cdot)$ in the original problem is bounded below by y' whenever $y_0 \geqslant y'$. ∎

The term $\delta f'(x')(1 + s(y, x'))$ represents the minimum discounted marginal value of investment in the stock as consumption varies between $y - x'$ and $\xi(y)$, normalized by the marginal return from consumption. If this lower bound on the discounted marginal value of investment is greater than 1 for all $y \geqslant f(x')$, then $f(x')$ is a safe standard of conservation. When (R.5) is relaxed the loss of monotonicity requires placing a lower bound on the discounted marginal value of investment over a range of possible consumption. In comparison, when the return function is supermodular the discounted marginal value of investment need only be greater than 1 from *some* stock level.

Even in the more general case, our sufficient conditions for the existence of a safe standard always hold if f is δ-productive. Further, if $R_y = 0$, then $s(y, x^*) = 0$ and condition (a) of Theorem 4.2 reduces to the δ-productivity requirement for a safe standard of conservation with a stock-independent return. Theorem 4.2.b arises purely from the fact that the return function may be nonmonotonic. If (b) holds not even a myopic agent will consume beyond the amount needed to regenerate the current stock.

The fundamental conclusion is that, regardless of the stock dynamics, the conditions under which a safe standard of conservation is efficient are weaker when the immediate return function depends on the stock as opposed to the stock-independent case. In particular, a safe standard of conservation may be optimal even when the natural growth rate of the resource is always less than the discount rate. The strength of the stock effect on immediate return vis-à-vis that of immediate consumption is crucial in determining the efficiency of conservation.

5. Global Conservation

In this section, we discuss conditions under which conservation of the resource is optimal from *all* positive initial stocks. When the return function is stock-independent, global conservation is efficient if and only if the production function is δ-productive at zero, that is, if and only if there exists some $\varepsilon > 0$ such that $\delta f(x) \geqslant x$ for all $x \in (0, \varepsilon)$ (e.g., [9]). This is equivalent to $\delta f'(0) \geqslant 1$ if f is S-shaped (i.e., $\gamma > 0$) and $\delta f'(0) > 1$ if f is strictly concave (i.e., $\gamma = 0$). In Section 4 we have seen that if one allows for stock-dependence then the welfare effect from investment in the stock may lead to conservation even if the technology is not δ-productive anywhere. By analogy, it is natural to conjecture that the efficiency of global conservation should depend on both the intrinsic productivity of the resource and the intrinsic welfare effect of investment, where intrinsic refers to behavior in a neighborhood of zero.

5.i. *Global Conservation with Complementarity between Current and Future Stocks*

We first examine the economic efficiency of global conservation under (R.5). Intuition suggests that complementarity between the current stock and investment in future stocks ought to enhance the efficiency of conservation and reduce the possibility of extinction as it places a higher marginal value on large stock sizes vis-à-vis current consumption, compared to the case where the immediate return is independent of the stock.

Let $m(y) = \inf\{R_y(c, y) \colon y - f^{-1}(y) < c \leqslant y\}$ and define

$$\alpha = \liminf_{x \downarrow 0} \delta f'(x)[1 + \{m(f(x))/R_c(f(x) - x, f(x))\}].$$

Theorem 5.1. *Assume* (R.5). (a) *Suppose there exists some* $\varepsilon > 0$ *such that* $R_c(f(x) - x, f(x)) > 0$ *for all* $x \in (0, \varepsilon)$. *If* $\alpha > 1$ *then every optimal program exhibits global conservation.* (b) *Suppose there exists some* $\varepsilon > 0$ *such that* $R_c(f(x) - x, f(x)) \leqslant 0$ *for all* $x \in (0, \varepsilon)$. *Then every optimal program exhibits global conservation.*

