Rolling stock rescheduling in passenger railway transportation using dead-heading trips and adjusted passenger demand

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A B S T R A C T

In this paper we introduce dead-heading trips and adjusted passenger demand in the Rolling Stock Rescheduling Problem (RSRP). Unfortunately, disruptions disturb passenger railway transportation on a daily basis. Such a disruption causes infeasibilities in the timetable, rolling stock circulation, and crew schedule. We propose a Mixed-Integer Linear Programming model to tackle the RSRP. This formulation includes the possibility of using dead-heading trips (moving empty trains) during, and after, a disruption. Furthermore, passenger flows are included to handle the adjusted passenger demand after the occurrence of a disruption. Many rolling stock rescheduling models are unable to cope with changing passenger demand. In this paper we include passenger demand on a more accurate level in the RSRP. We have tested the model on different cases from Netherlands Railways. The results show that dead-heading trips are useful to reduce the number of cancelled trips and that adjusted passenger demand has a large influence on the rescheduled circulation.

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1. Introduction and contributions

1.1. Introduction

The main focus of Netherlands Railways (NS) is to provide a good passenger service. Therefore, NS is constantly focusing on improving the quality of its services. An important measure for this quality is the ability to react to unforeseen events occurring during the day. Two kinds of unforeseen events are of interest to railway operators: disruptions and disturbances. A disruption causes the planned timetable, rolling stock circulation, and crew schedule to be infeasible. During a disturbance, however, a delay is either absorbed by the slack in the system or by rescheduling only the timetable: the rolling stock and crew schedule can absorb the disturbance and do not have to be rescheduled. The focus of this paper is on dealing with the first type of unforeseen events: disruptions.

All planned resource schedules (timetable, rolling stock circulation, and crew schedule) have to be adapted as soon as a disruption occurs in order to secure their feasibility. In practice, the first step is to update the original timetable. In the Netherlands, more than a thousand different, so called, contingency plans exist to update the timetable. These contingency plans contain a number of rules stating which trains have to be cancelled, rerouted, or delayed in case of a specific disruption. After the end of a disruption, railway operators usually want to have their timetable to be as much alike the original

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timetable as possible. This is already taken into account in the prescheduled contingency plans. Secondly, based on the new timetable, the original rolling stock circulation has to be rescheduled. Finally, with the new timetable and the new rolling stock schedule as input, the crew schedule must be modified.

In this paper we focus on the second step: rescheduling the planned rolling stock circulation given the rescheduled timetable as input. There already exist models for the Rolling Stock Rescheduling Problem (RSRP), see for instance Nielsen (2011); Nielsen et al. (2012); Haahr et al. (2014); and Sato et al. (2009). These models are currently not applied in practice yet. One of the reasons is that not all details of the real world are taken into account. In this paper we introduce an extension of the RSRP model which includes two of these practical details: dead-heading trips and adjusted passenger demand.

Due to the disruption, it may be that certain stations have a surplus of rolling stock units while other stations have a shortage of rolling stock units to execute the updated timetable. A shortage of rolling stock units can lead to additional cancelled trains, because there is no rolling stock unit available for at least one of the trips in the updated timetable. Furthermore, a shortage of rolling stock units possibly leads to appointing trains with too little capacity for the passenger demand to a trip. That is why NS has the possibility to schedule empty trains, called dead-heading trips, from one station to another to increase the local inventory during a disruption. By using dead-heading trips, NS wants to decrease the number of cancelled trains and to increase the customer satisfaction during and after a disruption. In this paper we adapt the current RSRP models, by adding the possibility to use dead-heading trips during a disruption.

The major objective of the new rolling stock circulation is to uphold a good passenger service. In other words, to cancel as few trains as possible and to use trains with enough capacity for all passenger demand. Most of the current rescheduling models assume passenger demand to be static. However, passenger demand depends on the appointed rolling stock units to trips (e.g. cancelling a trip leads to a demand increase on the next trip with the same origin and destination). In the Netherlands, more information about passengers is currently available due to smart card data. With this data, we were able to identify the incoming and outgoing demand on a trip in the undisrupted situation. These are defined as the number of passengers that enter the railway system at the start of a trip and the number of passenger that leave the railway system after a trip. With the incoming and outgoing demand we are able to take adjusted passenger demand into account in the RSRP.

