Inventory Competition with Yield Reliability Improvement

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Abstract: This article studies the inventory competition under yield uncertainty. Two firms with random yield compete for substitutable demand: If one firm suffers a stockout, which can be caused by yield failure, its unsatisfied customers may switch to its competitor. We first study the case in which two competing firms decide order quantities based on the exogenous reliability levels. The results from the traditional inventory competition are generalized to the case with yield uncertainty and we find that quantity and reliability can be complementary instruments in the competition. Furthermore, we allow the firms to endogenously improve their yield reliability before competing in quantity. We show that the reliability game is submodular under some assumptions. The results indicate that the competition in quantity can discourage the reliability improvement. With an extensive numerical study, we also demonstrate the robustness of our analytical results in more general settings.

Keywords: inventory competition; random yield; reliability improvement

1. INTRODUCTION

Inventory competition (also referred to as newsvendor game) is first studied by Parlar [19]. In the traditional problem, each firm has an initial stochastic demand and decides the procurement or production quantity. If one firm fails to satisfy all of its demand, some of the unsatisfied customers will switch to another firm. The firms compete with each other through order quantities.

Yield reliability plays a critical role in today's increasingly competitive market. In many industries such as the electronics and the auto industry, component makers or original equipment manufacturers usually have production processes that are subject to random yield. The yield uncertainty can be caused by unreliable equipment, deficient technology, and so forth. As a result, a manufacturer's actual output would suffer a random shortage, which may lead to the loss of its regular customers. The unsatisfied customers may then decide to buy substitutable products from other firms instead. For example, Hella and Valeo are two competing auto-part manufacturers who provide lighting equipments to automakers such as Nissan and Volkswagen [1]. Each component manufacturer normally supply its regular customers (i.e., automakers). Once the initial order cannot be satisfied (for example, due to quality issue), the customer may purchase substitutable items from an alternative manufacturer. The same thing happens to manufacturing firms that produce other components (e.g., stamping parts, wheels, and so forth.) to automakers. Hendricks and Singhal [13] point out that supply glitches can hinder the firms from capitalizing on the market demand. Emmelhainz et al. [8] report that a manufacturer can lose more than half of its customers to its competitors due to the stockout, whereas a retailer can lose up to 14% of its customers. Besides quantity, firms also compete in yield reliability. For example, auto-part manufacturers such as Hella and Valeo can improve their yield reliability by investing in technology innovation or adopting advanced equipments.

Motivated by the fact that manufacturing firms often suffer yield losses which can cause their regular customers to go elsewhere, we study the inventory competition between firms that are subject to yield uncertainty. We raise the following questions.

1. How will the results from the traditional inventory competition change when we take into account yield uncertainty? What is the effect of yield reliability on firms' order decisions and profits in the equilibrium?
2. As firms can also adopt advanced production equipment or processes to improve yield reliability, how do they decide what reliability improvements to make given a subsequent competition in inventory?

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To answer the questions in (1), we first generalize the traditional inventory competition (e.g., Parlar [19], Lippman and McCardle [14]) to the case in which two competing firms are subject to yield uncertainty. Each firm has an initial stochastic demand and decides the attempted production quantity based on the exogenous yield reliability levels. We find that the game is still submodular in the order quantities but there can be multiple equilibria due to the yield uncertainty. We show that under some conditions quantity and reliability serve as complementary instruments in the competition. A high-reliability level encourages the firm to go for a large production quantity and also prevents its competitor from doing so.

To study how the firms compete in reliability, we further allow them to endogenously improve their yield reliability before production. We hence have a two-stage game in which the firms determine the reliability levels at the first stage and then decide order quantities at the second stage. For analytical tractability, we first restrict our attention to the case where the firms' initial demand shares are deterministic and the yield outcomes follow a Bernoulli distribution. We show that the reliability game is submodular, and the competition in quantity may discourage a firm to pursue as high a reliability level as a monopoly would. In addition, by analyzing a special case in which improving reliability incurs a linear cost, we find that two firms with the same parameters may select different reliability levels in the equilibrium due to the subsequent quantity competition. Finally, our numerical results indicate that the basic insights derived analytically remain valid in more general settings with a variety of uncertain demand and other types of random yield.

The rest of this article is organized as follows. Section 2 reviews the related literature. Section 3 studies the scenario with exogenous yield reliability and Section 4 investigates the model with endogenous reliability. Finally, Section 5 summarizes the article.

2. LITERATURE REVIEW

The first line of research related to our work is on the inventory competition with demand substitution. Parlar [19] considers a duopoly model in which each firm chooses the inventory level of its product. The products of the two firms are substitutable in the sense that a deterministic fraction of one firm's excess demand will switch to the other firm. They show that the quantity game is submodular and there exists a unique equilibrium. Wang and Parlar [24] extend the model to a three-person game. Lippman and McCardle [14] also consider newsvendors who compete in their inventory levels but the excess demand is reallocated based on different rules. Netessine and Rudi [17] further study the inventory competition involving multiple firms. Zhao and Atkins [29] consider the case in which the firms simultaneously determine the price and inventory level and the initial demand share is sensitive to the prices quoted by all the firms. However, no one has yet considered yield uncertainty in the inventory competition, which has a critical effect for manufacturing firms.

Another line of related literature is on supply risk management. Many papers investigate the buyer's optimal procurement strategies when suppliers are unreliable (e.g., Anupindi and Akella [4], Tomlin [22], Dada et al. [6], Federgruen and Yang [11]). We refer interested readers to the review by Yano and Lee [28]. Some papers examine the effect of supply reliability improvement. Gupta and Cooper [12] study a single manufacturing firm and find that a stochastically larger yield rate is not necessarily beneficial. Liu et al. [15] study a retailer who can determine both inventory level and marketing effort. They analyze how the supply improvement affects marketing decisions. Recently, some authors consider the supply reliability improvement as an endogenous decision. Wang et al. [25] study a single firm that can source from multiple unreliable suppliers and/or make an effort to improve the supply reliability. Tang et al. [21] consider a decentralized supply chain where the buyer may provide subsidies for the supplier and the supplier then determines the reliability level. Nikoofal and Gümüş [18] study the incentive- and audit-based mechanisms for managing supplier's reliability improvement when the supplier has private information about its true reliability. Unlike these papers, we consider the yield reliability improvement in a competitive environment.

Some researchers have studied supply uncertainty in a competitive environment, but they focused on the different types of competition instead. Tang and Kouvelis [20] consider a Cournot-type competition between two end-market firms that source from two suppliers with random yield. They analyze the effect of different sourcing structures (dual/single sourcing) on the equilibrium. Deo and Corbett [7] also study a Cournot-type competition under yield uncertainty in analysing the influenza vaccine market. They investigate an entry game of multiple firms and analyze the effect of yield uncertainty on the number of firms entering the market. Federgruen and Yang [10] consider a model in which the suppliers with yield uncertainty compete by selecting the mean and standard deviation of the random yield factor. They use the attraction demand function and the market share is a function of the reliability level chosen by each firm. Wang et al. [26] study the case where two competing manufacturers source from an unreliable supplier and are able to improve supplier's yield process before making ordering decisions. In their model, the firms compete in the service level in the downstream market. The key difference between their paper and ours is that they consider the demand as a function of service levels provided by the competing firms, but in our article each firm has an initial demand and the unmet demand goes elsewhere after yield uncertainty is realized. In other words, our model does not require the customers to know anything about the firms' yield reliability levels ex ante.

3. THE MODEL WITH EXOGENOUS YIELD RELIABILITY

3.1. Formulation

We consider a classical setting of inventory competition but take into account yield uncertainty. Firm i ($i=1,2$) has an initial demand share D_i. Let q_i denote the stocking quantity of firm i. If firm i suffers a stock-out, that is, q_i turns out to be less than D_i, then a fixed fraction of the excess demand will switch to its competitor, firm j ($j \neq i$). Let D_i^s denote the effective demand of firm i and it can be expressed as

$$D_i^s = D_i + \gamma_{ji}(D_j - q_j)^+,$$

where γ_{ij} ($0 \leq \gamma_{ij} \leq 1$) is the given fraction of the unsatisfied customers of firm i switching to firm j and $[x]^+ = \max(0, x)$. γ_{ij} will hereafter be referred to as the switching rate. We will use $F_{D_i^s}$ and $f_{D_i^s}$ to denote the cumulative distribution function (cdf) and the probability density function (pdf) of D_i^s, respectively. For notational ease, $F_{D_i^s|\epsilon_j}$ and $f_{D_i^s|\epsilon_j}$ will, respectively, denote the conditional cdf and pdf of D_i^s on the realization of ϵ_j.

Each firm is subject to a random yield. If firm i attempts to produce a quantity Q_i, its stocking quantity will be the actual production output (or effective output), that is, $q_i = Q_i\epsilon_i$, where ϵ_i is the stochastic yield rate with support [0, 1]. This proportional yield is commonly used for modelling yield uncertainty in the literature. We assume that firms' yield processes are independent so that the ϵ_i are independent random variables. To quantify the yield reliability and facilitate our analysis, we borrow the modeling approach of Wang et al. [25]. Let $G_i(\cdot, a_i)$ and $g_i(\cdot, a_i)$ be the cdf and pdf of ϵ_i where a_i is a reliability index. If $a_i \leq \hat{a}_i$, then the random factor $\epsilon_i(a_i)$ is (first-order) stochastically smaller than $\epsilon_i(\hat{a}_i)$, that is, $G(x, a_i) \geq G(x, \hat{a}_i)$ for all x. That is, a larger index implies a higher reliability level.

If firm i places an initial quantity Q_i, it will incur a production cost $c_i Q_i$. The firm can obtain a unit refund $\delta_i c_i$ for defective items where $\delta_i \in [0, 1]$ is the refund rate. This formulation allows the defective items to incur a lower unit cost than the effective one. For example, the poor quality components may be sold or reused for remanufacturing. Also, this difference in unit cost arises from the cost structure. For instance, unlike the effective components, the defective components may or may not incur warehousing, packaging, and delivery costs. Consequently, the total production cost can be written as $c_i Q_i - \delta_i c_i Q_i(1 - \epsilon_i) = (1 - \delta_i)c_i Q_i(1 - \epsilon_i) + c_i Q_i\epsilon_i$, that is, firm i pays c_i for each effective unit and $(1 - \delta_i)c_i$ for each defective unit. If $\delta_i = 1$, the firm obtains a full refund so only the effective components incur a cost. If $\delta_i = 0$, the firm takes all the losses caused by random yield.

In addition, firm i's revenue for selling one unit of its product is p_i and the unit salvage value is s_i. As in the classical newsvendor problem, we assume $p_i > c_i > s_i$.

Given a fixed pair of reliability index, $\mathbf{a} = (a_1, a_2)$, the firms play a quantity game under yield uncertainty. The expected profit of firm i in the quantity game can be written as

$$\pi_i^q(Q_i|Q_j, \mathbf{a}) = E[p_i \min(D_i^s, Q_i\epsilon_i) + s_i(Q_i\epsilon_i - D_i^s)^+ - c_i Q_i + \delta_i c_i Q_i(1 - \epsilon_i)],$$

where we use the superscript "q" to denote the expected profit for the quantity game.

By determining the order quantity Q_i, firm i maximizes $\pi_i^q(Q_i|Q_j, \mathbf{a})$. Rearranging the terms, the problem for firm i can be written as

$$\begin{aligned}\max_{Q_i \geq 0} \quad & \pi_i^q(Q_i|Q_j, \mathbf{a}) \\ &= (p_i - \delta_i c_i)Q_i E[\epsilon_i] - (p_i - s_i)E[(Q_i\epsilon_i - D_i^s)^+] \\ &\quad - (1 - \delta_i)c_i Q_i. \end{aligned} \tag{1}$$

3.2. Characterization of the Quantity Game Under Yield Uncertainty

Note that if each firm has a completely reliable yield process, our problem boils down to the inventory competition studied by Parlar [19] and Lippman and McCardle [14]. However, yield uncertainty does affect both players' strategies and hence the equilibrium that is achieved. Due to yield uncertainty, the firm may not enter the market given a poor reliability level.

PROPOSITION 1: Firm i ($i = 1, 2$) will choose a positive order quantity if and only if

$$E[\epsilon_i] > \frac{(1 - \delta_i)c_i}{p_i - \delta_i c_i}. \tag{2}$$

Otherwise, it will choose $Q_i^* = 0$.

PROOF: All proofs are given in the appendix. □

Let $\psi_i = \frac{(1-\delta_i)c_i}{p_i-\delta_i c_i}$. Proposition 1 indicates that ψ_i is the lowest reliability level needed for firm i to enter the market. We will hence refer to ψ_i as firm i's entry reliability. First, firm i' entry reliability is independent of any parameter of its competitor as the firm has an initial demand D_i. Second, the entry reliability becomes higher as δ_i decreases. If $\delta_i = 1$, that is, the firm obtains a full refund for the defective units, then it is always willing to enter the market as long as $\epsilon_i > 0$ with a positive probability. Alternatively, one can rearrange the term and (2) becomes $p_i > \frac{(1-\delta_i)c_i}{E[\epsilon_i]} + \delta_i c_i$. The right-hand side is the expected effective cost for producing one good unit of product: For the fraction of cost with (respectively, without) refund, the firm pays c_i (respectively, $c_i/E[\epsilon_i]$) for each effective unit. The entry condition basically requires that the selling price is greater than the overall effective cost.

In traditional inventory competition, the quantity game is shown to be submodular [19]. One can show that this property still holds in the presence of yield uncertainty. The submodularity implies that a firm will reduce its order quantity if its competitor increases the order.

THEOREM 1: (i) The quantity game is submodular in (Q_1, Q_2). (ii) There exists a pure strategy Nash equilibrium in the quantity game. If (a_1, a_2) is such that the entry condition (2) is satisfied for both players, the equilibrium (Q_1^*, Q_2^*) solves the equations for $i, j = 1, 2$ and $i \neq j$:

$$\frac{(p_i - \delta_i c_i)E[\epsilon_i] - (1-\delta_i)c_i}{p_i - s_i} = \int_0^1\int_0^1 \epsilon_i F_{D_i^s|\epsilon_j}(Q_i^*\epsilon_i)dG_j(\epsilon_j, a_j)dG_i(\epsilon_i, a_i); \qquad (3)$$

(iii) If ϵ_i is a Bernoulli random variable with success probability a_i and $\gamma_{ij} < 1$ for $i, j = 1, 2$ and $i \neq j$, there exists a unique equilibrium in the quantity game.