Proof of Theorem 5.1. (a) There are 2 cases to consider, $\{\underline{y}_t\}\downarrow 0$ asymptotically, or there exists some t such that $\underline{x}_t = 0$. First, suppose $\underline{x}_t = 0$ and without loss of generality let t be the first such period. Then $\underline{c}_t = \underline{y}_t > 0$. By the principle of optimality, $0 \leqslant R(\underline{y}_t, \underline{y}_t) + (\delta/(1-\delta))\, R(0,0) - [R(\underline{y}_t - \varepsilon, \underline{y}_t) + (\delta/(1-\delta))\, R(f(\varepsilon)-\varepsilon, f(\varepsilon))] \leqslant R_c(\underline{y}_t - \varepsilon, \underline{y}_t)\,\varepsilon - (\delta/(1-\delta))\,[R_c(f(\varepsilon)-\varepsilon, f(\varepsilon))[f(\varepsilon)-\varepsilon] + R_y(f(\varepsilon)-\varepsilon, f(\varepsilon))\, f(\varepsilon)]$. This implies that for all $y \leqslant \underline{y}_t$, $R_c(y-\varepsilon, y) \geqslant R_c(\underline{y}_t - \varepsilon, \underline{y}_t) \geqslant (\delta/(1-\delta))[R_c(f(\varepsilon)-\varepsilon, f(\varepsilon))\,(f(\varepsilon)-\varepsilon) + R_y(f(\varepsilon)-\varepsilon, f(\varepsilon))\, f(\varepsilon)]/\varepsilon$, where the first inequality follows from (R.5). Choose small enough y and ε such that $y = f(\varepsilon)$. After some manipulation, this implies $1 \geqslant \delta[1 + \{R_y(f(\varepsilon)-\varepsilon, f(\varepsilon))/R_c(f(\varepsilon)-\varepsilon, f(\varepsilon))\}]\, f(\varepsilon)/\varepsilon \geqslant \delta[1 + \{m(f(\varepsilon))/R_c(f(\varepsilon)-\varepsilon, f(\varepsilon))\}]\, f(\varepsilon)/\varepsilon$. Taking $\liminf_{\varepsilon \to 0}$ then produces a contradiction.

Next, suppose $\{\underline{y}_t\}\downarrow 0$ asymptotically. There are two subcases to consider. In the first, $\limsup_{x\downarrow 0} R_c(f(x)-x, f(x)) < +\infty$. Under (R.5), $\{\underline{y}_t\}\downarrow 0$ implies $\underline{x}_t > \underline{x}_{t+1}$ and $\underline{c}_{t+1} > f(\underline{x}_t) - \underline{x}_t$ for all t. Concavity of R then yields

$$0 \leqslant R_c(\underline{c}_{t+1}, \underline{y}_{t+1}) = R_c(f(\underline{x}_t) - \underline{x}_{t+1}, f(\underline{x}_t)) \leqslant R_c(f(\underline{x}_t) - \underline{x}_t, f(\underline{x}_t)). \quad (5.1)$$

Hence, $\limsup_{x\downarrow 0} R_c(f(x)-x, f(x)) < +\infty$ implies $\lim_{t\uparrow +\infty} R_c(\underline{c}_{t+1}, \underline{y}_{t+1}) < +\infty$. From this $[R_c(\underline{c}_{t+1}, \underline{y}_{t+1})/R_c(\underline{c}_t, \underline{y}_t)] \to 1$ as $t \uparrow +\infty$. Since $\underline{c}_t > 0$ for all t the Ramsey–Euler equation holds. One can rewrite it as

$$\begin{aligned} &R_c(\underline{c}_t, \underline{y}_t)/R_c(\underline{c}_{t+1}, \underline{y}_{t+1}) \\ &\quad = \delta f'(\underline{x}_t)[1 + \{R_y(\underline{c}_{t+1}, \underline{y}_{t+1})/R_c(\underline{c}_{t+1}, \underline{y}_{t+1})\}] \\ &\quad \geqslant \delta f'(\underline{x}_t)[1 + \{m(\underline{y}_{t+1})/R_c(f(\underline{x}_t) - \underline{x}_t, f(\underline{x}_t))\}] \qquad \text{(using (5.1))} \\ &\quad = \delta f'(\underline{x}_t)[1 + \{m(f(\underline{x}_t))/R_c(f(\underline{x}_t) - \underline{x}_t, f(\underline{x}_t))\}]. \qquad (5.2) \end{aligned}$$

Taking the liminf as $t \uparrow +\infty$ on both sides of (5.2), noting that $\underline{x}_t \downarrow 0$ and $R_c(\underline{c}_t, \underline{y}_t)/R_c(\underline{c}_{t+1}, \underline{y}_{t+1}) \to 1$, we obtain $\alpha \leqslant 1$, which is a contradiction.