1.2. Contributions and structure of the paper

The contributions of this paper can be summarized as follows. First of all, this paper is, to the best of our knowledge, the first to include dead-heading trips in a formulation to tackle the rolling stock rescheduling problem. By including dead-heading trips, possibly less additional trains get cancelled and passenger satisfaction will increase. Secondly, an efficient preprocessing method is applied to select potential dead-heading trips from the complete set of possible dead-heading trips. In this way, dead-heading trips can adequately be included in the rolling stock rescheduling formulation. Thirdly, to the best of our knowledge, this paper is the first to include dynamic passenger flows directly in the formulation for rolling stock rescheduling. Finally, different boarding strategies for passengers in case the appointed capacity to a trip is less than the actual passenger demand are proposed and formulated.

Three important assumptions are taken into account while designing the models:

1. The order of train units within a composition is of importance (e.g. the composition ab differs from the composition ba).
2. The incoming passenger demand does not change due to a disruption.
3. Passengers do not leave the railway system prematurely and do not take a detour to their destination during a disruption.

The first assumption is of importance, because it is defined at which side of an incoming train rolling stock units may be (un)coupled. As a consequence, we need to keep track of which unit is in front of the incoming train and which unit resides at the back of the incoming train. The second assumption is used, because there is no information available about the change in the incoming demand due to the disruption. This assumption can be relaxed as soon as this information becomes available. The third is used, because the objective of using adjusted passenger demand is to appoint rolling stock compositions with a large capacity to trips where the actual passenger demand is large. The capacities of the compositions differ significantly (e.g. the smallest composition has capacity for 405 passengers and the largest compositions for 1407 passengers). As a consequence, it is not important to predict the passenger demand precisely, but to be near the exact number.

The remainder of this paper is structured as follows: in the next section a literature overview is given. In Section 3 the models used to solve the RSRP are shown and discussed. In Section 4 the model is tested on real life instances of NS. Finally, in Section 5 conclusions and remarks on further research are given.

2. Literature review

Cacchiani et al. (2014) give an overview of related articles on disruption management in general. We refer to them for the reader interested in papers for timetable and crew rescheduling. In this paper we focus on rolling stock rescheduling. So, the remainder of the papers discussed are on (re)scheduling the rolling stock.
2.1. Scheduling

Fioole et al. (2006) formulated a model to assign rolling stock to the timetable in the scheduling phase. The model is able to handle the order of rolling stock units within compositions. Furthermore, it is able to handle complicated line structures, such as combining and splitting of trains. NS uses this model to generate the rolling stock schedules since 2004.

Cacchiani et al. (2012) investigate the problem of designing an original rolling stock circulation in such a way that the recoverability of the circulation in case of large disruptions is most effective. This model is an extension of the model used by Fioole et al. (2006). They include different disruption scenarios in their model and minimize the total number of cancelled trips, the additional shunting operations, and the deviations from the end-of-day rolling stock balances for all reallocation plans. Next to those targets, their objective focuses on the number of seat shortages, carriage kilometers, and the complexity and risk of shunting operations as well. Their model is tested on the 3000 line of NS between Den Helder and Nijmegen. They simulate different disruption scenarios and show that in their robust solution less trains have to be cancelled and the recovery costs are lower than in the originally planned schedule.

Lingaya et al. (2002) studies the problem of scheduling locomotives and carriages. They describe a train as a Last-In-First-Out (LIFO) stack, where carriages can be coupled or uncoupled from the rear part of the train in LIFO order only.

Borndörfer et al. (2015) uses a hypergraph formulation to come up with a rolling stock circulation where certain practical requirements (e.g. maintenance) are taken into account. Their morel is tested on real life instances of the German railway company Deutsche Bahn. Circulations for a generic week are found in between 10 minutes and 4 days of computation time.

2.2. Rescheduling

In this section we discuss papers studying the rolling stock rescheduling problem. Note that these papers do not include the possibility of scheduling dead-heading trips to reduce the number of additionally cancelled trains. Furthermore, the papers also do not take dynamic passenger flows directly into account.