However, although a unique equilibrium is always guaranteed in the case of a completely reliable yield as shown by Parlar [19], the yield uncertainty gives rise to multiple equilibria in general (See Example 1). If the random yield factor follows a Bernoulli distribution and the switching rate is strictly less than one, we can then show that there always exists a unique equilibrium.

For the submodular game with two players, we can consider it as a supermodular game by redefining the strategies as $(Q_1, -Q_2)$. In a supermodular game, there exist the component-wise largest and smallest equilibria. This can be translated in our problem: Among all equilibria (Q_1^*, Q_2^*), there are an equilibrium $(\bar{Q}_1^*, \underline{Q}_2^*)$ containing the largest value of component Q_1^* and the smallest value of component Q_2^*, and an equilibrium $(\underline{Q}_1^*, \bar{Q}_2^*)$ containing the smallest value of component Q_1^* and the largest value of component Q_2^*. In addition, if $\bar{Q}_i^* = \underline{Q}_i^*$ for all i, there exists a unique equilibrium.

Furthermore, we can use the tâtonnement scheme [23] to compute these two equilibria numerically. The main idea of the tâtonnement scheme is to exploit the monotonicity of the best response. By setting the initial strategy of firm 2 as $Q_2^0 = 0$, we solve firm 1's problem and obtain $Q_1^0 = Q_1^*(Q_2)$. Given Q_1^0, we next solve for $Q_2^1 = Q_2^*(Q_1^0)$ and then $Q_1^1 = Q_1^*(Q_2^1)$. Iteratively, (Q_1^k, Q_2^k) will converge to the equilibrium $(\bar{Q}_1^*, \underline{Q}_2^*)$ as the best response is decreasing in the competitor's strategy. Analogously, if we start with $Q_1^0 = 0$, the approach will converge to $(\underline{Q}_1^*, \bar{Q}_2^*)$. If two equilibria happen to be the same, the game has a unique equilibrium.

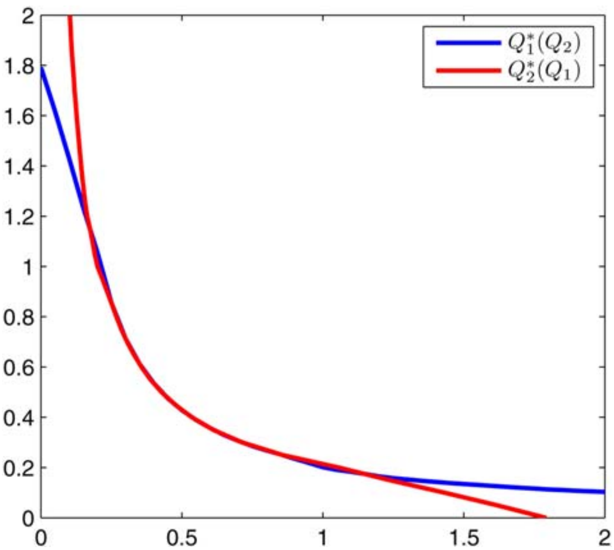

Figure 1. Best response functions of example 1. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

In Example 1, we demonstrate the existence of multiple equilibria using the tâtonnement scheme.

EXAMPLE 1 (Multiple Equilibria): We consider two identical firms with parameters $p = 6$, $c = 5$, $\delta = 0.5$, $s = 0$. Each firm faces an initial demand which follows a uniform distribution $U[0, 2]$. Each firm has a uniform random yield factor $\epsilon \sim U[0.5, 1]$. Implementing the tâtonnement scheme with a tolerance 10^{-6}, we find two quantity equilibria: (1.112, 0.1814) and (0.1814, 1.112). Because it is a symmetric game, we can also find another equilibrium with the same quantities (0.463, 0.463). Figure 1 plots the best response functions and the two best response curves have multiple intersections.

We close this subsection by noting that the preceding results (i.e., Proposition 1 and Theorem 1) can be extended when random yield factors are dependent. If the ϵ_i are dependent with a joint density function $g(\epsilon_1, \epsilon_2)$, by the similar derivation we can rewrite the first-order condition (3) for the interior optimum as, for $i, j = 1, 2$ and $i \neq j$,

$$\frac{(p_i - \delta_i c_i)E[\epsilon_i] - (1-\delta_i)c_i}{p_i - s_i} = \int_0^1\int_0^1 \epsilon_i F_{D_i^s|\epsilon_j}(Q_i^*\epsilon_i)g(\epsilon_1, \epsilon_2)d\epsilon_1 d\epsilon_2. \qquad (4)$$

To show the unique equilibrium for the Bernoulli case, we can define four probabilities to describe the joint yield distribution: $\lambda_{11} = \Pr\{\epsilon_1 = 1, \epsilon_2 = 1\}, \lambda_{10} = \Pr\{\epsilon_1 = 1, \epsilon_2 = 0\}, \lambda_{01} = \Pr\{\epsilon_1 = 0, \epsilon_2 = 1\}$, and $\lambda_{00} = \Pr\{\epsilon_1 = 0, \epsilon_2 = 0\}$. Invoking these λ's in the first-order condition, one can show that the best response is a contraction mapping if both switching rates are strictly less than one as we did in the case of independent Bernoulli yield.

3.3. Comparative Statics

In this subsection, we mainly investigate how the equilibrium quantities and firms' expected profits change as one firm's yield reliability level varies. In the case of multiple equilibria, we focus on the equilibria $(\bar{Q}_1^*, \underline{Q}_2^*)$ and $(\underline{Q}_1^*, \bar{Q}_2^*)$.

PROPOSITION 2: For all $i, j = 1, 2$ and $i \neq j$, (i) $\bar{Q}_i^*$ and $\underline{Q}_i^*$ are increasing in γ_{ji}, and $\underline{Q}_j^*$ and $\bar{Q}_j^*$ are decreasing in γ_{ji}. (ii) If $\delta_i \leq \frac{s_i}{c_i}$ and $G_i(x, a_i)$ is a truncated distribution in $[a_i, 1]$ from a general distribution $G_i^0(x)$ which is independent of a_i, then $\bar{Q}_i^*$ and $\underline{Q}_i^*$ are increasing in a_i, and $\underline{Q}_j^*$ and $\bar{Q}_j^*$ are decreasing in a_i. (iii) If $\delta_i \leq \frac{s_i}{c_i}$ and ϵ_i is a Bernoulli distribution with parameter $(1, a_i)$, then for the unique equilibrium (Q_1^*, Q_2^*), Q_i^* is increasing in a_i and Q_j^* is decreasing in a_i.

Part (i) states that if a firm can absorb more unsatisfied customers from its competitor, in the equilibrium this firm would choose a larger quantity and its competitor would order less. For the inventory competition with perfect yield reliability, a similar result has been shown in the literature. In the presence of yield uncertainty, there can be multiple equilibria, and we generalize this result to the equilibria $(\bar{Q}_1^*, \underline{Q}_2^*)$ and $(\underline{Q}_1^*, \bar{Q}_2^*)$.

Parts (ii) and (iii) state, for certain random yield distributions[1], how yield reliability levels affect the quantity equilibrium when $\delta_i \leq \frac{s_i}{c_i}$. When the refund rate is sufficiently low such that $\delta_i c_i \leq s_i$, that is, the unit refund for defective items is not greater than the salvage value, a higher reliability level always encourages the firm to choose a larger production quantity and also prevents the competitor from doing so. Besides reducing yield failures, our result implies that an improvement in yield reliability can also bring additional benefit for a competing firm through inducing its competitor to choose a small quantity.

[1] In part (ii), the condition for distribution $G_i(\cdot, a_i)$ requires the yield factor to have a support between a_i and 1. A simple example is the uniform distribution $[a_i, 1]$. Such truncated distributions are observed in many production processes. If a product fails to meet a certain quality standard, the product will be scrapped or reworked. Also, in a serial production line, only qualified items will be processed in the subsequent steps. The output of the above two kinds of production process follows a truncated distribution [16]. In this case, improving the index a_i can be interpreted as adopting the production equipment or process with a high-quality standard.

In fact, we have $\frac{\partial^2 \pi_i^q}{\partial Q_i \partial a_i} \geq 0$ for all (Q_1, Q_2) if the conditions in part (ii) or part (iii) hold. The interpretation of this is that the yield reliability level is complementary to the order quantity. We highlight this point in the following corollary.

COROLLARY 1: For $i = 1, 2$, if $G_i(x, a_i)$ is a truncated distribution in $[a_i, 1]$ from a general distribution $G_i^0(x)$ or $G_i(x, a_i)$ follows a Bernoulli distribution with success probability a_i, the inequality $\delta_i \leq \frac{s_i}{c_i}$ implies that the quantity Q_i and reliability level a_i are complementary instruments for firm i in the competition. That is, for any fixed Q_j and a_j $(j \neq i)$, the expected profit of firm i is supermodular in (Q_i, a_i).

The next proposition states the effect of yield reliability on the firm's profit in the equilibrium.

PROPOSITION 3: Suppose $\delta_i \leq \frac{s_i}{c_i}$. For $i, j = 1, 2$ and $j \neq i$, (i) If $G_i(x, a_i)$ is a truncated distribution in $[a_i, 1]$ from a general distribution $G_i^0(x)$, under the equilibria $(\bar{Q}_1^*, \underline{Q}_2^*)$ and $(\underline{Q}_1^*, \bar{Q}_2^*)$, the expected profit of firm i is increasing in its reliability index a_i and decreasing in its competitor's reliability index a_j; (ii) If ϵ_i is a Bernoulli distribution with parameter $(1, a_i)$, the expected profit of firm i is increasing in its reliability index a_i and decreasing in its competitor's reliability index a_j.

Under the same conditions as in Proposition 2, with a higher reliability level, a firm is able to earn more profit, whereas its competitor's profit will be reduced. We can take a closer look at the marginal profit by increasing the reliability level. Taking the total derivative of π_i^q with respect to a_i, we have

$$\frac{\partial \pi_i^q(Q_i{}^*(\mathbf{a}) | Q_j{}^*(\mathbf{a}), \mathbf{a})}{\partial a_i} = \underbrace{\frac{\partial \pi_i^q}{\partial Q_i^*}\frac{\partial Q_i^*}{\partial a_i}}_{0} + \underbrace{\frac{\partial \pi_i^q}{\partial Q_j^*}}_{-}\underbrace{\frac{\partial Q_j^*}{\partial a_i}}_{-} + \underbrace{\frac{\partial \pi_i^q}{\partial a_i}\Big|_{\mathbf{Q}=\mathbf{Q}^*(\mathbf{a})}}_{+}. \quad (5)$$

The first term is zero by the first-order optimality condition. One can show $\frac{\partial Q_j^*}{\partial a_i} \leq 0$ and $\frac{\partial \pi_i^q}{\partial a_i} \geq 0$ if the conditions in Proposition 3 are satisfied. From the above expression, we find that improving the reliability has two effects which contribute together to the firm's profit. The second term indicates that a higher reliability level reduces the competitor's equilibrium quantity and hence enlarges the firm's effective demand share, which leads to an increase in its expected profit. The third term indicates that a high-reliability level, in a more direct way, increases the expected profit with the quantity being fixed.

Note that $\delta_i \leq \frac{s_i}{c_i}$ provides a sufficient (but not necessary) condition under which equilibrium quantities have the

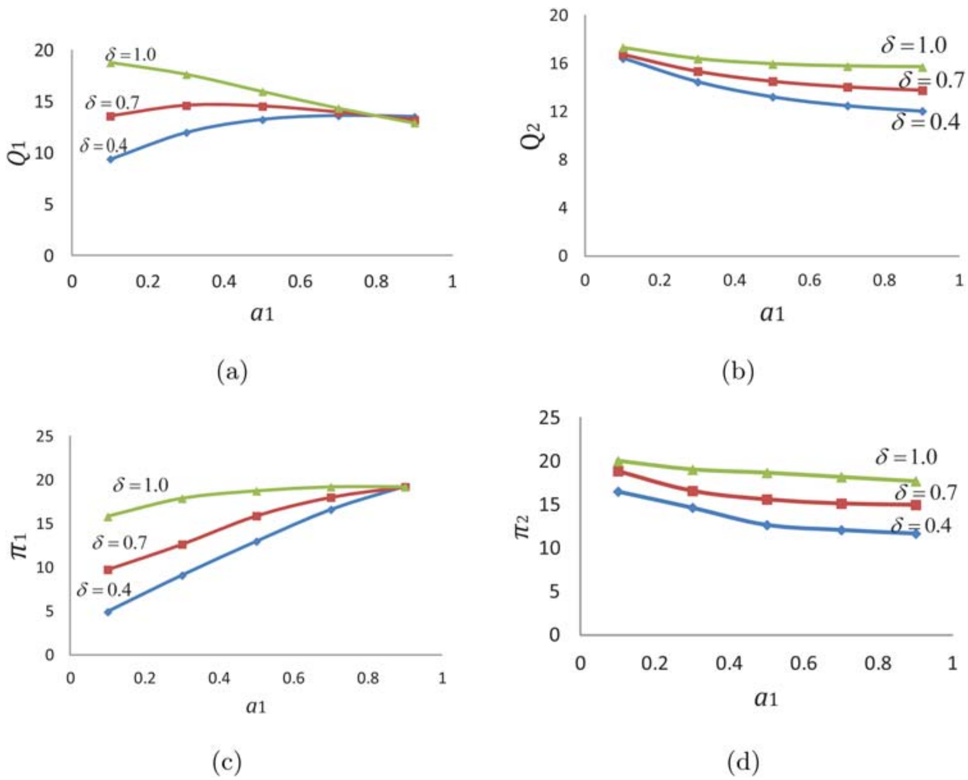

Figure 2. Effect of yield reliability with $\delta = 0.4, 0.7, 1.0$: uniform yield $[a_i, 1]$. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

monotone property discussed above. When $\delta_i > \frac{s_i}{c_i}$, this monotone property, however, may or may not hold, as shown by Example 2.

