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Afriat in the lab ☆

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Abstract

Varian (1988) showed that the utility maximization hypothesis cannot be falsified when only a subset of goods is observed. We show that this result does not hold under the assumptions that unobserved prices and expenditures remain constant. These assumptions are naturally satisfied in laboratory settings where the world outside the lab remains unchanged during the experiment. Hence for so-called induced budget experiments the Generalized Axiom of Revealed Preference is a necessary and sufficient condition for utility maximization in general, not just over lab goods. Lab experiments are therefore a valid tool to put the utility maximization hypothesis to the test.

JEL classification: C14; C91; D11; D12

Keywords: Afriat's Theorem; Experimental economics; GARP; Revealed preference; Utility maximization

1. Introduction

In the past twenty years, laboratory experiments have become an important tool for economists to test theories and elicit preferences. Induced budget experiments, in which subjects are asked to make choices from budgets provided by the experimenter, make particular use

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of the opportunity to collect data that is otherwise difficult to come by.[1] Such experiments have become increasingly popular.[2]

Choices on such budgets can be tested for consistency with the Generalized Axiom of Revealed Preference (GARP), which is a necessary and sufficient condition for the existence of a utility function that rationalizes the observed choices (Afriat, 1967; Varian, 1982). Choices on budgets with many different prices collected under clean laboratory conditions provide well-suited data for this test. Experiments therefore seem to offer a unique opportunity to put the utility maximization hypothesis to the test as observing a violation of GARP falsifies the hypothesis.

However, testing a data set for consistency with GARP only characterizes utility maximization when the demand for all available goods is observed. Varian (1988) shows that if we only observe demand for a subset of goods, then GARP is no longer necessary. In his conclusion, Varian (1988) calls his finding "a negative result, similar in spirit to the Sonnenschein–Mantel–Debreu results" (p. 184) and laments "[t]he sad fact" that unless the entire demand is observed, the utility maximization hypothesis imposes no restrictions on observable data. Based on the same result, Cox (1997) argues that if only demand data on a subset of goods is available, tests "cannot discriminate between inconsistencies with the utility hypothesis and inconsistencies with weak separability" (p. 1055).

Clearly even the best laboratory experiments can only include a subset of the set of goods available to subjects before, during, and after the experiment. It therefore seems necessary to include the caveat that the analysis of experimental data is only about a sub-utility function for goods in the lab. However, we will show that this is not the case: Our theorem shows that consistency of the observed data with GARP is still a necessary and sufficient condition for utility maximization over all (observed and unobserved) goods if unobserved prices and expenditure remain constant. In particular, these conditions are naturally satisfied in the lab, as the world outside the lab typically remains unchanged during the course of an experiment. Thus, consistency with GARP of the choice set collected in the lab or under similar conditions is still a necessary and sufficient condition for the maximization of a utility function over all goods, and the utility maximization hypothesis can be falsified using laboratory experiments.

2. Testing utility maximization with subsets of goods

Let $\mathbb{R}^k_+$ be the *consumption space*, where $k \geq 2$ is the number of different goods. A decision maker *demands* a bundle of goods $\mathbf{x}^i \in \mathbb{R}^k_+$ when facing the *price vector* $\mathbf{p}^i \in \mathbb{R}^k_{++}$ such that *expenditure* equals $\mathbf{p}^i\mathbf{x}^i$. We then say that $(\mathbf{x}^i, \mathbf{p}^i)$ constitutes one *observation*, although we will later assume that we do not necessarily observe all parts of $\mathbf{x}^i$ and $\mathbf{p}^i$. We assume that we have N observations, and the entire set of observations is denoted by $\Omega = \{(\mathbf{p}^i, \mathbf{x}^i)\}_{i=1}^N$.

An observation $\mathbf{x}^i$ is *directly revealed preferred* to $\mathbf{x}$, written $\mathbf{x}^i \, \mathrm{R}^0 \, \mathbf{x}$, if $\mathbf{p}^i\mathbf{x}^i \geq \mathbf{p}^i\mathbf{x}$. It is *revealed preferred* to $\mathbf{x}$, written $\mathbf{x}^i \, \mathrm{R} \, \mathbf{x}$, if $\mathbf{x}^i \, \mathrm{R}^0 \, \mathbf{x}^a$, $\mathbf{x}^a \, \mathrm{R}^0 \, \mathbf{x}^b, \ldots, \mathbf{x}^c \, \mathrm{R}^0 \, \mathbf{x}$; in that case, R is called the *transitive closure* of R^0. It is *strictly directly revealed preferred* to $\mathbf{x}$, written $\mathbf{x}^i \, \mathrm{P}^0 \, \mathbf{x}$, if $\mathbf{p}^i\mathbf{x}^i > \mathbf{p}^i\mathbf{x}$.