Budai et al. (2010) introduce the Rolling Stock Rebalancing Problem. This problem is faced during short-term planning and during real time rescheduling. The problem they face is that a rescheduled rolling stock circulation is feasible, but there still exist off-balances in the rolling stock inventory at the end of the planning period. An off-balance is the difference between the original planned end-of-day balance and the rescheduled end-of-day balance. Their objective is to change the rolling stock circulation to solve the off-balance. The authors propose two heuristics. The first is a simple greedy approach: construct small feasible transformations to the rolling stock allocation iteratively until no more improvements are possible. The other is a two phase heuristic, where in the first phase a number of feasible transformations are selected and in the second phase an integer linear program is used to determine which transformations are selected. Their model is tested on the 3000 line of NS. It is shown that both heuristics can be used fast and effectively. The problem of this formulation is that it requires a rescheduling circulation as input and thereafter slight modifications are applied to reduce the off-balances. In our formulation, dead-heading trips might be used in order to decrease these off-balances during rescheduling.

Sato et al. (2009) propose a formulation for reallocating resources to trips in a railway network in case of large disruptions. Resources can either refer to rolling stock units or to crew members. The objective of the formulation is to produce a schedule that differs as little as possible from the original schedule. They use a two phase algorithm: in the first phase they attempt to resolve conflicts generated by the disruption. These conflicts are resolved through small changes to the original schedule. In the second phase a local search heuristic is used to improve the rescheduled plan. Their formulation is tested on a Japanese railway line. It is shown that feasible solutions arise in an acceptable amount of time for usage in practice. A difference with our approach is that this formulation does not take adjusted passenger demand or dead-heading trips into account.

Sato and Fukumura (2012) tackle the problem of reassigning locomotives to tasks during a disruption. Here, a task is defined as hauling freight carriages from one station to another. They define a MIP model that assigns the locomotives to sequences of tasks with minimum cost. This MIP model is then solved using a column generation technique. The authors conclude that the problem can be solved within a practical amount of time.

Jespersen-Groth et al. (2009) investigates problems related to rolling stock rescheduling at DSB S-Tog. They appoint anonymous train units of two types to the trains in the rescheduled timetable. Furthermore, they formulate a complementary train unit sequencing and routing model. Here, actual physical train units are assigned to the positions in the trains. The results demonstrate that the models cannot always be solved to optimality in a short computation time. However, within only a couple of minutes the optimality gap is within a few percent.

Nielsen (2011) extended the model of Fioole et al. (2006) to cope with rescheduling. He formulated an integer programming problem with the adjusted timetable and the original rolling stock schedule as input and an adjusted rolling stock circulation as output. The formulation used in this paper is extended in our paper to cope with adjusted passenger demand and dead-heading trips.

Nielsen et al. (2012) propose a rolling horizon to solve the rolling stock rescheduling problem. The idea behind the rolling horizon is that at the beginning of the disruption not all information about the duration of the disruption is known: this information becomes gradually available. The rescheduling is periodically performed within a limited rolling horizon
length, possibly taking new information into account. At each time instant where an updated timetable becomes available, or when a certain amount of time has passed without any update, the MIP of Nielsen (2011) is solved again for a certain time horizon. This model is tested on instances of NS. Solutions with small deviations from the original plan are found in short computation times.

Haahr et al. (2014) make use of a column generation approach to solve the RSRP. This method does not take the order of the rolling stock units within a composition into account. The method makes use of a decomposition method based on individual paths for the units. As a consequence, unit specific constraints could be applied to them. Haahr et al. (2016) compares two different approaches for the RSRP. The first approach is based on the model of Fioole et al. (2006) and the second approach extended the model proposed by Haahr et al. (2014) by including the order of rolling stock units within a composition. The models are compared on instances from different railway operators in different countries. Results demonstrate that the model of Fioole et al. (2006) is on average faster to solve the rescheduling instances.

Wagenaar et al. (2016) introduce scheduled maintenance appointments in the RSRP. Certain rolling stock units have a maintenance appointment during the day at one of the stations. If this is not included in the model for rescheduling, these rolling stock units will most likely miss their appointments. They introduce three different extensions of the model introduced by Fioole et al. (2006). Results demonstrate that their models are able to efficiently take maintenance appointments into account in the RSRP.

All of the above models assume passenger demand to be static. That means that every trip has a predefined passenger demand, which is not influenced by the compositions appointed to other trips. In the extreme case that a trip gets cancelled, this means that the passenger demand for the next trip with the same origin and destination does not change, even though most passengers wait to board the next train to their destination.