In the second subcase $\limsup_{x\downarrow 0} R_c(f(x)-x, f(x)) = +\infty$ and $\alpha = \delta f'(0)$ since R_y is bounded above. Under T.3 there exists $\tilde{x} > 0$ such that $\delta f'(x) > 1$ for all $x \in (0, \tilde{x})$. Choose y_0 close enough to 0 such that $\hat{x} = f^{-1}(y_0) < \tilde{x}$. Since $\underline{y}_t$ is strictly decreasing in t, it follows that $\underline{x}_t < \tilde{x}$ for all t. Further, $\delta f'(x) > 1$ on $(0, \tilde{x})$, so $f(x) - (x/\delta)$ is strictly increasing on $(0, \tilde{x})$. Therefore, $[f(\underline{x}_t - (\underline{x}_t/\delta)] < [f(\hat{x}) - (\hat{x}/\delta)]$ for all t. Let $C = \sum_{t=0}^{\infty} (1-\delta)\, \delta^t \underline{c}_t$. Using arguments analogous to [9, Lemma 1] it can be shown that $f(\hat{x}) - \hat{x} \leqslant C$;[8] however, following the arguments of [9, Proof of Lemma 2] it must be that $C < [f(\hat{x}) - \hat{x}]$ and this yields a contradiction.

[8] This relies on the fact that $\underline{y}_1 = f(\underline{x}_0) < y_0$ so that $\underline{c}_0 > [f(\hat{x}) - \hat{x}]$. Hence, $R(\cdot, f(\hat{x}))$ is strictly increasing on $(0, [f(\hat{x}) - \hat{x}])$ so that for all $c < [f(\hat{x}) - \hat{x}]$, $R(f(\hat{x}) - \hat{x}, f(\hat{x})) > R(c, f(\hat{x}))$.

(b) If $R_c(f(x)-x, f(x)) \leqslant 0$ then the agent always invests at least $f^{-1}(y)$ from an initial stock y. The result then follows from the monotonicity of $\underline{y}_t$. ∎

In this theorem there are two ways for $\alpha > 1$ to reduce to $\delta f'(0) > 1$, or δ-productivity at 0. The first is the traditional case where the return function is independent of the stock. The second is when the stock effect valued in terms of the marginal return from consumption becomes negligible as consumption goes to zero. In either case it is an insignificant intrinsic welfare effect that results in δ-productivity as a condition for conservation. This provides a natural economic interpretation to δ-productivity that does not arise in the usual decentralized approach. Note that if $R_c(f(x)-x, f(x))$ stays bounded above as x converges to zero, then global conservation can be optimal even if $\delta f'(0) < 1$, that is, even if the intrinsic growth rate of the resource is less than the discount rate.

It is worth pointing out that global conservation in the stock-independent case is typically established for interior optimal programs (an exception is Mitra and Ray [20] who assume a unique nontrivial steady state). In characterizing possibilities for conservation and extinction it is especially important to consider noninterior actions as they may be more likely to lead to extinction. This is one strength of Theorem 5.1.

5.ii. *Global Conservation: The General Case*

We now dispense with assumption (R.5) and analyze the efficiency of global conservation when the return function is not necessarily supermodular. As we have seen, this allows the possibility of complex nonlinear dynamics. Let $s(y, x)$ be defined as in Section 4.ii. Recall that $\delta f'(x)(1 + s(y, x))$ represents the minimum discounted marginal value of investment in the stock as consumption varies between $y - x$ and $\xi(y)$, normalized by the marginal return from consumption.

In the case where f is concave a lower bound on the intrinsic discounted marginal value of investment leads to the following result on the efficiency of global conservation.

THEOREM 5.2. *Assume optimal programs are interior and that $\gamma = 0$, that is, f is strictly concave.*[9] *If either* (a) *there exists some $\varepsilon > 0$ such that $R_c(f(x)-x, f(x)) > 0$ for all $x \in (0, \varepsilon)$ and* $\liminf_{x \downarrow 0} \delta f'(0)(1 + s(f(x), x)) > 1$, *or* (b) *there exists some $\varepsilon > 0$ such that $R_c(f(x)-x, f(x)) \leqslant 0$ for all $x \in (0, \varepsilon)$, then every optimal program exhibits global conservation.*

[9] The proof of this theorem only requires that f is concave. However assumption (T.5) implies that in our framework, f can be concave only if $\gamma = 0$, in which case it must be strictly concave.