EXAMPLE 2: Two firms have identical parameters: $p_1 = p_2 = 6, c_1 = c_2 = 3, s_1 = s_2 = 1, \gamma_{12} = \gamma_{21} = 0.9$, and $\delta_1 = \delta_2 = \delta$. D_1 and D_2 independent and identically follow a gamma distribution with mean of 10 and cv (coefficient of variation) = 0.6. The reliability factor ϵ_i follows a uniform distribution $[a_i, 1]$ for both $i = 1, 2$. For $\delta \in \{0.4, 0.7, 1.0\}$, when a_1 varies and a_2 is fixed as 0.5, the equilibrium quantities[2] are reported in Figs. 2a and 2b, and the expected profits are plotted in Figs. 2c and 2d.

When $\delta = 0.4$, although it is greater than $\frac{s_i}{c_i}$, the equilibrium quantities still exhibit the monotone property as stated in Proposition 2. However, when $\delta = 0.7$, Q_1^* first increases and then decreases as a_1 becomes larger. When $\delta = 1$, that is, the firm gets a full refund for defective items, Q_1^* is in fact decreasing in a_1. Intuitively, there are two opposite effects of a_i on Q_i^*.[3] First, a higher value of a_i incentivizes firm i to increase Q_i as the effective cost is lower. Second, a greater value of a_i enables firm i to choose a lower initial quantity to obtain the same number of effective units. With a small δ_i, the former effect dominates the latter. This is because the effective cost, which is given be $\frac{(1-\delta_i)c_i}{E[\epsilon_i]} + \delta_i c_i$, becomes more dependent on $E[\epsilon_i]$ with a lower refund rate. When $\delta = 1$, the effective cost is simply c_i and independent of $E[\epsilon_i]$. Hence, the latter effect becomes dominant, so the firm reduces the initial quantity as the reliability level increases. Conversely, Q_2^* exhibits a decreasing trend for all values of δ. In particular, when $\delta = 1$, both Q_1^* and Q_2^* are decreasing in a_1. This implies that a higher a_1 can lower the competitor's initial quantity Q_2^* even though firm 1 also chooses a smaller quantity. In addition, we find for all values of δ, firm 1's expected profit is increasing in a_1, whereas firm 2's expected profit is decreasing in a_1 (see Figs. 2c and 2d). In particular, the marginal benefit for firm 1 from improving a_1 turns out to be smaller as the refund rate becomes higher.

In the case of Bernoulli yield, however, a high success probability does not induce firm 1 to lower initial quantity

[2] They are computed with the tâtonnement scheme and we report the equilibrium $(\bar{Q}_1^*, \underline{Q}_2^*)$ for instance. For the equilibrium $(\underline{Q}_1^*, \bar{Q}_2^*)$ we have similar observations.

[3] We graciously thank the review team for prompting the following discussion.

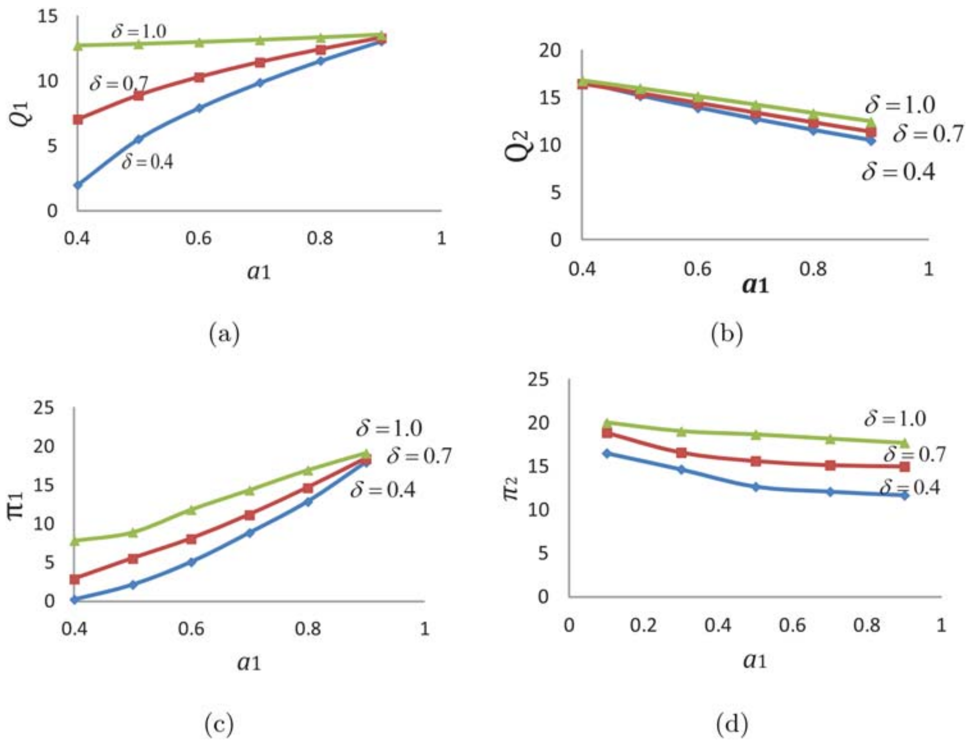

Figure 3. Effect of yield reliability with $\delta = 0.4, 0.7, 1.0$: Bernoulli yield. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

even when $\delta = 1$, as shown in Fig. 3a. This is because a higher reliability level only reduces the effective cost but cannot enable the firm to receive the same number of good units with a smaller initial quantity as the actual output is always "all or nothing".

4. THE MODEL WITH ENDOGENOUS YIELD RELIABILITY

4.1. Description and Assumptions

In this section, we allow the reliability levels to be endogenously determined by the firms. Let a_i^0 be the initial reliability level of firm i. Before placing the initial order, each firm may invest in its production reliability such that the initial index a_i^0 is increased to a_i ($a_i \geq a_i^0$). Firm i has to pay a cost $I_i(a_i)$ for improving its reliability level to a_i. Assume that $I_i(a_i)$ is increasing and convex in a_i and $I_i(a_i^0) = 0$.

We consider the following sequence of events. At the beginning, the firms select their desired reliability levels (a_1, a_2) to improve and incur the improvement cost. We assume that once these levels are reached, the reliability levels are observable. For example, a firm's reliability level can be learned by others through the quality certificates (e.g., ISO9000) it obtained. Based on these reliability levels, the firms decide the initial order quantities (Q_1, Q_2). The actual output is then realized and unsatisfied customers switch to the other firm. Therefore, we have a two-stage game as illustrated in Fig. 4 .

Let $(Q_1^*(\mathbf{a}), Q_2^*(\mathbf{a}))$ be the equilibrium quantities in the second stage, then firm i maximizes the first-stage profit by choosing a reliability level a_i. The first-stage problem of firm i can be written as

$$\max_{a_i \geq a_i^0} \quad \pi_i^r(a_i|a_j) = \pi_i^q(Q_i^*(\mathbf{a})|Q_j^*(\mathbf{a}), \mathbf{a}) - I_i(a_i),$$

which equals the expected profit in the equilibrium of the second-stage quantity game minus the cost for reliability improvement. Note that we use superscript "r" to denote the expected profit in the reliability game, and the second-stage profit π_i^q is the same as the profit function defined in the quantity game with exogenous yield reliability.

For the analytical tractability, we need the following assumptions. Numerical experiments in Section 4.4 will investigate more general cases where these assumptions are relaxed.

ASSUMPTION 1: For firm $i = 1, 2$, the random yield factor ϵ_i follows a Bernoulli distribution with success probability a_i.

As shown in Theorem 1, the Bernoulli-type random yield guarantees the uniqueness of the quantity equilibrium, which

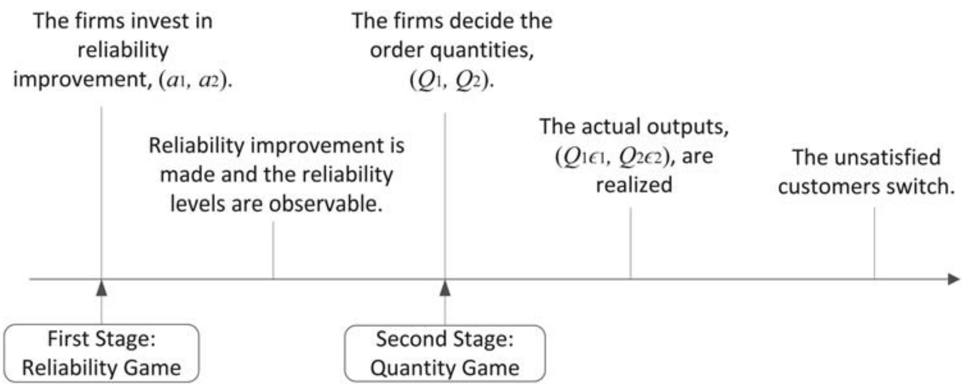

Figure 4. Sequence of events

enables us to derive analytical results for the first-stage reliability game. This assumption corresponds to the situation where the production is completely shut down due to catastrophic events. For chemical manufacturers, unreliable yield process may lead to production accidents which result in a complete shutdown of the production. For instance, in 2012 the chemical manufacturer, Evonik Industries, suffered an explosion at one of its nylon resin plants supplying PA12, a key component for auto fuel lines and brake lines [3]. This led to a production shutdown for months. The explosion was caused by an overdosage of a catalyst, and to prevent such production accidents from happening again, Evonik started to implemented additional safety measures [2]. In pharmaceutical industry, plants can be shut down due to violations of the U.S. Food and Drug Administration standards (e.g., product contamination [9, 27]). The companies may consider adopting high-quality technology and process to prevent product contamination. In our model, $1-a_i$ represents the probability of production accidents or contamination occurring, which can be reduced by adopting more reliable safety measures and production process.

ASSUMPTION 2: For firm $i=1, 2$, the initial demand share D_i is deterministic.

Assumption 2 excludes any uncertainty irrelevant to yield reliability so that we can restrict our attention to the reliability decisions. In other words, all the randomness of effective demand D_i^S arises from the competitor's yield uncertainty. For automotive parts manufacturers, it is possible to know their regular customers' (i.e., automakers) order before production and choosing the reliability mode.

4.2. Analysis

4.2.1. The Second-Stage Problem

We solve the two-stage game backward. The second stage of the game can be regarded as a restricted version of the model with exogenous reliability. Under Assumptions 1 and 2, the expected profit in the quantity game becomes piecewise linear. We can write it as

$$\begin{aligned}
\pi_i^q(Q_i|Q_j,\mathbf{a}) &= ((p_i-\delta_i c_i)a_i-(1-\delta_i)c_i)Q_i \\
&\quad -(p_i-s_i)a_i E_{\epsilon_j}[(Q_i-D_i^S)^+] \\
&= ((p_i-\delta_i c_i)a_i-(1-\delta_i)c_i)Q_i \\
&\qquad -(p_i-s_i)a_i a_j(Q_i-D_i-\gamma_{ji}(D_j-Q_j)^+)^+ \\
&\qquad -(p_i-s_i)a_i(1-a_j)(Q_i-D_i-\gamma_{ji}D_j)^+. \qquad (6)
\end{aligned}$$

The best response of the second-stage quantity game then relies on the slope of each piece in the payoff function. If $(p_i-\delta_i c_i)a_i-(1-\delta_i)c_i \le 0$, the firm chooses to produce nothing. Note that it just fails the entry condition stated in the Proposition 1. If $(p_i-\delta_i c_i)a_i-(1-\delta_i)c_i > 0$, the optimal order quantity for firm i could be either $D_i+\gamma_{ji}D_j$ or $D_i+\gamma_{ji}(D_j-Q_j)^+$ and is determined by whether $(p_i-\delta_i c_i)a_i-(1-\delta_i)c_i-(p_i-s_i)a_i a_j > 0$ or not.

We define

$$\phi_i(a_i) = \frac{(p_i-\delta_i c_i)a_i-(1-\delta_i)c_i}{(p_i-s_i)a_i}. \qquad (7)$$

For any given (a_1, a_2), the best response quantity of firm i can then be written as

$$Q_i^*(Q_j) = \begin{cases} 0 & \text{if } \phi_i(a_i) \le 0 \\ D_i+\gamma_{ji}(D_j-Q_j)^+ & \text{if } 0 < \phi_i(a_i) \le a_j \\ D_i+\gamma_{ji}D_j & \text{if } \phi_i(a_i) > a_j \end{cases}. \qquad (8)$$

Note that $\phi_i(a_i)$ reflects the trade-off between shortage cost and overage cost, like the critical fractile in the standard newsvendor problem. A large value of $\phi_i(a_i)$ implies that shortage is relatively expensive so the firm tends to produce more. For convenience, we define $\overline{\phi}_i = \frac{p_i-c_i}{p_i-s_i}$. Then,

Table 1. Summary of threshold values.

$\psi_i = \frac{(1-\delta_i)c_i}{p_i-\delta_i c_i}$	Entry reliability: determining whether to choose a positive quantity
$\phi_i(a_i) = \frac{(p_i-\delta_i c_i)a_i-(1-\delta_i)c_i}{(p_i-s_i)a_i}$	Determining the initial quantity as $D_i + \gamma_{ji}(D_j - Q_j)^+$ or $D_i + \gamma_{ji}D_j$
$\overline{\phi}_i = \phi_i(1) = \frac{p_i-c_i}{p_i-s_i}$	The largest value of $\phi_i(a_i)$
$a_j^A = \phi_i(a_i^*)$	Determining whether a_i^* is attained as an interior solution
a_j^B such that $a_j^B = \phi_i(a_i^{**}(a_j^B))$	Determining whether $a_i^{**}(a_j)$ is attained as an interior solution

for $\delta_i < 1, \phi_i(a_i)$ is a concave increasing function with $\phi_i(\psi_i) = 0$ and $\phi_i(1) = \overline{\phi}_i$. For $\delta_i = 1, \phi_i(a_i)$ is always equal to $\overline{\phi}_i$. For expositional clarity, all threshold values defined in the article are summarized in Table 1.

The best response function is characterized by one of the three cases based on the value of $\phi_i(a_i)$. If $\phi_i(a_i) < 0$ which implies a_i is smaller than the lowest necessary reliability level ψ_i, then the firm will refuse to enter the market and will produce nothing. If $0 < \phi_i(a_i) \leq a_j$, it is optimal to order a quantity of $D_i + \gamma_{ji}(D_j - Q_j)^+$, which is dependent on the competitor's strategy. Finally, if $\phi_i(a_i) > a_j$, the firm will order a quantity of $D_i + \gamma_{ji}D_j$. Note that in the third case, the best response of firm *i* is always $D_i + \gamma_{ji}D_j$, regardless of the strategy of firm *j*. This is because from firm *i*'s perspective, the competitor's reliability a_j is low enough such that it makes sense to try to take over the competitor's market share.