Kroon et al. (2015) consider real-time rescheduling of rolling stock during large disruptions while taking dynamic passenger flows into account. They use a two-stage feedback loop, where in one stage the rolling stock allocation is optimized by using the model of Nielsen (2011) and in the other stage the effect of passenger flows on the allocation of the rolling stock is determined by means of a passenger simulation. This simulation provides feedback in terms of passenger delays due to limited capacity of the assigned rolling stock. This feedback is then used in the optimization model in order to reallocate the rolling stock again, in such a way that the passenger delay is reduced. Given the reallocation of the rolling stock, the passenger simulation is performed again and feedback is given to the optimization model and this loop continues for a number of iterations. In our model, we take changing passenger flows directly into the formulation into account.

3. Model

In this section we introduce the model we use to reschedule the rolling stock after the occurrence of a disruption given the modified timetable. In this model we take the order of the rolling stock units in a train into account. In Section 3.1 we start with explaining the algorithm that selects potential dead-heading trips from the set of all possible dead-heading trips. Thereafter, in Section 3.2 we include adjusted passenger demand in the RSRP directly, as opposed to Kroon et al. (2015) who use a two-stage feedback loop. Finally, in Section 3.4 we show the complete model that is used to solve the RSRP. In this model both dead-heading trips and adjusted passenger demand are included.

3.1. Dead-heading trips

Dead-heading trips may be used in practice to transfer rolling stock units from stations with a surplus of train units to stations with a shortage of train units. Ideally, we would like to take into account all possible dead-heading trips, in order to reschedule the rolling stock. It is possible to schedule a dead-heading trip at every time instance, so there is a long list of potential dead-heading trips. Therefor, it is unrealistic to take all possible dead-heading trips into account. We can limit this number by defining a potential departing station for a dead-heading trip to be the arrival station of a regular trip in the timetable, and a potential arrival station of a dead-heading trip to be the departing station of a different regular trip. This can be done without loss of information, because all possibilities in between do not make a difference for the inventory registration. Therefore, all the dead-heading trips in between can be aggregated into one dead-heading trip. See, for instance, Fig. 1 where four potential deadheading trips between the stations B and G are displayed. Note that a dead-heading trip could depart at any time instance between the departure at B and the arrival at G, these four are just shown as an example. These dead-heading trips are all aggregated into one dead-heading trip that departs the earliest directly after trip A – B and arrives the latest just before the start of trip G – H. This means that, if this dead-heading trip is scheduled, at the start of trip G – H station G has at least one additional rolling stock unit in its inventory, and station B has at least one rolling stock unit less in inventory at the start of trip B – C.

There are certain practical rules that potential dead-heading trips have to satisfy. In this section we propose a preprocessing module that retrieves only those dead-heading trips that satisfy the imposed rules from the set of all possible dead-heading trips. The imposed rules are the following:

1. A dead-heading trip can only be scheduled after the disruption has occurred and only until a certain amount of time after the disruption is over.
2. The travel time of the dead-heading trip may not be longer than a threshold value of time set by the operator.
3. The track that is disturbed due to the disruption may not be used by the dead-heading trip.
4. The dead-heading trip does not cause a conflict with the timetable.

Furthermore, there is one obvious basic constraint which the empty train unit sent via a dead-heading trip has to fulfill: The train unit can only be used in further operations after the dead-heading trip arrived at its destination.

To this end, let $T$ be the set of trips in the adapted timetable and $S$ the set of stations in the railway network. A trip $t \in T$ is then defined by a tuple $(s^d_t, s^a_t, \tau^d_t, \tau^a_t)$. Here, $s^d_t(s^a_t)$ is defined as the station where trip $t \in T$ departs (arrives). Furthermore, let $\tau^d_t$ be the departure time of trip $t \in T$ and $\tau^a_t$ the arrival time. Then, we define $\sigma(t) = \arg \max_{u \in T} \{ \tau^d_u : \tau^d_u < \tau^d_t, s^d_u = s^d_t \}$, thus $\sigma(t)$ is the predecessor trip of trip $t \in T$. Furthermore, we define $\lambda(t) = \arg \max_{u \in T} \{ \tau^d_u : \tau^d_u < \tau^d_t, s^d_u = s^d_t \}$, meaning that $\lambda(t)$ is the previous trip with the same origin and destination station on the same line as trip $t \in T$. Fig. 2 shows the relation between $t$, $\sigma(t)$, and $\lambda(t)$. Here, trip $t$ is the second trip between station $B$ and $C$. Its predecessor trip $\sigma(t)$ is the trip before between station $A$ and $B$, and its previous trip $\lambda(t)$ is the first trip between stations $B$ and $C$.