Proof of Theorem 5.2. For interior optimal programs and f strictly concave, the Ramsey–Euler equation holds, and the value function is concave and differentiable. Suppose that $\liminf\{y_t\}=0$. Then there exists a subsequence of time periods such that $x_{\tau+1}<x_\tau$ and $f(x_\tau)-x_\tau<c_{\tau+1}\leqslant \xi(f(x_\tau))$. First consider case (a). Following the logic of (4.3) one obtains $V'(f(x_\tau))=R_c(c_{\tau+1}, f(x_\tau))+R_y(c_{\tau+1}, f(x_\tau))\geqslant[1+s(f(x_\tau), x_\tau)]\,\delta f'(x_\tau)\,V'(f(x_{\tau+1}))$. Since V is concave and $x_\tau>x_{\tau+1}$ this implies $1\geqslant[1+s(f(x_\tau), x_\tau)]\,\delta f'(x_\tau)$. Letting $\tau\to\infty$ then produces a contradiction to $\text{limin}_{x\downarrow 0}\,\delta f'(0)(1+s(f(x), x))>1$. In case (b), one obtains a contradiction when $x_\tau<\varepsilon$, since $R_c(f(x_\tau)-x_\tau, f(x_\tau))\leqslant 0$ implies $c_{\tau+1}\leqslant f(x_\tau)-x_\tau$. ∎

The conditions for global conservation in Theorem 5.2 are weaker than the requirement that $\delta f'(0)>1$, or δ-productivity at 0. Hence, if the production function is concave, the return function can have an important influence on the efficiency of global conservation.

The next lemma is useful for a further examination of the global properties of conservation and extinction.

LEMMA 5.3. *For interior optimal programs if* $\{y_t\}$ *is an optimal stock sequence such that* $\liminf_{t\to\infty} y_t=0$, *then* $\limsup_{t\to\infty} y_t=0$.

In models without stock-dependence a necessary condition for global conservation is $\delta f'(0)\geqslant 1$. Theorem 5.2 shows that with stock-dependence global conservation can occur even if $\delta f'(0)<1$, provided f is concave. We have not been able to derive a similar result when f is nonconcave. The best we can ensure is the following.

THEOREM 5.4. *Assume optimal programs are interior,* $\gamma>0$, *and* $\delta f'(0)\geqslant 1$. *Then global conservation is efficient and extinction does not occur from any strictly positive initial stock.*

Proof of Theorem 5.4. Suppose that from some initial stock $y_0>0$ there is a subsequence of the optimal stock program that converges to zero. Let x^* be the input level where the average product $[f(x)/x]$ is maximized. Then, $x^*>\gamma$ and $f(x^*)/x^*=f'(x^*)$. Further, since $\delta f'(0)=\lim_{x\downarrow 0}\delta[f(x)/x]\geqslant 1$, we have $\delta f'(x^*)>1$. Let $x^{**}=\text{argmax}\{f(x)-x/\delta\colon x\geqslant 0\}$. Then, $\delta f'(x^{**})=1$ and $x^{**}>x^*>\gamma$. Since $\liminf_{t\to\infty} y_t=0$, Lemma 5.3 implies that the entire optimal stock sequence $\{y_t\}\to 0$. Hence, there exists an initial stock y_0 such that (a) $y_t<y_0$ for all t, (b) $\delta f'(y_t)>1$ for all t, and (c) $y_0<f(x^*)$. Together (a) and (c) imply that $x_t\leqslant\hat{x}<x^*<x^{**}$, for all $t\geqslant 0$, where $\hat{x}=f^{-1}(y_0)$. Since $[f(x)-(x/\delta)]$ is strictly increasing on $(0, x^{**})$, we have that $[f(x_t)-(x_t/\delta)]<[f(\hat{x})-(\hat{x}/\delta)]$ for all t. The remainder of the proof follows identical arguments to those used in the last stage of the Proof of Theorem 5.1.a. ∎

To conclude, in Section 5 we have derived sufficient conditions for the efficiency of global conservation in a general setting. When the return function is supermodular we have shown that global conservation is efficient under weaker conditions than in models with stock-independent return. Even if the intrinsic growth rate of the resource is less than the discount rate, it may be optimal to conserve the species from all positive stock levels, provided the intrinsic welfare effect is strong enough. However, if the welfare effect becomes small as the stock converges to zero (either because the marginal utility of current consumption becomes unbounded relative to the marginal stock effect or because return function is independent of the stock), then the conditions for global conservation reduce to $\delta f'(0) > 1$. When the return function is not necessarily supermodular the welfare effect is still important provided f is strictly concave. Further, even in the general case our results encompass the δ-productivity at zero requirement used to ensure the efficiency of global conservation in stock-independent models.