The following theorem characterizes the quantity equilibrium which can be achieved for different pairs of reliability indices (a_1, a_2).

THEOREM 2: For $i, j = 1, 2$ and $i \neq j$, the quantity equilibrium in the second-stage game is one of the five cases: (I) (0, 0), (II) $(D_i + \gamma_{ji}D_j, 0)$ or $(0, D_j + \gamma_{ij}D_i)$, (III) (D_i, D_j), (IV) $(D_i + \gamma_{ji}D_j, D_j)$ or $(D_i, D_j + \gamma_{ij}D_i)$, and (V) $(D_i + \gamma_{ji}D_j, D_j + \gamma_{ij}D_i)$.

1. The Case of $\delta_i < 1$ and $\delta_j < 1$
    - 1.1. If $\overline{\phi}_i > \psi_j$ and $\overline{\phi}_j > \psi_i$, the quantity equilibrium can be (I), (II), (III), (IV), or (V) as shown in Figs. 5a and 5b.
    - 1.2. If $\overline{\phi}_i \leq \psi_j$ and $\overline{\phi}_j \leq \psi_i$, the quantity equilibrium can be (I), (II), or (III) as shown in Fig. 5c.
    - 1.3. If $\overline{\phi}_i > \psi_j$ and $\overline{\phi}_j \leq \psi_i$, the quantity equilibrium can be (I), (II), (III), or (IV) as shown in Fig. 5d.
2. The Case of $\delta_i = 1$ or $\delta_j = 1$
    - 2.1. If $\delta_i = 1$ and $\delta_j = 1$, the quantity equilibrium can be (III), (IV), or (V) as shown in Fig. 5e.
    - 2.2. If $\delta_i < 1, \delta_j = 1$ and $\overline{\phi}_j > \psi_i$, the quantity equilibrium can be (II), (III), (IV), or (V) as shown in Fig. 5f.
    - 2.3. If $\delta_i < 1, \delta_j = 1$ and $\overline{\phi}_j \leq \psi_i$, the quantity equilibrium can be (II), (III), or (IV) as shown in Fig. 5g.

Region (I) represents the case in which neither firm is willing to enter the market, which occurs if the reliability levels of both firms are lower than the entry requirements. In Region (II), only one firm's reliability fails to meet the entry requirement; the other firm chooses to order its entire potential demand, $D_i + \gamma_{ji}D_j$. Region (III) indicates the case in which each firm orders its initial demand. It happens when neither of the firms has a significant advantage in yield reliability. In Region (IV), one firm is significantly ahead in terms of reliability. This firm would order more than its initial demand in the hopes of taking a portion of its competitor's demand, even if its competitor also orders the initial demand. Region (V) appears if Regions (IV) overlap. In this region, both firms are aggressive in their ordering decisions.

4.2.2. The First-Stage Problem

We are now ready to analyze the first-stage reliability game. Recall that the reliability cost $I_i(a_i)$ is assumed to be convex increasing. Consider the first-stage profit $\pi_i^r(a_i|a_j) = \pi_i^q(\mathbf{Q}^*(\mathbf{a}), \mathbf{a}) - I_i(a_i)$. If $\mathbf{Q}^*(\mathbf{a})$ is fixed, $\pi_i^r(a_i|a_j)$ is concave in a_i. The best response $a_i(a_j)$ is then obtained by

$$a_i(a_j) = \begin{cases} a_i^0, \text{if } \frac{\partial \pi_i^q}{\partial a_i} < \frac{dI_i(a_i)}{da_i} \text{ for all } a_i \\ \text{the unique solution such that } \frac{\partial \pi_i^q}{\partial a_i} = \frac{dI_i(a_i)}{da_i} \\ 1, \text{if } \frac{\partial \pi_i^q}{\partial a_i} > \frac{dI_i(a_i)}{da_i} \text{ for all } a_i. \end{cases} \tag{9}$$

However, π_i^q depends on which quantity equilibrium is achieved in the second stage so the marginal gain for improving reliability $\frac{\partial \pi_i^q}{\partial a_i}$ differs. Therefore, we have to find the candidate solution for each quantity equilibrium and select the one that generates the highest profit as the best response in the first stage.

Invoking Theorem 2 and the second-stage payoff function (6), we have four possible values of $\frac{\partial \pi_i^q}{\partial a_i}$ for different quantity equilibria. Table 2 lists all the possibilities, where $a_i^*, a_i^{**}(a_j)$, and a_i^{***} are defined as the optimal solutions if the

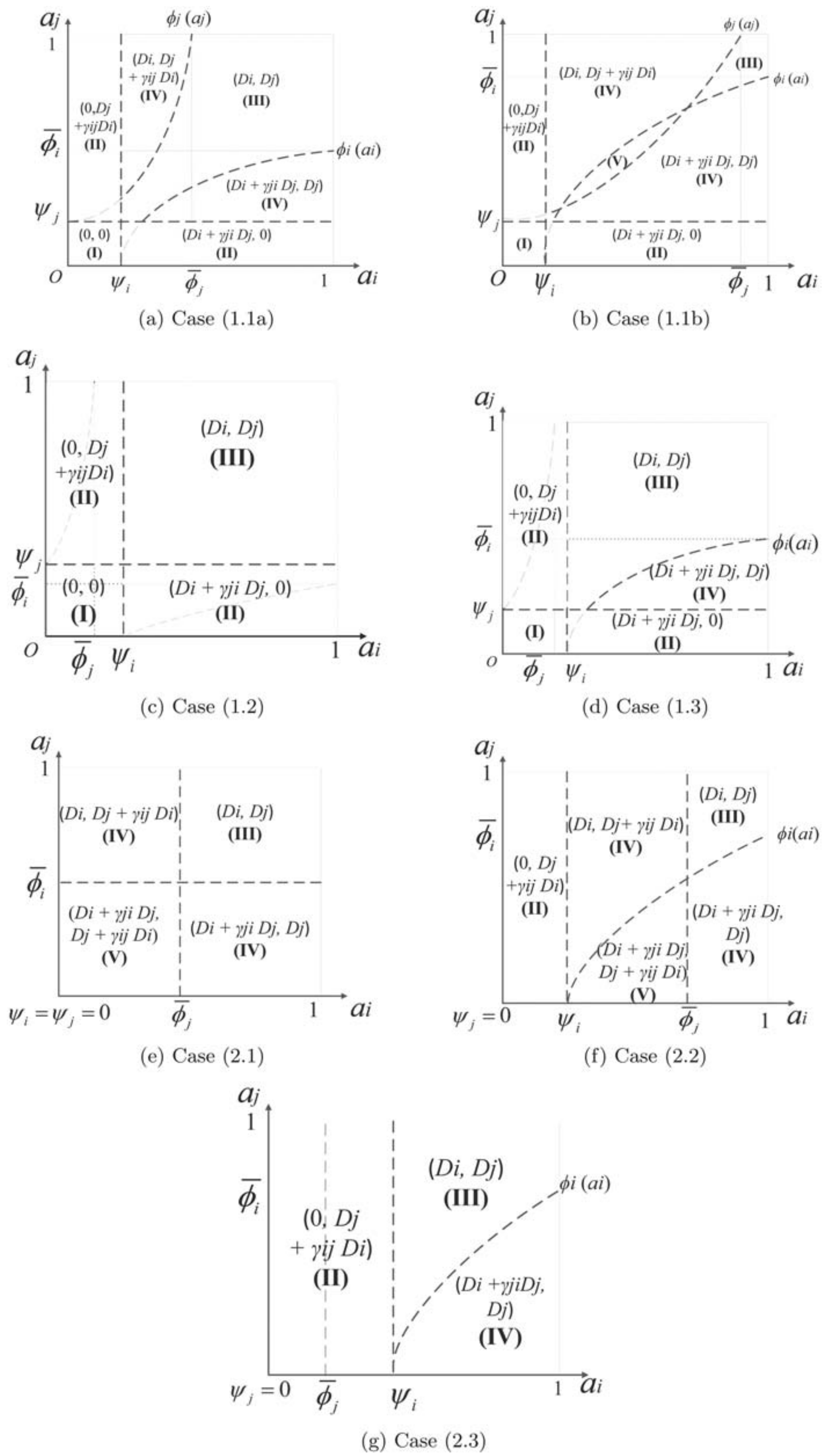

Figure 5. Quantity equilibrium with deterministic initial demand

Table 2. Candidate solutions to the first-stage problem.

Quantity equilibrium (Q_i^*, Q_j^*)	Marginal gain $\frac{\partial \pi_i^q}{\partial a_i}$	Optimal reliability
$(0,0), (0, D_j + \gamma_{ij} D_i)$	0	a_i^0
$(D_i, D_j), (D_i, D_j + \gamma_{ij} D_i)$	$(p_i - \delta_i c_i) D_i$	a_i^*
$(D_i + \gamma_{ji} D_j, D_j), (D_i + \gamma_{ji} D_j, D_j + \gamma_{ij} D_i)$	$(p_i - \delta_i c_i) D_i + [(p_i - \delta_i c_i) - (p_i - s_i) a_j] \gamma_{ji} D_j$	$a_i^{**}(a_j)$
$(D_i + \gamma_{ji} D_j, 0)$	$(p_i - \delta_i c_i)(D_i + \gamma_{ji} D_j)$	a_i^{***}

corresponding (Q_i^*, Q_j^*) is achieved and can be calculated by substituting the corresponding $\frac{\partial \pi_i^q}{\partial a_i}$ into (9). Note that some of the candidate solutions may not be attained given a particular a_j as whether the quantity equilibrium is achievable also hinges on a_j.

a_i^* is the solution under the equilibrium where firm i chooses the order quantity D_i. As a_i^* is chosen for the equilibrium where firm i only obtains the initial demand D_i, we name it as the prude strategy. $a_i^{**}(a_j)$, which decreases in a_j, is the solution for the equilibrium where firm i chooses the order quantity $D_i + \gamma_{ji} D_j$ in the hopes of occupying parts of the competitor's initial demand, so we label $a_i^{**}(a_j)$ as the aggressive strategy. Finally, a_i^{***} is the solution under the quantity equilibrium where firm j does not enter the market so we name it as the monopolistic strategy.

The following theorem characterizes the best response function in the first-stage game.

THEOREM 3: For $i, j = 1, 2$ and $j \neq i$, (i) if $\overline{\phi}_i \leq \psi_j$, the best response of firm i is given by

$$a_i(a_j) = \begin{cases} a_i^* & \text{if } a_j > \psi_j, \\ a_i^{***} & \text{if } a_j \leq \psi_j; \end{cases} \tag{10}$$

(ii) if $\overline{\phi}_i > \psi_j$, there exist two thresholds a_j^A and a_j^B $(a_j^A \leq a_j^B \leq \overline{\phi}_i)$, where $a_j^A = \phi_i(a_i^*)$ and a_j^B is the unique solution such that $a_j^B = \phi_i(a_i^{**}(a_j^B))$. The best response function of firm i is given by

$$a_i(a_j) = \begin{cases} a_i^* & \text{if } a_j > a_j^B, \\ a_i^* \text{ or } a_i^{**}(a_j), \text{ chosen from max} & \\ \quad (\pi_i^r(a_i^* | a_j), \pi_i^r(a_i^{**} | a_j)) & \text{if } a_j^A < a_j \leq a_j^B, \\ a_i^{**}(a_j) & \text{if } \psi_j < a_j \leq a_j^A, \\ a_i^{***} & \text{if } a_j \leq \psi_j. \end{cases} \tag{11}$$

The monopolistic strategy is evidently dominant given $a_j \leq \psi_j$, that is, when firm j does not meet the entry requirement. If firm j chooses a high-reliability level such that $a_j > \overline{\phi}_i$, firm i would choose the prude strategy as the chance of occupying its competitor's market is quite slim.

For a_j between ψ_j and $\overline{\phi}_i$, we have to consider the two threshold values defined in Theorem 3. a_j^B gives the turning point above which $a_i^{**}(a_j)$ is not attained (given $a_j > a_j^B$, the quantity equilibria associated with $a_i^{**}(a_j)$ are not achievable at $a_i = a_i^{**}(a_j)$). Similarly, a_j^A serves as the turning point below which the prude strategy a_i^* is not attained. If firm j's reliability level lies between a_j^A and a_j^B, both prude and aggressive strategies are feasible for firm i. Then, the firm has to compare the payoffs with the strategies a_i^* and $a_i^{**}(a_j)$. The complication is essentially due to that the first-stage decision will affect the quantity equilibrium that will be achieved in the second stage. The firms, thus, need to look ahead when they make the reliability improvement.

REMARK 1: In the best response function, we have $a_i^* \leq a_i^{**}(a_j) \leq a_i^{***}$.

The above inequalities can be verified by comparing the values of $\frac{\partial \pi_i^q}{\partial a_i}$ in Table 2 and noticing that $a_i^{**}(a_j)$ is chosen only if $\psi_j < a_j \leq \overline{\phi}_i$. This is basically because the marginal gain for improving reliability is higher when the firm has a larger market share in the quantity equilibrium.

REMARK 2: The first-stage payoff function may not be quasiconcave.

One may have observed that given $a_j^A < a_j \leq a_j^B$, the optimal payoff function is given by $\max(\pi_i^r(a_i^* | a_j), \pi_i^r(a_i^{**}(a_j) | a_j))$. That is, the payoff function with respect to a_i is the maximum of two different concave functions of a_i. Hence, the payoff function is not unimodal. The reason for this is that the firm can choose the most profitable quantity equilibrium by strategically selecting the reliability level in the first stage.