An important note to make is that we assume that trains with the same origin and destination station on the same line do not overtake each other anywhere on the track. Then, we introduce $v_{s,s'}$ as the time it takes to transfer an empty train unit from station $s \in S$ to $s' \in S$. Note that all scheduled trips $t \in T$ have a travel time longer or equal to $v_{s,s'}$. With respect to the dead-heading trips, we define $D$ as the set of potential dead-heading trips satisfying all imposed rules. Here, $d \in D$ is defined to be a dead-heading trip scheduled to depart from station $s^d_d$ and arriving at station $s^a_d$, departing the earliest at time $\tau^d_d$ and arriving the latest at time $\tau^a_d$. The minimum travel time of a dead-heading trip $d \in D$ equals $v_{s^d_d,s^a_d}$.

As mentioned before, to limit the amount of potential dead-heading trips, we aggregate all possible dead-heading trips that depart from the arrival station of a trip to a departure station of a different trip in the timetable into one. We introduce a set $D'$ containing all aggregated potential dead-heading trips. Algorithm 1 defines the set $D'$. Note that the set $D'$ also contains the dead-heading trips which violate the imposed rules.

The first rule (1.) states that the dead-heading trip can only be used after the disruption has occurred and until a certain amount of time after the disruption is over (denoted by $\zeta$). The second rule (2.) imposes that a dead-heading trip in the set
Algorithm 1 The set $D'$.

1: for $t_1 \in T$ do
2:    for $t_2 \in T$ do
3:        if $s^1_{t_1} \neq s^2_{t_2} \land \tau^d_{t_1} + v^d_{s^1_{t_1}s^2_{t_2}} \leq \tau^d_{t_2}$ then
4:            Create new dead-heading trip $d$, with:
5:                $s^d_d = s^1_{t_1}$
6:                $s^d_d = s^2_{t_2}$
7:                $\tau^d_d = \tau^a_{t_1}$
8:                $\tau^d_d = \tau^a_{t_2}$
9:                Minimum traveltime $= v^d_{s^1_{t_1}s^2_{t_2}}$
10:       $D' = D' \cup \{d\}$

$s$ (dis) and $s'$ (dis).

![Fig. 3. Four different forbidden dead-heading trips.](image)

$D'$ can only be added to the set $D$ if the travel time is not larger than a threshold value $\gamma$. To check whether one of these rules is violated, denote $\text{dis}^\text{start}$ as the start time of the disruption and $\text{dis}^\text{end}$ as the predicted end time of the disruption. A potential dead-heading trip $d \in D$ has thus to satisfy conditions (3.1)–(3.3):

$$\tau^d_d \geq \text{dis}^\text{start}$$  \hspace{1cm} (3.1)

$$\tau^d_d \leq \text{dis}^\text{end} + \zeta$$  \hspace{1cm} (3.2)

$$v^d_{s^d_d s^d_d} \leq \gamma$$  \hspace{1cm} (3.3)

Note that the values $\zeta$ and $\gamma$ are set by the operator and can obtain any possible positive value.

The third rule (3.2) states that the scheduled dead-heading trip may not use the disturbed track. Denote $s$ (dis) and $s'$ (dis) as the two stations between which the disruption occurs. Then, Fig. 3 shows the four different cases which may not occur. These cases can all be prevented by simply checking whether a dead-heading trip uses the track between stations $s$ (dis) and $s'$ (dis).

The fourth rule (4.) states that a dead-heading trip should not cause a conflict with the timetable. To facilitate the exposition, we make the simplifying assumption that trains which do not have the same departure and arrival station, do not share any common track during their trips. Although this holds not true in reality, it does not affect the validity of our methodology. An extension of our methodology is possible if the information stating which trains makes use of which track is available. Currently, we define a dead-heading trip to be conflict-free if the following conditions hold:

1. The time between the departure of the dead-heading trip and the departure of other trips with the same origin and destination station is larger than the minimum headway time $H$. 