6. The Possibility of Extinction

In this section, we analyze situations under which it may be optimal to consume the resource to extinction. If the immediate return is independent of the stock then it is known that $\delta f'(0) < 1$ implies extinction is optimal from stocks close to zero. Further, global extinction is optimal if the technology is nowhere δ-productive (see [9]). In the previous two sections we have seen that the introduction of a stock-dependent return function increases the class of environments and discount factors for which conservation is efficient. It follows that stronger conditions should be required to obtain extinction as an efficient outcome when returns are stock-dependent.

Establishing the optimality of extinction under a reasonable set of general conditions turns out to be quite difficult when the optimal policy can exhibit nonmonotonic behavior. As our purpose is to show how the conditions for extinction are modified by stock-dependent returns, we restrict our analysis to the supermodular case where optimal stock programs are monotonic over time. In this case, global extinction can only be an efficient outcome if there does not exist a strictly positive steady state.

Theorem 6.1. *Assume optimal programs are interior. If* $\delta f'(x)[R_c(f(x)-x, f(x)) + R_y(f(x)-x, f(x))] < R_c(f(x)-x, f(x))$ *holds for all* $x \in (0, K]$ *then there does not exist any safe standard of conservation and every optimal program exhibits global extinction.*

The conditions of Theorem 6.1 directly rule out the existence of a positive stock that satisfies the definition of a steady state given in Section 2. One may compare Theorem 6.1 to the corresponding result in models where the immediate return is independent of the stock. There extinction occurs from all stocks if $\delta f'(x) < 1$ for all $x \in (0, K)$. The condition imposed in Theorem 6.1 is a stronger requirement. This is in accordance with our general claim that stock-dependence of the return function (for the supermodular case) makes the sufficient condition for conservation weaker and that for global extinction stronger.

There is also a larger class of situations where extinction is an efficient outcome only if the stock is sufficiently small. The next theorem characterizes some of these situations. Recall the definition of α from Section 5.i where it was shown that global conservation is optimal (that is extinction is never optimal from any strictly positive stock) if $\alpha > 1$. Our condition for extinction from small stocks is close to a negation of this condition.

THEOREM 6.2. *Assume optimal programs are interior and* (R.6). *If* $\lim_{x \downarrow 0} R_c(f(x) - x, f(x)) = +\infty$ *and* $\alpha < 1$, *then there exists some* $z > 0$ *such that extinction occurs from all stocks* $y_0 \in (0, z)$.

Proof of Theorem 6.2. Let $M(y)$ be defined by $M(y) = \sup\{R_y(c, y): 0 < c \leqslant y\}$. Since $M(y)$ is bounded and $\lim_{x \downarrow 0} R_c(f(x) - x, f(x)) = +\infty$, it follows that

$$\limsup_{x \downarrow 0} \delta f'(x)[1 + \{M(f(x))/R_c(f(x) - x, f(x))\}] = \delta f'(0) = \alpha < 1. \qquad (6.1)$$

Hence, there exists some $z > 0$, such that for all $x \in (0, z)$

$$\delta f'(x)[1 + \{M(f(x))/R_c(f(x) - x, f(x))\}] < 1. \qquad (6.2)$$

Suppose the theorem is not true. Then, by the monotonicity of optimal programs it must be that extinction never occurs from any $y_0 > 0$ and optimal stocks from all positive initial stocks must converge to a strictly positive steady state. For $y_0 \in (0, z)$, optimal stocks must be strictly increasing so long as $y_t < z$ because from (6.1) no steady state lies in $(0, z)$. Let y_0 be any stock small enough such that if $\{y_t\}$ is the optimal program from y_0, then $y_0 < y_1 < z$. Consider any $y \in [y_0, y_1]$. We claim that $c^* = \inf\{C(y): y \in [y_0, y_1]\} > 0$. Suppose the contrary. Then, there exists a sequence $\{y^n\}$ such that $\{y^n - \bar{X}(y^n)\} \to 0$. Since $\{y^n\}$ and $\{\bar{X}(y_n)\}$ are bounded there is subsequence $\{n'\}$ of $\{n\}$ such that $\{y^{n'}\}$ and $\{\bar{X}(y^{n'})\}$ converge to, say, $y^+ \in [y_0, y_1]$. Taking limits in the optimality equation $V(y^{n'}) = R(y^{n'} - \bar{X}(y^{n'}), y^{n'}) + \delta V(f(\bar{X}(y^{n'})))$ as $n' \to +\infty$, we obtain $V(y^+) = R(0, y^+) + \delta V(f(y^+))$. This contradicts the fact that the right-hand side of the optimality equation has an interior solution. Thus, $c^* > 0$.