We then want to fully characterize the reliability game. Note that $a_i^* \leq a_i^{**}(a_j) \leq a_i^{***}$ and $a_i^{**}(a_j)$ is decreasing in a_j. In fact, we can also show that within the interval $a_j^A < a_j \leq a_j^B$ the best response $a_i(a_j)$ is nonincreasing in a_j. As a result, the best response function characterized in Theorem 3 is (weakly) decreasing. Hence, the reliability game is submodular, which implies that a higher reliability level induces the competitor to choose a lower reliability level in the equilibrium. Like the quantity game with given reliability levels, among all reliability equilibria (a_1, a_2), there

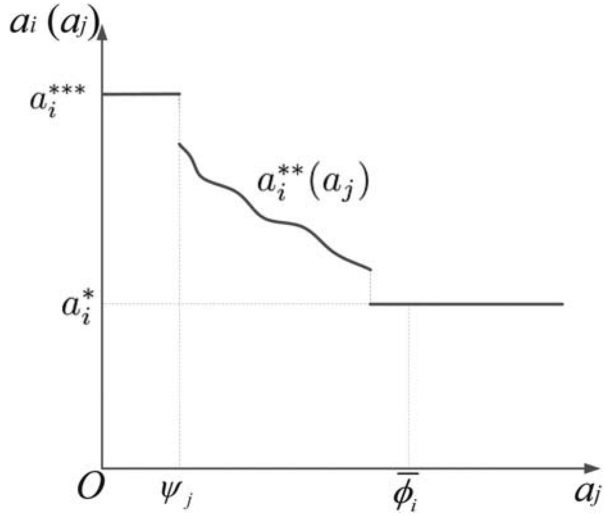

Figure 6. Best response function of reliability level $a_i(a_j)$

exists an equilibrium $(\bar{a}_1, \underline{a}_2)$ $[(\underline{a}_1, \bar{a}_2)]$ containing the highest [lowest] equilibrium level of a_1 and the lowest [highest] equilibrium level of a_2.

THEOREM 4: The first-stage game is submodular in (a_1, a_2). There exist Nash equilibria $(\bar{a}_1, \underline{a}_2)$ and $(\underline{a}_1, \bar{a}_2)$ where $\bar{a}_i$ and $\underline{a}_i$ are, respectively, the highest and lowest equilibrium reliability levels for firm i ($i = 1, 2$). If $\bar{a}_i = \underline{a}_i$ for all i, there is a unique equilibrium.

In general, the best response function is discontinuous, as illustrated in Fig. 6. For example, it is not continuous at $a_j = \psi_j$ unless $\delta_j = 1$, which implies $\psi_j = 0$. The other discontinuous point occurs when the optimal strategy jumps from $a_i^{**}(a_j)$ to a_i^*.

By the best response function, if the initial reliability levels for both firms are already high enough, a_i^* will be the dominant strategy. Consequently, the resulting equilibrium is unique.

PROPOSITION 4: (a_1^*, a_2^*) is the unique equilibrium provided that the initial reliability level $a_i^0 > a_i^B$ for all $i = 1, 2$. Moreover, a_i^* only depends on firm i's own characteristics.

Proposition 4 describes a mature market where the firms' production technologies or processes are already good. In such a market, the firms' reliability levels are high enough to deter each other from adopting an aggressive strategy. As a result, each firm chooses to maintain its own business and only pursues a "prude" reliability a_i^* rather than an "aggressive" or "monopolistic" reliability. In other words, the competition weakens the firm's incentive to improve the yield process. The rationale behind is that the competing firm has a limited market share and so the marginal gain from improving reliability is relatively small. Consequently, they are reluctant to pursue as high a reliability level as a monopoly does.

Consider a special case where one unreliable firm competes with one perfectly reliable firm. The firm with perfect yield reliability, say firm j (that is, $a_j^0 = 1$), can at least fulfill its initial demand share. As a result, the unreliable firm would never choose an aggressive strategy to produce $D_i + \gamma_{ji} D_j$. According to Theorem 2, the quantity equilibrium in the second stage then depends only on firm i' reliability level a_i.

PROPOSITION 5: For $i, j = 1, 2$ and $j \neq i$, provided that $a_j^0 = 1$, in the second-stage game with any a_i, the quantity equilibrium is one of the following three cases:

$$(Q_i^*, Q_j^*) = \begin{cases} (0, D_j + \gamma_{ij} D_i) & \text{if } a_i < \psi_i, \\ (D_i, D_j + \gamma_{ij} D_i) & \text{if } \psi_i \le a_i < \overline{\phi}_j, \\ (D_i, D_j) & \text{if } a_i \ge \overline{\phi}_j. \end{cases} \quad (12)$$

In the first-stage game, firm i will always choose the reliability level a_i^*.

As summarized in Proposition 5, in the second stage firm i only chooses from 0 and D_i according to its entry reliability, while the reliable firm j chooses either $D_j + \gamma_{ij} D_i$ or D_j, depending on whether its competitor is reliable enough. In the first stage, the reliability choice of firm i is rather straightforward. The prude reliability level is a dominant strategy for firm i, because with $a_j^0 = 1$ firm i cannot end up with a market share $D_i + \gamma_{ji} D_j$.

Finally, the following proposition gives the comparative statics based on the submodular property.

PROPOSITION 6: For $i, j = 1, 2$ and $i \neq j$, the equilibrium reliability levels $\bar{a}_i$ and $\underline{a}_i$ are nondecreasing in γ_{ji}, and nonincreasing in γ_{ij}.

Proposition 6 implies that the equilibrium reliability levels are sensitive to the customer switching rate in the inventory competition. A firm will select a high-reliability level if it has a chance to absorb a large portion of its competitor's unmet demand and vice versa. This result suggests that it would be conductive to the yield reliability improvement if a manufacturer can attract more of other firm's customers to consider it as a backup vendor.

4.3. Special Case: The Symmetric Game with Linear Improvement Cost

To gain more insights from the reliability game, we consider a symmetric game in which two firms have the same parameters and improving reliability incurs a linear cost. Let $I(a_i) = (a_i - a^0)m$ where m is the cost rate. With this linear cost function, the best response in the reliability game is either to improve the reliability to $a_i = 1$ or to simply adopt $a_i = a^0$. As the two firms are ex ante identical, we can

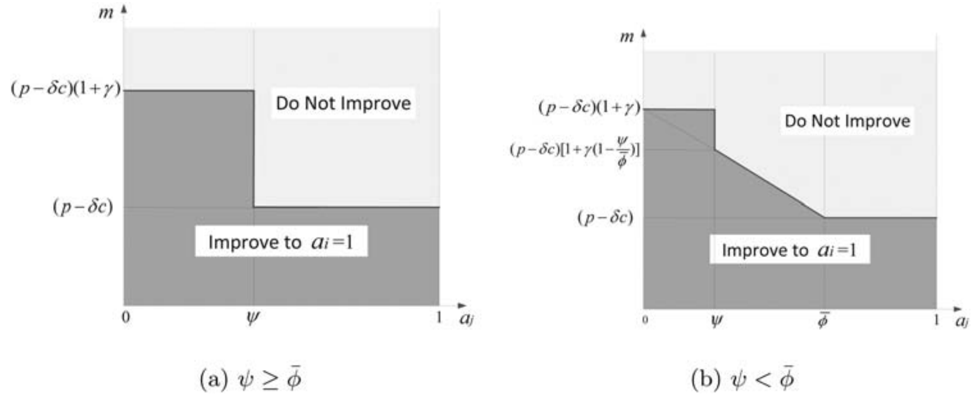

Figure 7. Best response of reliability investment with linear improvement costs

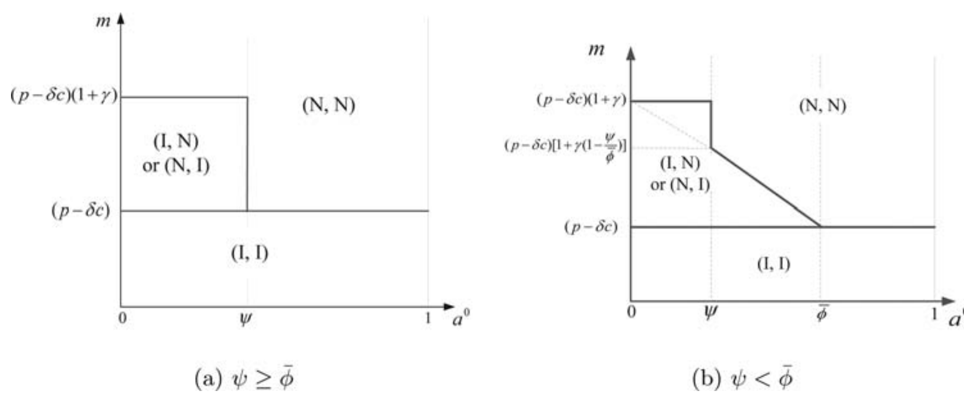

Figure 8. Pure strategy Nash equilibria in the symmetric game with linear improvement costs (I = Improve; N = Not Improve).

normalize their initial demand to one, that is, $D = 1$. By applying Theorem 3, we can derive the best response function, which is illustrated in Fig. 7 for any given cost rate m. The highest cost margin for a firm to make the improvement is decreasing in the competitor's action a_j. This is because firm i's marginal gain from improvement is decreasing in a_j. Intuitively, when firm j is reliable enough, the expected market share of firm i will be small because of the slim possibility of obtaining the extra demand γD from firm j. Hence, even though firm i purchases a perfect yield process, the expected reward can be limited if a_j is sufficiently high.

Based on the best responses, we can characterize the possible pure strategy Nash equilibria with any given cost rate m and initial reliability level a^0 in Fig. 8. Recall that $p - \delta c$ equals the marginal gain from improving when the firm only obtains its own demand D (see Table 2). Hence, when $m < p - \delta c$, the improvement will be a dominant strategy whether the competitor chooses to improve or not. As a result, (I, I) will be the unique equilibrium. Similarly, $(p-\delta c)(1+\gamma)$ is the marginal gain if the competitor does not enter the market. Thus, when $m > (p - \delta c)(1 + \gamma)$, (N, N) will be a dominant equilibrium.

For $p - \delta c \leq m \leq (p - \delta c)(1 + \gamma)$, the results are slightly different in the cases of $\psi \geq \bar{\phi}$ and $\psi < \bar{\phi}$. However, in both scenarios there exists a region in which the Nash equilibrium is either (I, N) or (N, I) (i.e., only one firm chooses to improve). This happens when a firm thinks the improvement worthwhile unless the competitor also chooses to improve. If firm j does not improve, that is, $a_j = a^0$, firm i will enjoy a high probability of taking over the additional demand γD. This gives firm i an expected market share which is large enough to induce improvement. Conversely, if firm j chooses to improve, that is, $a_j = 1$, firm i can only get its own demand and the resulting marginal gain from improvement can no longer compensate the marginal cost m. As a result, only one firm would choose to improve in the equilibrium. In addition, the asymmetric equilibrium only occurs when a^0 is relatively low. When $a^0 > \bar{\phi}$, even if firm j does not improve, $a_j = a^0$ is high enough to prevent firm i from improving.

Our results shed light on why there often exist both high-reliability and low-reliability firms supplying similar products to the market. For a newly developed product (e.g., some electronic components for automobiles), the production technology can be unreliable among all firms in the

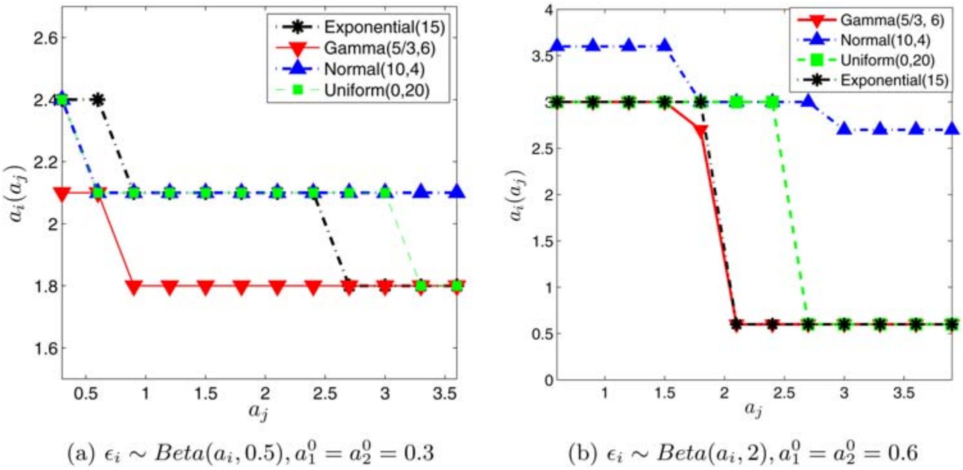

Figure 9. Best responses with uncertain demand and Beta random yield. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

market. All firms can then be regarded as identical ones with low initial reliability levels. Some firms expecting large market shares will invest more in the reliability, whereas others may be reluctant to do so because they think the improvement not worthwhile if other firms choose to retain customers by improving the yield reliability. As a result, the equilibrium outcome will be asymmetric as predicted by our model.[4]

4.4. Numerical Study

Thus far, we have derived analytical results under Assumptions 1 and 2. In this subsection, we conduct numerical experiments to test whether the basic results obtained also apply in more general settings with uncertain demand and other yield distributions.

Consider two firms that are ex ante identical. Set $p = 6, c = 4, s = 1, \delta = 0.6$ and $\gamma_{21} = \gamma_{12} = 0.9$. Improving the reliability level from a_i^0 to a_i incurs a cost $I(a_i) = m(a_i - a_i^0)^2$, where $m = 0.8$. In the experiments, we consider four types of demand distributions: exponential, normal, uniform, and gamma distributions. The yield factor ϵ_i follows the beta distribution with parameters (a_i, b). Firm i can improve the yield reliability by increasing the value of a_i. We consider two groups of experiments with $b = 0.5$ and 2, respectively, so as to include different shapes of density curves.

The second-stage equilibrium is computed with the tâtonnement scheme. As stated in Remark 2, the first-stage payoff function may not be unimodal. Hence, the strategy space of a_i is equally discretized into twelve values, and we conduct an exhaustive search to solve the first-stage game. The first-stage best response functions are reported in Fig. 9a.

In eight scenarios with various combinations of yield and demand distributions, we find that all best response functions are nonincreasing. Although the submodularity of the first-stage game is proved in the case of deterministic demand and Bernoulli yield, this property appears to be valid in more general settings.[5] Next, we consider the effect of the demand switching rate on first-stage equilibria. In Proposition 6, we have shown the basic monotone properties of equilibrium reliability levels with respect to γ_{ij}. We examine if these properties also apply in more general conditions. We consider the Gamma demand for instance. All parameters remain the same as in the preceding experiments, except that we vary γ_{21} from 0.1 to 0.9 while fixing $\gamma_{12} = 0.9$. With the tâtonnement scheme,[6] we compute $(\bar{a}_1, \underline{a}_2)$ and $(\underline{a}_1, \bar{a}_2)$ in each instance and the results are presented in Table 3 . The numerical observations conform to the result in Proposition 6.