There are many potential dead-heading trips possible between the arrival of one trip and the departure of another trip in the timetable. All those potential dead-heading trips are aggregated into a single dead-heading trip departing at $\tau_d^d$ the earliest and arriving at $\tau_d^a$ the latest. Most likely the difference between the earliest departure time and the latest arrival time is larger than the actual travel time of the dead-heading trip. Therefore, we define $m_d$ as the amount of time slack in an aggregated dead-heading trip $d \in D'$:

$$m_d = \tau_d^a - \tau_d^d - \nu_{s^d_d, a^d_d}$$

Remember, $\lambda(t)$ is defined as the previous trip with the same origin and destination station as trip $t \in T$. Furthermore, $\chi_t$ is the time in between trip $t \in T$ and its previous trip with same origin and destination $\lambda(t)$. This time is the smallest time between either the arrivals at station $s^d_d$ or the departures at station $s^d_a$ of the two trips:

$$\chi_t = \min(\tau_d^d - \tau_{\lambda(t)}^d, \tau_d^a - \tau_{\lambda(t)}^a)$$

To check whether there is room available on the track for one of the potential dead-heading trips in the aggregated dead-heading trip $d \in D'$, we define $\alpha_d$ as the set of potentially conflicting scheduled trips $t \in T$ with the aggregated dead-heading trip $d \in D'$. If the maximum time gap between the potentially conflicting trips ($\chi_t$) is larger than twice the headway time $H$, then there is room in the network for at least one of the dead-heading trips in the aggregated dead-heading trip $d$ to be scheduled. To this end $\alpha_d$ is defined as the set of trips $t \in T$ taking place on the same track as where we want to schedule the dead-heading trip $d \in D'$. See for example Fig. 4. Here we have displayed the travel time of an aggregated dead-heading trip, indicated by a black line. This dead-heading trip can depart at $\tau_d^d$ the earliest and arrives at $\tau_d^a$ the latest. All dashed lines represent scheduled trips that possibly are in conflict with the potential dead-heading trip.

In order to check whether a trip $t \in T$ conflicts with the dead-heading trip, we first check whether they the same departure and arrival station ($s^d_d = s^d_t$, and $s^d_a = s^d_t$). Secondly, we check whether the departure time of the scheduled trip $t$ is later than the earliest departure time plus the minimum headway time ($\tau_d^d + H$) and before the latest possible departure time plus the minimum headway time ($\tau_d^a + m_d + H$) of the dead-heading trip. Finally, the arrival time of the trip $t$ has to be before the latest possible arrival time plus the minimum headway time ($\tau_d^a + H$) and after the earliest possible arrival time plus the minimum headway time ($\tau_d^d - m_d + H$) of the dead-heading trip. This is also visualized in Fig. 4.

$$\alpha_d = \{ t \in T | s^d_d = s^d_t \land s^d_a = s^d_t \land \tau_d^d + H \leq \tau_d^a \leq \tau_d^a + m_d + H \land \tau_d^a - m_d + H \leq \tau_d^a \leq \tau_d^d + H \}$$

(3.4)

Thus, Eq. (3.4) defines the set of trips $t \in T$ that have a potential conflict with the aggregated dead-heading trip $d \in D'$.

In order to test whether there is room for the dead-heading trip to be scheduled, we need to check whether the maximum time gap of the trips in the set $\alpha_d$ is larger than twice the minimum allowed headway time $H$. So, if inequality (3.5) is satisfied, then there is room for at least one of the dead-heading trips in the aggregated dead-heading trip $d \in D'$ to be scheduled.

$$\max_{t \in \alpha_d} \chi_t \geq 2H$$

(3.5)

To conclude, a dead-heading trip $d \in D'$ is added to the set of potential dead-heading trips $D$ if and only if the dead-heading trip satisfies conditions (3.1)–(3.3) and condition (3.5), and does not use the track between stations $s(dis)$ and $s'(dis)$. 