[1] To the best of the authors' knowledge, the term 'induced budget experiment' was introduced by Banerjee and Murphy (2011) "[t]o contrast them from *induced value* experiments, i.e. those in which demand and supply are determined by the experimenter and the object of interest is the performance of an allocation mechanism" (p. 3864).

[2] Examples include Sippel (1997), Harbaugh and Krause (2000), Mattei (2000), Andreoni and Miller (2002), Février and Visser (2004), Fisman et al. (2007), Choi et al. (2007), Banerjee and Murphy (2011), Dawes et al. (2011), Visser and Roelofs (2011), Bruyneel et al. (2012), Becker et al. (2013), Burghart et al. (2013), Ahn et al. (2014), and Choi et al. (2014).

A utility function $u : \mathbb{R}^k_+ \to \mathbb{R}$ *rationalizes* Ω if $u(\mathbf{x}^i) \geq u(\mathbf{x})$ whenever $\mathbf{x}^i \,\mathrm{R}\, \mathbf{x}$. The set Ω satisfies the *Generalized Axiom of Revealed Preference* (GARP) if $\mathbf{x}^i \,\mathrm{R}\, \mathbf{x}^j$ implies $\left[\text{not } \mathbf{x}^j \,\mathrm{P}^0\, \mathbf{x}^i\right]$ for all $i, j \in \{1, \ldots, N\}$. GARP completely characterizes the utility maximization hypothesis, as Afriat's Theorem shows.

Afriat's Theorem *(Afriat, 1967; Diewert, 1973; Varian, 1982). The following conditions are equivalent:*

1. *The set of observations Ω satisfies GARP.*
2. *There exists a non-satiated utility function that rationalizes Ω.*
3. *There exists a continuous, monotonic, and concave utility function that rationalizes Ω.*

However, Varian (1988) found that if demand for even just one good is not observed, GARP loses all bite. To state this formally, let us partition the set of goods and the set of prices into two sets each, with the first subsets consisting of $\ell \geq 1$ goods and prices, respectively, and the second subsets consisting of $m \geq 1$ goods and prices, respectively, with $\ell + m = k$. For the goods, let

$$\mathbf{y}^i = (y^i_1, \ldots, y^i_\ell),$$
$$\mathbf{z}^i = (z^i_1, \ldots, z^i_m),$$
$$\mathbf{x}^i = (y^i_1, \ldots, y^i_\ell, z^i_1, \ldots, z^i_m),$$

and for the prices, let

$$\mathbf{q}^i = (q^i_1, \ldots, q^i_\ell),$$
$$\mathbf{r}^i = (r^i_1, \ldots, r^i_m),$$
$$\mathbf{p}^i = (q^i_1, \ldots, q^i_\ell, r^i_1, \ldots, r^i_m).$$

From now on, $\mathbf{y}^i$ and $\mathbf{q}^i$ will be observed demand and prices, while $\mathbf{z}^i$ and $\mathbf{r}^i$ may or may not be observed. Let $\Omega_O = \{(\mathbf{q}^i, \mathbf{y}^i)\}_{i=1}^N$. We define GARP for Ω_O similarly to GARP for Ω.

Theorem 1 *(Varian, 1988). Suppose we observe Ω_O and $\{\mathbf{r}^i\}_{i=1}^N$ but not $\{\mathbf{z}^i\}_{i=1}^N$. Then we can always find $\{\mathbf{z}^i\}_{i=1}^N$ such that Ω satisfies GARP regardless of whether or not Ω_O satisfies GARP.*

Varian's (1988) proof of Theorem 1 was incomplete; recently van Bruggen (2016) provided a new proof. Note that Theorem 1, as well as Theorem 2 below, are slightly more general versions of the ones stated by Varian (1988) who formulates the results in terms of a single unobserved commodity (i.e., $m = 1$). The versions here follow from simple extensions of Varian's (1988) proof.

Suppose demand for all goods is observed but the prices for some of the goods are unobserved. In that case, GARP only maintains its bite for subsets of the data where demand is the same for all goods with unknown prices, as the next theorem shows. This condition is very strong; it seems fairly implausible that a researcher would observe demand without observing prices and that this demand remains constant. In any case, researchers will typically not know in advance whether demand will be constant and can therefore not rely on it.

Theorem 2 *(Varian, 1988). Suppose we observe Ω_O and $\{\mathbf{z}^i\}_{i=1}^N$ but not $\{\mathbf{r}^i\}_{i=1}^N$. For every subset $\mathcal{I}$ of indices $\{1, \ldots, N\}$ such that $\mathbf{z}^i = \mathbf{z}^j$ for all $i, j \in \mathcal{I}$, $\{(\mathbf{p}^i, \mathbf{x}^i)\}_{i \in \mathcal{I}}$ satisfies GARP if and only* *if* $\{(\mathbf{q}^i, \mathbf{y}^i)\}_{i\in\mathcal{I}}$ *satisfies GARP. For every* $\mathcal{J} \subseteq \{1, \dots, N\}$ *such that* $\mathbf{z}^i \neq \mathbf{z}^j$ *for all* $i \neq j$, $i, j \in \mathcal{J}$, *we can always find* $\{\mathbf{r}^i\}_{i\in\mathcal{J}}$ *such that* $\{(\mathbf{p}^i, \mathbf{x}^i)\}_{i\in\mathcal{J}}$ *satisfies GARP regardless of whether or not* $\{(\mathbf{q}^i, \mathbf{y}^i)\}_{i\in\mathcal{J}}$ *satisfies GARP.*

In what follows, we assume that unobserved prices and unobserved expenditure are the same across observations, while allowing for unobserved demand to change. Our theorem shows that these assumptions restore the power of GARP.