5. CONCLUSION

In the traditional inventory competition, firms compete by selecting their stocking levels and a firm's initial customers will switch to the competitor if a stockout occurs. For manufacturing firms that are subject to random yield,

[4] For example, among automotive electronics manufacturers, one of well-known giant firms is Foxconn. Unlike Foxconn serving the worldwide market, some small and less reliable firms in China also supply automotive electronic parts to their local market. This fact is consistent with our equilibrium outcome.

[5] In addition, we also tested the cases where ϵ_i follows uniform distributions $[a_i, 1]$ and $[0, a_i]$, though the results are not reported here. In all those instances, the submodularity is also observed.

[6] The convergence of the tâtonnement scheme here is not guaranteed as it is intractable to prove the submodularity in the general settings. Nevertheless, in all numerical instances we tested the submodularity still holds and hence the tâtonnement scheme successfully converged to the first-stage equilibria.

Table 3. Effect of γ_{21} on the first-stage equilibria.

	γ_{21}	$(\bar{a}_1, \underline{a}_2)$	$(\underline{a}_1, \bar{a}_2)$
$\epsilon_i \sim \text{Beta}(a_i, 0.5)$	0.1	(1.5,1.8)	(1.5, 1.8)
	0.3	(1.8,1.8)	(1.8, 1.8)
	0.5	(1.8,1.8)	(1.8, 1.8)
	0.7	(1.8,1.8)	(1.8, 1.8)
	0.9	(1.8,1.8)	(1.8, 1.8)
$\epsilon_i \sim \text{Beta}(a_i, 2)$	0.1	(0.6,3)	(0.6,3)
	0.3	(0.6,3)	(0.6,3)
	0.5	(0.6,3)	(0.6,3)
	0.7	(2.7,0.6)	(0.6,3)
	0.9	(3,0.6)	(0.6,3)

stockouts can also be caused by yield failure. In this article, we study the inventory competition in the presence of yield uncertainty.

We first study the case where two competing firms decide the order quantities based on the exogenous yield reliability levels. Our analysis answers the questions raised in Section 1.

How do the results from the traditional inventory competition change in the presence of yield uncertainty? Due to yield uncertainty, there is a lowest reliability level required for the firm to enter the market (Proposition 1). Like the traditional inventory competition with completely reliable supply, the game is submodular (Theorem 1) but yield uncertainty gives rise to multiple equilibria (Example 1).

What is the effect of yield reliability on the equilibrium in the inventory competition? Under some conditions, we show that for the competing firms, quantity and yield reliability serve as complementary instruments (Proposition 2 and Corollary 1). The firm can increase its expected profit with a higher reliability level, and its competitor's profit is also reduced in the equilibrium (Proposition 3). In fact, a high-reliability level has two effects in increasing the firm's expected profit: a direct effect that reduces the firm's risk of yield failure and an indirect effect that lowers the competitor's production quantity in the equilibrium. In addition, possessing a high-reliability level does not necessarily lead to a large production quantity, especially when the firm can obtain a very high refund for defective items (Example 2).

We further consider the case in which the firms can endogenously determine the yield reliability levels before production. The problem then becomes a two-stage game. In the first stage, the firms simultaneously select their reliability levels to improve toward. In the second stage, the initial order quantities are determined.

Our analysis of the reliability game answers the question: How do firms make the reliability improvement given a subsequent competition in quantity? If a firm's initial demand is deterministic and random yield follows a Bernoulli distribution, we show that the first-stage game is submodular (Theorem 4).[7] We find that competing firms can be reluctant to pursue a high-reliability level as a monopoly does (Proposition 4). This result indicates that the competition in quantity can discourage yield reliability improvement. The reason is that the potential market share of a competitive firm is smaller than that of a monopoly, so the marginal gain from improving reliability is relatively small for the competitive firm. The equilibrium reliability levels are also sensitive to the customer switching rate. The firm would choose a higher reliability level if more customers can switch to it from its competitor and vice versa (Proposition 6). Hence, it would be conductive to a firm's reliability improvement if more of its competitor's customers consider it as a backup vendor. By analyzing a symmetric game where improving reliability incurs a linear cost, we find that due to the inventory competition, two firms with identical parameters may choose different reliability levels in the equilibrium and this happens in a nascent market where the initial reliability is relatively low. By numerical experiments, we demonstrate that the main results, though derived in a restrictive setting, are robust when the random yield and demand follow more general distributions.

Several extensions may be investigated in the future research. In the model with endogenous reliability, we have assumed that the initial demand share is exogenously given. If the customers have some knowledge about the firms' reliability, they may prefer one firm over the other ex ante. In this case, the initial demand can also depend on the reliability levels that are selected in the first-stage game. This situation can be studied in future research. Furthermore, in the reliability game, we have assumed the firms' yield processes are independent. In some cases, firms may have some common supply sources. If so, the yield processes could be correlated and improving the yield reliability may also benefit its competitor. However, it would be technically challenging to incorporate the yield dependence in the two-stage game framework. Finally, the demand substitution makes transshipment possible: Two competing firms may exchange the items by some transfer payment. The inventory decision and reliability improvement under this situation are worth studying in the future.

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[7] Wang et al. [26] find a similar result for yield reliability improvement when two firms compete in their service levels. In their model, market share is determined by the service levels, whereas in our problem a firm's effective demand is determined based on the realization of random yield.

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APPENDIX: PROOFS

PROOF OF PROPOSITION 1: Taking the first-order derivative of firm i's payoff function (1) with respect to Q_i, we have

$$\frac{\partial \pi_i^q(Q_i|Q_j,\mathbf{a})}{\partial Q_i} = (p_i - \delta_i c_i)E[\epsilon_i] - (p_i - s_i)\frac{\partial E_{D,\epsilon}[(Q_i\epsilon_i - D_i^s)^+]}{\partial Q_i} - (1-\delta_i)c_i.$$

To calculate the derivative of the expectation in the second term, one convenient way is to interchange the expectation and the derivative, which is justified by the fact that the function inside the expectation is integrable and its derivative is bounded (See Ref. [17]). Hence, we have

$$\frac{\partial E_{D,\epsilon}[(Q_i\epsilon_i - D_i^s)^+]}{\partial Q_i} = E_{D,\epsilon}[\frac{\partial (Q_i\epsilon_i - D_i^s)^+}{\partial Q_i}] = E_{D,\epsilon}[\epsilon_i 1_{\{Q_i^*\epsilon_i \ge D_i^s\}}] = E_\epsilon[\epsilon_i F_{D_i^s|\epsilon_j}(Q_i\epsilon_i)],$$

where $F_{D_i^s|\epsilon_j}(Q_i\epsilon_i) = \Pr(D_i + \gamma_{ji}(D_j - Q_j\epsilon_j)^+ \le Q_i\epsilon_i|\epsilon_j)$. Thus,

$$\frac{\partial \pi_i^q(Q_i|Q_j,\mathbf{a})}{\partial Q_i} = (p_i - \delta_i c_i)E_\epsilon[\epsilon_i] - (p_i - s_i)\int_0^1\int_0^1 \epsilon_i F_{D_i^s|\epsilon_j}(Q_i\epsilon_i)dG_j(\epsilon_j,a_j)dG_i(\epsilon_i,a_i) - (1-\delta_i)c_i. \quad \text{(A.1)}$$

Then, by checking the second-order derivative,

$$\frac{\partial^2 \pi_i^q(Q_i|Q_j,\mathbf{a})}{\partial {Q_i}^2} = -(p_i - s_i)\int_0^1\int_0^1 \epsilon_i^2 f_{D_i^s|\epsilon_j}(Q_i\epsilon_i)dG_j(\epsilon_j,a_j)dG_i(\epsilon_i,a_i) \le 0,$$

that is, $\pi_i^q(Q_i|Q_j,\mathbf{a})$ is concave in Q_i. By concavity, we know that a positive Q_i maximizes $\pi_i^q(Q_i|Q_j,\mathbf{a})$ if and only if (A.1) is greater than zero at $Q_i = 0$. Substituting $Q_i = 0$ into (A.1) yields the entry condition given by (2). □

PROOF OF THEOREM 1: We prove part (i) by checking the cross partial derivative of π_i^q,

$$\frac{\partial^2 \pi_i^q(Q_i|Q_j,\mathbf{a})}{\partial Q_i \partial Q_j} = -(p_i - s_i)\gamma_{ji}\int_0^1\int_0^1 \epsilon_i\epsilon_j \frac{\partial F_{D_i^s|\epsilon_j}(Q_i\epsilon_i)}{\partial Q_j} dG_j(\epsilon_j,a_j)dG_i(\epsilon_i,a_i) \le 0.$$

The inequality holds as

$$\frac{\partial F_{D_i^s|\epsilon_j}(Q_i\epsilon_i)}{\partial Q_j} = f_{D_i^s|D_j>Q_j\epsilon_j,\epsilon_j}(Q_i\epsilon_i)\Pr\left(D_j > Q_j\right) > 0.$$

For part (ii), the existence of an equilibrium follows by the submodularity. It has been shown that the payoff function $\pi_i^q(Q_i|Q_j\mathbf{a})$ is concave in Q_i. Hence, the equilibrium (Q_i^*, Q_j^*) solves Eq. (3) given by the first-order conditions.

In part (iii), we assume that the random reliability factor follows a Bernoulli distribution. The uniqueness of the equilibrium can be shown by verifying that the best response function is contraction mapping (See Ref. [5]). We check the absolute value of the slope of the best response function:

$$\left|\frac{\partial Q_i{}^*(Q_j)}{\partial Q_j}\right| = \left|-\frac{\partial^2 \pi_i^q(Q_i|Q_j,\mathbf{a})/\partial Q_i \partial Q_j}{\partial^2 \pi_i^q(Q_i|Q_j,\mathbf{a})/\partial {Q_i}^2}\right| = \frac{\gamma_{ji}\int_0^1\int_0^1 \epsilon_i\epsilon_j f_{D_i^s|D_j>Q_j\epsilon_j,\epsilon_j}(Q_i\epsilon_i)\Pr\left(D_j > Q_j\epsilon_j\right)dG_j(\epsilon_j,a_j)dG_i(\epsilon_i,a_i)}{\int_0^1\int_0^1 \epsilon_i^2 f_{D_i^s|\epsilon_j}(Q_i\epsilon_i)dG_j(\epsilon_j,a_j)dG_i(\epsilon_i,a_i)}.$$

Substituting the Bernoulli distribution into the above equation yields

$$\left|\frac{\partial Q_i{}^*(Q_j)}{\partial Q_j}\right| = \frac{\gamma_{ji}a_i a_j f_{D_i^s|D_j>Q_j}(Q_i)\Pr(D_j > Q_j)}{a_i a_j f_{D_i^s|\epsilon_j=1}(Q_i) + a_i(1-a_j)f_{D_i^s|\epsilon_j=0}(Q_i)} \le \frac{\gamma_{ji} f_{D_i^s|D_j>Q_j,\epsilon_j=1}(Q_i)\Pr(D_j > Q_j)}{f_{D_i^s|\epsilon_j=1}(Q_i)} < 1. \quad \text{(A.2)}$$

The last inequality holds as $\gamma_{ij} < 1$. □

PROOF OF PROPOSITION 2: According to Theorem 9 of Ref. 5, the largest and smallest equilibria in the supermodular game are increasing in a parameter if for every player the payoff functions is supermodular in this player's strategy and the parameter.

To prove part (i), it suffices to show $\partial \pi_i^q/\partial Q_i \partial \gamma_{ji} \ge 0$ and $\partial \pi_j^q/\partial Q_j \partial \gamma_{ji} \le 0$, as the game is supermodular in $(Q_i, -Q_j)$. We have

$$\frac{\partial^2 \pi_i^q(Q_i|Q_j,\mathbf{a})}{\partial Q_i \partial \gamma_{ji}} = -(p_i - s_i)\frac{\partial E_\epsilon[\epsilon_i F_{D_i^s|\epsilon_j}(Q_i\epsilon_i)]}{\partial \gamma_{ji}} \ge 0,$$

as

$$\frac{\partial E_\epsilon[\epsilon_i \Pr(D_i + \gamma_{ji}(D_j - Q_j\epsilon_j)^+ \le Q_i\epsilon_i)]}{\partial \gamma_{ji}} \le 0.$$

Together with

$$\frac{\partial^2 \pi_j^q(Q_j|Q_i,\mathbf{a})}{\partial Q_j \partial \gamma_{ji}} = 0 \le 0,$$

one can conclude that part (i) holds.