Define $G = \sup\{R_c(c^*, y): y_0 \leqslant y \leqslant y_1\}$. Then, $0 < G < +\infty$ and for all $y \in [y_0, y_1]$, $R_c(c, y) \leqslant R_c(c^*, y) \leqslant G$, where, $c \in C(y)$. Next we claim that there exists $y' < y_0$ such that

$$R_c(c', y') > G, \tag{6.3}$$

for all $c' \in C(y')$. To see this note that the optimal stock sequence from y' is increasing over time so that $c' \in C(y') \leqslant y' - f^{-1}(y')$ and $R_c(c', y') \geqslant R_c(y' - f^{-1}(y'), y') \to +\infty$ as $y' \downarrow 0$. Therefore, (6.3) must hold for some y' small enough.

Let $\{y'_t\}$ be an optimal program from y'. This programs converges to a positive steady state and therefore in finite time $y'_t \geqslant z \geqslant y_0$. Let $T > 0$ be the first period such that $y'_T \geqslant y_0$. Obviously, $y'_{T-1} < y_0$. Monotonicity then implies $y'_T \leqslant y_1$. Thus, $y'_T \in [y_0, y_1]$, and by the definitions of c^* and G,

$$R_c(c'_T, y'_T) \leqslant G. \tag{6.4}$$

For any $t < T$, the Ramsey–Euler equation gives

$$\begin{aligned}
&[R_c(c'_t, y'_t)/R_c(c'_{t+1}, y'_{t+1})] \\
&\quad = \delta f'(x'_t)[1 + \{R_y(c'_{t+1}, y'_{t+1})/R_c(c'_{t+1}, y'_{t+1})\}] \\
&\quad \leqslant \delta f'(x'_t)[1 + \{M(f(x'_t))/R_c(f(x'_t) - x'_t, f(x'_t))\}] < 1.
\end{aligned} \tag{6.5}$$

In (6.5) the last inequality follows from (6.2) and $x'_t \leqslant y_1 < z$, for $t \leqslant T$, and the first inequality uses the definition of $M(\cdot)$ and the fact that $R_c(c'_{t+1}, y'_{t+1}) = R_c(f(x'_t) - x'_{t+1}, f(x'_t)) \geqslant R_c(f(x'_t) - x'_t, f(x'_t))$ since $x'_{t+1} \geqslant x'_t$ and R is concave. By applying (6.5) recursively and then using (6.3), one obtains $R_c(c'_T, y'_T) > R_c(c'_{T-1}, y'_{T-1}) \cdots > R_c(c'_0, y'_0) \geqslant G$, which contradicts (6.4). ∎

This result says that if the intrinsic welfare effect is zero and if the intrinsic growth rate of the resource is sufficiently low, then from small stock levels it is efficient to harvest the resource to extinction. Here, the intrinsic welfare effect becomes insignificant because the marginal value of sustainable consumption grows arbitrarily large as the stock becomes smaller; i.e., $R_c(f(x) - x, f(x)) \to +\infty$ as $x \downarrow 0$. This is a strong assumption; however, even in a framework where return is independent of the stock, the proof of the possibility of efficient extinction uses the fact that the marginal utility of consumption is infinite at zero (e.g., [9]).

7. Conclusions

This paper has derived conditions under which a dynamically efficient management policy leads to conservation or extinction of a renewable resource. Three potential extensions are worth noting. First, our proof of Theorems 4.1 and 4.2 are based on the convex hull of the technology. This implies that we lose information about the nonconvex section. As a consequence it may be possible to strengthen these results. Second, we do not consider the efficiency of extinction when optimal resource stocks behave nonmonotonically over time. Third, the analysis is entirely in a deterministic setting. The existing literature on stochastic resource models with stock-dependent returns focuses on issues other than conservation and extinction, and assumptions are normally imposed that insure global conservation (e.g., [19, 23, 24]). Recently, however, there has been an interest in extinction and survival in stochastic models where utility depends only on consumption (e.g., [21]). Therefore, it would be interesting to examine the problem of conservation or extinction of optimally managed resources when there is uncertainty over resource productivity.

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