Theorem 3. *Suppose we only observe* Ω_O*, and that* $\mathbf{r}^i = \mathbf{r}^j = \mathbf{r}$ *and* $\mathbf{r}\mathbf{z}^i = \mathbf{r}\mathbf{z}^j$ *for all* $i, j \in \{1, \dots, N\}$*. Then* Ω *satisfies GARP if and only if* Ω_O *satisfies GARP.*

Proof of Theorem 3. Let R_y^0 be the directly revealed preference relation on $\mathbb{R}_+^\ell \times \mathbb{R}_+^\ell$ constructed using Ω_O, that is, $\mathbf{y}^i \,\mathrm{R}_y^0\, \mathbf{y}^j$ if $\mathbf{q}^i\mathbf{y}^i \geq \mathbf{q}^i\mathbf{y}^j$, and let R_y be the transitive closure of R_y^0. Let P_y^0 be the corresponding strictly directly revealed preference relation, that is, $\mathbf{y}^i \,\mathrm{P}_y^0\, \mathbf{y}^j$ if $\mathbf{q}^i\mathbf{y}^i > \mathbf{q}^i\mathbf{y}^j$. We have that $\mathbf{x}^i \,\mathrm{R}^0\, \mathbf{x}^j$ if

$$\begin{aligned} &\mathbf{p}^i\mathbf{x}^i \geq \mathbf{p}^i\mathbf{x}^j \\ \Leftrightarrow\ &\mathbf{q}^i\mathbf{y}^i + \mathbf{r}\mathbf{z}^i \geq \mathbf{q}^i\mathbf{y}^j + \mathbf{r}\mathbf{z}^j, \end{aligned}$$

and with $\mathbf{r}\mathbf{z}^i = \mathbf{r}\mathbf{z}^j$ we obtain $\mathbf{q}^i\mathbf{y}^i \geq \mathbf{q}^i\mathbf{y}^j$ which is the condition for $\mathbf{y}^i \,\mathrm{R}_y^0\, \mathbf{y}^j$. Thus, $\mathbf{x}^i \,\mathrm{R}^0\, \mathbf{x}^j$ if and only if $\mathbf{y}^i \,\mathrm{R}_y^0\, \mathbf{y}^j$, and similarly, $\mathbf{x}^i \,\mathrm{P}^0\, \mathbf{x}^j$ if and only if $\mathbf{y}^i \,\mathrm{P}_y^0\, \mathbf{y}^j$. Then a violation of GARP based on R and P^0 (i.e., Ω violates GARP) implies a violation of GARP based on R_y and P_y^0 (i.e., Ω_O violates GARP) and vice versa. Thus, [Ω violates GARP] $\Leftrightarrow$ [Ω_O violates GARP]. □

Our assumptions on unobserved prices and expenditures are typically satisfied in laboratory experiments. For all practical purposes, the world outside the lab remains unchanged during the course of an experiment. It is therefore reasonable to assume that prices for goods outside the lab remain constant. Furthermore, even if subjects plan to buy different bundles of goods outside the lab depending on which lab budget is implemented, their choices in the lab do not influence unobserved *expenditure* outside the lab.

To have multiple observations we also need to assume that subjects choose bundles from each budget separately instead of making one choice on an aggregated budget. If subjects are expected utility maximizers, a random lottery incentive mechanism guarantees this. Empirically, Hey and Lee (2005) found generally reassuring evidence suggesting that subjects do indeed make each choice “as if it were a separate question—in isolation from all the other questions in the experiment” (p. 233).

Finally, note that if subjects can take money with them from the lab, we know exactly how much it is and can therefore account for it. Ultimately, the crucial point of Theorem 3 is not that expenditure on unobserved demand is constant, but that the *unobserved* component is constant.

3. Conclusion

Much of the recent revitalization of and increased interest in revealed preference theory appears to be the consequence of the new tools offered by experimental economics. Indeed, we find that there are good reasons to be optimistic about applying revealed preference theory to experimental data. While it remains lamentable that we can technically never falsify utility maximization with typical household demand data, the problem is ameliorated for experimental data.

Laboratory experiments are therefore a uniquely powerful tool to test the hypothesis of utility maximization.

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