The distribution of $G_i(\cdot, a_i)$ in part (ii) implies

$$\frac{\partial G_i(x,a_i)}{\partial a_i} = \frac{\partial}{\partial a_i}\left(\frac{G_i^0(x) - G_i^0(a_i)}{G_i^0(1) - G_i^0(a_i)}\right) = -\frac{g_i^0(a_i)(G_i^0(1) - G_i^0(x))}{(G_i^0(1) - G_i^0(a_i))^2} \le 0$$

and

$$\frac{\partial g_i(x,a_i)}{\partial a_i} = \frac{\partial}{\partial a_i}(\frac{g_i^0(x)}{G_i^0(1) - G_i^0(a_i)}) \ge 0.$$

Then, we can check the cross partial derivatives as we did in the proof of part (i).

$$\frac{\partial^2 \pi_i^q(Q_i|Q_j,\mathbf{a})}{\partial Q_i \partial a_i} = (p_i - \delta_i c_i)\frac{\partial E_\epsilon[\epsilon_i]}{\partial a_i} - (p_i - s_i)\frac{\partial^2 E_{D,\epsilon}[(Q_i\epsilon_i - D_i^s)^+]}{\partial Q_i \partial a_i}$$
$$= \int_0^1\int_0^1 [(p_i - \delta_i c_i) - (p_i - s_i)F_{D_i^s|\epsilon_j}(Q_i\epsilon_i)]\epsilon_i \frac{\partial g_i(\epsilon_i, a_i)}{\partial a_i} d\epsilon_i dG_j(\epsilon_j, a_j).$$

As $\delta_i \le \frac{s_i}{c_i}$, we have $(p_i - \delta_i c_i) - (p_i - s_i)F_{D_i^s|\epsilon_j}(Q_i\epsilon_i) \ge (p_i - \delta_i c_i) - (p_i - s_i) \ge 0$. As $\partial g_i(\epsilon_i, a_i)/\partial a_i \ge 0$, we have $\partial^2\pi_i^q/\partial Q_i \partial a_i \ge 0$. Also,

$$\frac{\partial^2 \pi_j^q(Q_j|Q_i,\mathbf{a})}{\partial Q_j \partial a_i} = -(p_j - s_j)\frac{\partial^2 E_{D,\epsilon}[(Q_j\epsilon_j - D_j^s)^+]}{\partial Q_j \partial a_i} = -(p_j - s_j)\frac{\partial E_\epsilon[\epsilon_j F_{D_j^s|\epsilon_i}(Q_j\epsilon_j)]}{\partial a_i} \le 0.$$

The inequality holds as $\frac{\partial E_\epsilon[\epsilon_j F_{D_j^s|\epsilon_i}(Q_j\epsilon_j)]}{\partial a_i} \ge 0$, which can be checked by conditioning on whether D_i is entirely satisfied:

$$\frac{\partial E_\epsilon[\epsilon_j F_{D_j^s|\epsilon_i}(Q_j\epsilon_j)]}{\partial a_i} = \begin{cases} \frac{\partial E_\epsilon[\epsilon_j \Pr(D_j \le Q_j\epsilon_j)]}{\partial a_i} = 0 & \text{if } D_i \le Q_i\epsilon_i \\ \frac{\partial E_\epsilon[\epsilon_j \Pr(D_j + \gamma_{ij}D_i \le \gamma_{ij}Q_i\epsilon_i + Q_j\epsilon_j)]}{\partial a_i} \ge 0 & \text{if } D_i > Q_i\epsilon_i \end{cases}$$

Now, we are going to prove part (iii) using the Implicit Function Theorem. Let H be the Hessian matrix of the first-order conditions. One can verify that

$$|H| = \det\begin{vmatrix} \frac{\partial^2\pi_i^q}{\partial Q_i^2} & \frac{\partial^2\pi_i^q}{\partial Q_i \partial Q_j} \\ \frac{\partial^2\pi_j^q}{\partial Q_i \partial Q_j} & \frac{\partial^2\pi_j^q}{\partial Q_j^2} \end{vmatrix} > 0.$$

According to the Implicit Function Theorem, we have

$$\frac{\partial Q_i^*}{\partial a_i} = -\det\begin{vmatrix} \frac{\partial^2\pi_i^q}{\partial Q_i \partial a_i} & \frac{\partial^2\pi_i^q}{\partial Q_i \partial Q_j} \\ \frac{\partial^2\pi_j^q}{\partial Q_j \partial a_i} & \frac{\partial^2\pi_j^q}{\partial Q_j^2} \end{vmatrix} \Big/ |H|,$$

$$\frac{\partial Q_j^*}{\partial a_i} = -\det\begin{vmatrix} \frac{\partial^2\pi_i^q}{\partial Q_i \partial a_i} & \frac{\partial^2\pi_i^q}{\partial Q_i \partial Q_j} \\ \frac{\partial^2\pi_j^q}{\partial Q_j \partial a_i} & \frac{\partial^2\pi_j^q}{\partial Q_j^2} \end{vmatrix} \Big/ |H|.$$

Substituting the Bernoulli distribution into the second-order derivatives, we have

$$\det\begin{vmatrix} \frac{\partial^2\pi_i^q}{\partial Q_i \partial a_i} & \frac{\partial^2\pi_i^q}{\partial Q_i \partial Q_j} \\ \frac{\partial^2\pi_j^q}{\partial Q_j \partial a_i} & \frac{\partial^2\pi_j^q}{\partial Q_j^2} \end{vmatrix} = \underbrace{\frac{\partial^2\pi_i^q}{\partial Q_i \partial a_i}}_{+}\underbrace{\frac{\partial^2\pi_j^q}{\partial Q_j^2}}_{-} - \underbrace{\frac{\partial^2\pi_i^q}{\partial Q_i \partial Q_j}}_{-}\underbrace{\frac{\partial^2\pi_j^q}{\partial Q_j \partial a_i}}_{-} \le 0,$$

$$\det\begin{vmatrix} \frac{\partial^2\pi_i^q}{\partial Q_i^2} & \frac{\partial^2\pi_i^q}{\partial Q_i \partial a_i} \\ \frac{\partial^2\pi_j^q}{\partial Q_i \partial Q_j} & \frac{\partial^2\pi_j^q}{\partial Q_j \partial a_i} \end{vmatrix} = \underbrace{\frac{\partial^2\pi_i^q}{\partial Q_i^2}}_{-}\underbrace{\frac{\partial^2\pi_j^q}{\partial Q_j \partial a_i}}_{-} - \underbrace{\frac{\partial^2\pi_i^q}{\partial Q_i \partial a_i}}_{+}\underbrace{\frac{\partial^2\pi_j^q}{\partial Q_i \partial Q_j}}_{-} \ge 0.$$

Note that the condition $\delta_i \le \frac{s_i}{c_i}$ is required for $\partial^2\pi_i^q/\partial Q_i \partial a_i \ge 0$. Then, we have $\partial Q_i^*/\partial a_i \ge 0$ and $\partial Q_j^*/\partial a_i \le 0$. □

PROOF OF PROPOSITION 3: We prove this proposition by checking the total derivatives of π_i^q and π_j^q with respect to a_i.

$$\frac{\partial \pi_i^q(Q_i{}^*(\mathbf{a})|Q_j{}^*(\mathbf{a}),\mathbf{a})}{\partial a_i} = \frac{\partial \pi_i^q}{\partial Q_i^*}\frac{\partial Q_i^*}{\partial a_i} + \frac{\partial \pi_i^q}{\partial Q_j^*}\frac{\partial Q_j^*}{\partial a_i} + \frac{\partial \pi_i^q}{\partial a_i}.$$

The first term is equal to zero by the first-order optimality condition. For the second term, we have already shown $\partial Q_j^*/\partial a_i \le 0$ in Proposition 3. And

$$\frac{\partial \pi_i^q}{\partial Q_j}\bigg|_{Q_i = Q_i^*(\mathbf{a})} = -(p_i - s_i)\frac{\partial E_{D,\epsilon}[(Q_i^*\epsilon_i - D_i^s)^+]}{\partial Q_j} \le 0,$$

where the inequality holds as

$$\frac{\partial E_{D,\epsilon}[(Q_i^*\epsilon_i - D_i^s)^+]}{\partial Q_j} = \gamma_{ji}\int_0^1\int_0^1 \epsilon_j F_{D_i^s|D_j > Q_j\epsilon_j, \epsilon_j}(Q_i^*\epsilon_i) \times \Pr\left(D_j > Q_j\epsilon_j\right) dG_i(\epsilon_i, a_i) dG_j(\epsilon_j, a_j) \ge 0.$$

Then, we have shown that the second term is nonnegative. The remaining work is to show that the third term is also nonnegative.

$$\frac{\partial \pi_i^q}{\partial a_i}\bigg|_{\mathbf{Q}=\mathbf{Q}^*(\mathbf{a})} = (p_i - \delta_i c_i)Q_i^*\frac{\partial E[\epsilon_i]}{\partial a_i} - (p_i - s_i)\frac{\partial E[(Q_i^*\epsilon_i - D_i^s)^+]}{\partial a_i}$$
$$= \int_0^1\int_0^1 [(p_i - \delta_i c_i) - (p_i - s_i)F_{D_i^s(Q_j^*)|\epsilon_j}(Q_i^*\epsilon_i)] \times Q_i^*\left(-\frac{\partial G_i(\epsilon_i, a_i)}{\partial a_i}\right) d\epsilon_i dG_j(\epsilon_j, a_j).$$

The inside term is no less than zero if $\delta_i \le \frac{s_i}{c_i}$, so the third term is also nonnegative. Thus, $\pi_i^q(\mathbf{Q}^*(\mathbf{a}), \mathbf{a})$ is increasing in a_i.

Similarly, we have

$$\frac{\partial \pi_i^q(Q_i{}^*(\mathbf{a})|Q_j{}^*(\mathbf{a}),\mathbf{a})}{\partial a_j} = \frac{\partial \pi_i^q}{\partial Q_i^*}\frac{\partial Q_i^*}{\partial a_j} + \frac{\partial \pi_i^q}{\partial Q_j^*}\frac{\partial Q_j^*}{\partial a_j} + \frac{\partial \pi_i^q}{\partial a_j}.$$

The first term is still zero and the second term is negative as this time we have $\partial Q_j^*/\partial a_j \ge 0$. The third term is

$$\frac{\partial \pi_i^q}{\partial a_j}\bigg|_{\mathbf{Q}=\mathbf{Q}^*(\mathbf{a})} = -(p_i - s_i)\frac{\partial E[(Q_i^*\epsilon_i - D_i^s)^+]}{\partial a_j}$$
$$= -(p_i - s_i)\gamma_{ji}Q_j^*\int_0^1\int_0^1 F_{D_i^s|D_j > Q_j^*\epsilon_j, \epsilon_j}(Q_i^*\epsilon_i)\Pr\left(D_j > Q_j^*\epsilon_j\right) \times \left(-\frac{\partial G_j(\epsilon_j, a_j)}{\partial a_j}\right) d\epsilon_j dG_i(\epsilon_i, a_i) \le 0,$$

which completes the proof of part (i).

In part (ii), $\partial Q_i^*/\partial a_i \ge 0$ and $\partial Q_j^*/\partial a_i \le 0$ by Proposition 2. The remaining proof is analogous to that of part (i). □

PROOF OF THEOREM 2: We prove the theorem by analysing regions (I)–(V) one by one, which correspond to different possible equilibria. First, we consider the case when $\delta_i < 1$ and $\delta_j < 1$.

In region (I), $a_i < \psi_i$ and $a_i < \psi_i$, which is equivalent to $\phi_i(a_i) < 0$ and $\phi_j(a_j) < 0$. Thus, the best responses of both firms are zero, i.e., $Q_i^* = Q_j^* = 0$.

In region (II), we only show the equilibrium $(D_i + \gamma_{ji} D_j, 0)$, as the equilibrium $(0, D_j + \gamma_{ij} D_i)$ is symmetric. For the regions corresponding to $(D_i + \gamma_{ji} D_j, 0)$, we have $\phi_j(a_j) < 0$ ($\phi_j(a_j) < 0$ if and only if $a_j < \psi_j$), which implies $Q_j^* = 0$. And $\phi_i(a_i) > 0$ implies that the best response $Q_i^*(Q_j) = D_i + \gamma_{ji}(D_j - Q_j)^+$ or $D_i + \gamma_{ji} D_j$ [see Eq. (8)]. Yet given $Q_j^* = 0$, all these possibilities lead to $Q_i^* = D_i + \gamma_{ji} D_j$.

In region (III), we have $0 < \phi_i(a_i) \le a_j$ and $0 < \phi_j(a_j) \le a_i$. Then, according to Eq. (8), the best responses are $Q_i^*(Q_j) = D_i + \gamma_{ji}(D_j - Q_j)^+$ and $Q_j^*(Q_i) = D_j + \gamma_{ij}(D_i - Q_i)^+$. Solving the two equations yields the equilibrium solution $Q_i^* = D_i$ and $Q_j^* = D_j$.

Consider region (IV). We only show the equilibrium $(D_i + \gamma_{ji} D_j, D_j)$ as $(D_i, D_j + \gamma_{ij} D_i)$ follows by symmetry. $(D_i + \gamma_{ji} D_j, D_j)$ is achieved when $\phi_i(a_i) > a_j > \psi_j$, which implies the best response $Q_i^*(Q_j) = D_i + \gamma_{ji} D_j$. Also, in the region corresponding to $(D_i + \gamma_{ji} D_j, D_j)$, we have $0 < \phi_j(a_j) \le a_i$, which implies the best response $Q_j^*(Q_i) = D_j + \gamma_{ij}(D_i - Q_i^*)^+$. These two best response functions immediately give the equilibrium $(D_i + \gamma_{ji} D_j, D_j)$.

As for region(V), we have $\phi_i(a_i) > a_j$ and $\phi_j(a_j) > a_i$. The best response function directly gives the equilibrium, $Q_i^* = D_i + \gamma_{ji} D_j$ and $Q_j^* = D_j + \gamma_{ij} D_i$.

Next, we consider the case when $\delta_i = 1$ or $\delta_j = 1$. If both δ_i and δ_j are equal to one, then $\psi_i = \psi_j = 0$. As a result, the curves $\phi_i(a_i)$ and $\phi_j(a_j)$ becomes horizontal and vertical straight lines, respectively. Consequently, region (I) and region (II) shrink and disappear (see Fig. 5e). If only one of δ_i and δ_j is one, say $\delta_j = 1$, then $\psi_j = 0$ and $\phi_j(a_j)$ becomes a vertical line. This leaves two possibilities. When $\overline{\phi}_j > \psi_i$, region (V) shows up (Fig. 5f) and when $\overline{\phi}_j \le \psi_i$ there is no region (V) (Fig. 5g). □

PROOF OF THEOREM 3: Consider part (i) where $\overline{\phi}_i \le \psi_j$. Given $a_j \le \psi_j$, the first-stage equilibrium will be $(D_i + \gamma_{ji} D_j, 0)$ (region (II)). Plugging the equilibrium into π_i^r, the first-order condition yields $a_i(a_j) = a_i^{***}$. If $a_j > \psi_j$, there will be two possible equilibria in the second-stage game, (D_i, D_j) (region (III)) or $(D_i, D_j + \gamma_{ij} D_i)$ (region (IV)). But due to the operator $(D_j - Q_j)^+$, $Q_j^* = D_j$ or $D_j + \gamma_{ij} D_i$ has the same impact on the first-stage payoff function $\pi_i^r(a_i|a_j)$. So given $Q_i^* = D_i$, we have the best response $a_i(a_j) = a_i^*$ by the first-order condition.

Then, we consider part (ii). If $a_j > \overline{\phi}_i$ or $a_j \le \psi_j$, following the same logic as in part (i) we have $a_i(a_j) = a_i^*$ when $a_j > \overline{\phi}_i$ and $a_i(a_j) = a_i^{***}$ when $a_j \le \psi_j$. We then focus on $\psi_j < a_j \le \overline{\phi}_i$.

We first show that $a_i^* \le a_i^{**}(a_j) \le a_i^{***}$ for $\psi_j < a_j \le \overline{\phi}_i$. Note that $a_i^{**}(a_j)$ is (weakly) decreasing in a_j, as the corresponding marginal gain for improving reliability $\frac{\partial \pi_i^q}{\partial a_i}$ is $(p_i - \delta_i c_i)D_i + ((p_i - \delta_i c_i) - (p_i - s_i)a_j)\gamma_{ji} D_j$, which decreases with a_j. On the one hand, we have $a_i^{**}(a_j) \ge a_i^{**}(\overline{\phi}_i)$. For $a_j = \overline{\phi}_i$, the marginal gain $\frac{\partial \pi_i^q}{\partial a_i} = (p_i - \delta_i c_i)D_i + (1 - \delta_i)c_i \gamma_{ji} D_j) \ge (p_i - \delta_i c_i)D_i$. Recall that the marginal gain $(p_i - \delta_i c_i)D_i$, which is smaller, induces the optimal solution a_i^*. As a result, $a_i^{**}(a_j) \ge a_i^{**}(\overline{\phi}_i) \ge a_i^*$. On the other hand, we have $a_i^{**}(a_j) \le a_i^{**}(\psi_j) \le a_i^{**}(0) = a_i^{***}$. The last equality follows as the marginal gain for $a_j = 0$ is given by $(p_i - \delta_i c_i)(D_i + \gamma_{ji} D_j)$, which induces the optimal solution a_i^{***}.

Let $\hat{a}_i(a_j) = \phi_i^{-1}(a_j)$, the inverse function of curve $\phi_i(a_i)$. By definition, a_j^B is the value such that $a_i^{**}(a_j) = \hat{a}_i(a_j)$. Now, we show the following lemma first. □

LEMMA A.1: There is a unique a_j^B such that $a_i^{**}(a_j^B) = \hat{a}_i(a_j^B)$, and $a_j^A \le a_j^B \le \overline{\phi}_i$.

PROOF: Note that $\hat{a}_i(a_j)$ is increasing in a_j and $a_i^{**}(a_j)$ is decreasing in a_j. Because a_j^A is defined to be $\phi_i(a_i^*)$, we have, at the point $a_j^A, \hat{a}_i(a_j^A) = a_i^* < a_i^{**}(a_j^A)$. At the point $a_j = \overline{\phi}_i$, we have $\hat{a}_i(\overline{\phi}_i) = 1 \ge a_i^{**}(\overline{\phi}_i)$. Thus, there is a unique threshold value a_j^B between a_j^A and $\overline{\phi}_i$. □

The interval $\psi_j < a_j < \overline{\phi}_i$ can then be divided into at most three subintervals by the two threshold values a_j^A and a_j^B. In all these subintervals, we have $Q_j^* = D_j$ or $D_j + \gamma_{ij} D_i$, which has no effect on the first-stage payoff function π_i^r due to the "positive" operator $(D_j - Q_j)^+$. Thus, the best response of firm i only relies on Q_i^*. Recall that a_i^* is optimal for $Q_i^* = D_i$ and $a_i^{**}(a_j)$ is optimal for $Q_i^* = D_i + \gamma_{ji} D_j$. We can then suppress the arguments Q_j and a_j in the payoff functions for ease of notation.

First, consider $a_j^B < a_j \le \overline{\phi}_i$, which implies $a_i^* \le a_i^{**}(a_j) \le \hat{a}_i$. For $a_i < \hat{a}_i$, $Q_i^* = D_i$. So the interior solution a_i^* associated with $Q_i^* = D_i$ can be obtained. For $a_i \ge \hat{a}_i$, $Q_i^* = D_i + \gamma_{ji} D_j$. The interior solution $a_i^{**}(a_j)$ associated with $Q_i^* = D_i + \gamma_{ji} D_j$ cannot be obtained as $a_i^{**}(a_j) \le \hat{a}_i$. Firm i should compare the quantity equilibria: $Q_i^* = D_i$ induced by a_i^*, and $Q_i^* = D_i + \gamma_{ji} D_j$ induced by $\hat{a}_i$ ($\hat{a}_i$ is the best choice to induce $Q_i^* = D_i + \gamma_{ji} D_j$ as $a_i^{**}(a_j) \le \hat{a}_i$). The first-stage payoff function can be written as

$$\pi_i^r(a_i) = \max\{\pi_i^q(D_i + \gamma_{ji} D_j, \hat{a}_i) - I_i(\hat{a}_i), \pi_i^q(D_i, a^*) - I_i(a_i^*)\}.$$

Note that $\pi_i^q(D_i + \gamma_{ji} D_j, \hat{a}_i) = \pi_i^q(D_i, \hat{a}_i)$ as $\hat{a}_i$ is at the curve $\phi_i(a_i) = a_j$ where it is indifferent to order D_i or $D_i + \gamma_{ji} D_j$. Thus, the above maximum operator becomes a comparison between $\pi_i^r(\hat{a}_i | Q_i^* = D_i)$ and $\pi_i^r(a_i^* | Q_i^* = D_i)$. a_i^* is chosen because it is the global optimal point given $Q_i = D_i$.

Second, consider the subinterval $a_j^A < a_j \le a_j^B$, where $a_i^* \le \hat{a}_i \le a_i^{**}(a_j)$. In this case, both interior optimal solutions a_i^* and $a_i^{**}(a_j)$ can be obtained. Hence, firm i need to compare these solutions which induce the equilibrium quantities $Q_i^* = D_i$ and $Q_i^* = D_i + \gamma_{ji} D_j$, respectively. The best response a_i is then chosen by comparing $\pi_i^r(a_i^* | Q_i^* = D_i)$ and $\pi_i^r(a_i^{**}(a_j) | Q_i^* = D_i + \gamma_{ji} D_j)$.

Third, if $\psi_j < a_j \le a_j^A$ we have $\hat{a}_i \le a_i^* \le a_i^{**}(a_j)$. Following a similar logic to the first subinterval, one can show that $a_i^{**}(a_j)$ is the best response. The equilibrium quantity $Q_i^* = D_i$ is induced if $a_i < \hat{a}_i$. However, in the third subinterval, $a_i^* \ge \hat{a}_i$. This implies that the interior solution a_i^* that is associated with $Q_i^* = D_i$ cannot be obtained. Thus, the best choice to induce $Q_i^* = D_i$ is $\hat{a}_i$. Following the same logic as that regarding the first subinterval, we can show that choosing $a_i^{**}(a_j)$ to induce $Q_i^* = D_i + \gamma_{ji} D_j$ is better than choosing $\hat{a}_i$. □

PROOF OF THEOREM 4: Recall that $a_i^* \le a_i^{**}(a_j) \le a_i^{***}$ and $a_i^{**}(a_j)$ is decreasing in a_j. So according to the best response function characterized in Theorem 3, the best response function is (weakly) decreasing in the whole strategy space if it is (weakly) decreasing for $a_j^A < a_j \le a_j^B$. It suffices to show within the interval $a_j^A < a_j \le a_j^B$ that either there is a unique threshold above which a_i^* is preferred and below which $a_i^{**}(a_j)$ is preferred, or one of a_i^* and $a_i^{**}(a_j)$ is dominant within the whole interval. To do so, we only need to show that $\pi_i^r(a_i^{**}(a_j)|a_j)$ and $\pi_i^r(a_i^*|a_j)$ have at most one intersection within the interval $a_j^A < a_j \le a_j^B$.

Given $a_j^A < a_j \le a_j^B$, invoking the second-stage equilibrium corresponding to a_i^* (see Table 2), $Q_i^*(a_i^*, a_j) = D_i$ and $Q_j^*(a_i^*, a_j) = D_j$ or $D_j + \gamma_{ij} D_i$ in (6), we have

$$\pi_i^r(a_i^*|a_j) = ((p_i - \delta_i c_i)a_i^* - (1 - \delta_i)c_i)D_i - I_i(a_i^*),$$

which is constant in a_j.

Next, it suffices to show that $\pi_i^r(a_i^{**}(a_j)|a_j)$ is nonincreasing in a_j. $\pi_i^r(a_i^{**}(a_j)|a_j)$ can be found by invoking the corresponding quantity equilibria $Q_i^*(a_i^{**}(a_j), a_j) = D_i + \gamma_{ji} D_j$ and $Q_j^*(a_i^{**}(a_j), a_j) = D_j$ or $D_j + \gamma_{ij} D_i$

in (6). Applying the envelope theorem, we have

$$\frac{\partial \pi_i^r(a_i^{**}(a_j)|a_j)}{\partial a_j} = \left.\frac{\partial \pi_i^r(a_i|a_j)}{\partial a_j}\right|_{a_i=a_i^{**}(a_j)}$$
$$= -(p_i - s_i)a_i^{**}(a_j)\gamma_{ji}D_j \le 0.$$

Therefore, the best response of firm i is nonincreasing in firm j's strategy. That is, the reliability game is submodular.

The remainder is an immediate implication of submoduarity (See Ref. [23]). □

PROOF OF PROPOSITION 4: According to the best response function (11), given that a_j^0 is already higher than a_j^B, no matter which reliability level firm j chooses to improve, firm i will choose a_i^*. That is, a_i^* is a dominant strategy. So if $a_j^0 > a_j^B$ for both firm $j=1$, 2, the equilibrium (a_1^*, a_2^*) is unique. □

PROOF OF PROPOSITION 5: It immediately follows by Theorem 2 and Theorem 3. □

PROOF OF PROPOSITION 6: It suffices to show that π_i^r is supermodular in (a_i, γ_{ji}) and π_j^r is submodular in (a_j, γ_{ji}). The latter is trivially satisfied as π_j^r is actually independent of γ_{ji}. Now consider the former, that is, whether π_i^r is supermodular in (a_i, γ_{ji}). Let $\hat{a}_i > a_i$ and $\hat{\gamma}_{ji} > \gamma_{ji}$. We check the increasing differences of $\pi_i^r(a_i, \gamma_{ji})$. For any $a_i \le \psi_i, \pi_i^r(a_i, \gamma_{ji}) = 0$. We can then merely focus on the case $a_i > \psi_i$.

First, if $a_j \le \psi_j$, $Q_j^*(a_i, a_j) = 0$ and $Q_i^*(a_i, a_j) = D_i + \gamma_{ji}D_j$. Then,

$$\begin{aligned}\pi_i^r(\hat{a}_i, \hat{\gamma}_{ji}) - \pi_i^r(\hat{a}_i, \gamma_{ji}) &= ((p_i - \delta_i c_i)\hat{a}_i - (1-\delta_i)c_i)D_j(\hat{\gamma}_{ji} - \gamma_{ji})\\ &\ge ((p_i - \delta_i c_i)a_i - (1-\delta_i)c_i)D_j(\hat{\gamma}_{ji} - \gamma_{ji})\\ &= \pi_i^r(a_i, \hat{\gamma}_{ji}) - \pi_i^r(a_i, \gamma_{ji}).\end{aligned}$$

Second, if $\psi_j < a_j < \phi_i(a_i) < \phi_i(\hat{a}_i)$ (note that $\phi_i(a_i) < \phi_i(\hat{a}_i)$ as $\phi_i(x)$ is an increasing function), $Q_i^*(\hat{a}_i, a_j) = D_i + \gamma_{ji}D_j$ and $Q_j^*(\hat{a}_i, a_j) = D_j + \gamma_{ij}D_i$ or D_j, which leads to

$$\begin{aligned}\pi_i^r(\hat{a}_i, \hat{\gamma}_{ji}) - \pi_i^r(\hat{a}_i, \gamma_{ji}) &= (((p_i - \delta_i c_i) - (p_i - s_i)a_j)\hat{a}_i\\ &\quad -(1-\delta_i)c_i)D_j(\hat{\gamma}_{ji} - \gamma_{ji})\\ &\ge (((p_i - \delta_i c_i) - (p_i - s_i)a_j)a_i\\ &\quad -(1-\delta_i)c_i)D_j(\hat{\gamma}_{ji} - \gamma_{ji})\\ &= \pi_i^r(a_i, \hat{\gamma}_{ji}) - \pi_i^r(a_i, \gamma_{ji}),\end{aligned}$$

where the inequality holds because $(p_i - \delta_i c_i) - (p_i - s_i)a_j > 0$ by $a_j < \phi_i(\hat{a}_i)$.

Third, for $\phi_i(a_i) \le a_j < \phi_i(\hat{a}_i)$ we have

$$\begin{aligned}\pi_i^r(\hat{a}_i, \hat{\gamma}_{ji}) - \pi_i^r(\hat{a}_i, \gamma_{ji}) &= (((p_i - \delta_i c_i) - (p_i - s_i)a_j)\hat{a}_i\\ &\quad -(1-\delta_i)c_i)D_j(\hat{\gamma}_{ji} - \gamma_{ji})\\ &\ge 0 = \pi_i^r(a_i, \hat{\gamma}_{ji}) - \pi_i^r(a_i, \gamma_{ji}),\end{aligned}$$

where the inequality holds because $a_j < \phi_i(\hat{a}_i)$. The last equality follows as $\phi_i(a_i) \le a_j$ implies $Q_i^*(a_i, a_j) = D_i$ and substituting this into $\pi_i^r(a_i, \gamma_{ji})$ gives $\pi_i^r(a_i, \gamma_{ji})$ independent of γ_{ji}, which implies $\pi_i^r(a_i, \hat{\gamma}_{ji}) - \pi_i^r(a_i, \gamma_{ji}) = 0$.

Finally, if $a_j \ge \phi_i(\hat{a}_i)$, then $Q_i^*(\hat{a}_i, a_j) = D_i$. We have $\pi_i^r(\hat{a}_i, \hat{\gamma}_{ji}) - \pi_i^r(\hat{a}_i, \gamma_{ji}) = 0$ and this difference is also zero for any $a_i < \hat{a}_i$, so the increasing differences trivially follow.

Therefore, π_i^r is supermodular in (a_i, γ_{ji}). □

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