Fig. 4. Example of the potential conflicting trips with a dead-heading trip.
3.2. Modelling passenger demand by passenger flows

In the Netherlands smart cards are used since 2012 in public transport. A passenger checks in with his or her smart card at his or her origin and checks out again at his or her destination. Therefore, smart card data gives the time and location a passenger enters and leaves the railway system. The passenger demand per trip can be estimated by tracking the route passengers use from origin to destination during the undisrupted situation. This information enables us to take changing passenger flows (e.g. due to a disruption) into account in the RSRP.

With smart card data it is also possible to estimate the incoming and outgoing demand per trip, see van der Hurk et al. (2015). In the undisrupted situation, we define the incoming demand on trip $t \in T$, denoted by $ID_t$, as the number of passengers that want to enter the railway system by taking trip $t$. The number of passengers that want to leave the railway system after trip $t \in T$ in the undisrupted situation is defined as the outgoing demand and is denoted by $OD_t$. Additionally, the passenger flow on trip $t \in T$ (the number of passengers in the train) in the undisrupted situation is denoted by $F_t$. Remember that $\sigma(t)$ is defined as the predecessor of trip $t \in T$ and $\lambda(t)$ as the previous trip with the same origin and destination station as trip $t \in T$.

Before it is possible to model the passenger flow, we introduce the following decision variables:

- $F_t$ ∈ $\mathbb{R}_+$: the passenger flow sent through trip $t \in T$
- $SS_t$ ∈ $\mathbb{R}_+$: the total capacity shortage for passengers on trip $t \in T$
- $OD_t$ ∈ $\mathbb{R}_+$: the number of passengers that actually leave the railway system after trip $t \in T$

Fig. 5 provides a simple example of adjusted passenger demand. In the left part there are in total 4 trips scheduled: $(a-b), (b-c), (a'-b'),$ and $(b'-c')$. The two trips $(a-b)$ and $(b-c)$ are scheduled between the same two stations as the two trips $(a'-b')$ and $(b'-c')$, the only difference is that the trips $(a'-b')$ and $(b'-c')$ are scheduled later in time. The following parameters and variables are present in the left part of Fig. 5 for the trips $(a-b)$ and $(b-c)$:

- $ID_{(a-b)} = F_{(a-b)} = \bar{F}_{(a-b)} = 100$
- $OD_{(a-b)} = OD_{(a-b)} = 25$
- $ID_{(b-c)} = SS_{(b-c)} = 75$
- $F_{(b-c)} = \bar{F}_{(b-c)} = 150$

The same parameter and variable values hold for the trips $(a'-b')$ and $(b'-c')$.

In the right part of the figure we assume that a disruption occurs, leading to a complete blockage of the trip $(a-b)$. As a result, the passenger demand on all trips changes.

- First, it is assumed that the incoming demand stays the same for the trip $(a-b)$. However, due to the disruption no train can be appointed to the trip, causing a capacity shortage equal to the incoming demand: $SS_{(a-b)} = \bar{ID}_{(a-b)} = 100$.
- As a consequence, $F_{(a-b)} = OD_{(a-b)} = 0$.
- As a result, the passenger flow on trip $(b-c)$ only depends on its incoming demand: $F_{(b-c)} = \bar{ID}_{(b-c)} = 75$.
- The passenger flow on trip $(a'-b')$, however, now depends on the incoming demand and on the capacity shortage of its previous trip: $F_{(a'-b')} = \bar{ID}_{(a'-b')} = SS_{(a-b)} = 200$.
- That results in $OD_{(a'-b')} = OD_{(a-b)} + OD_{(a'-b')} = 50$.
- Finally, the passenger flow on trip $(b'-c')$ depends on its incoming demand and on the passenger flow of the previous trip: $F_{(b'-c')} = \bar{ID}_{(b'-c')} + F_{(a'-b')} - OD_{(a'-b')} = 225$. 

![Fig. 5. Example of taking adjusted passenger demand into account.](image-url)
We want to handle adjusted passenger demand as explained by means of the example. Remember that the following assumptions are included in the model with respect to passenger flows:

1. The incoming passenger demand does not change due to a disruption.
2. Passengers do not leave the railway system prematurely and do not take a detour to their destination. If a train is cancelled, passengers wait for the next trip with the same origin and destination.

This means that we do not model passenger demand as it would precisely happen in reality. However, we want to take the adjusted passenger demand into account such that the model appoints rolling stock compositions with enough capacity to trips. As a consequence, it does not matter whether the demand on a trip equals, for instance, 1050 or 1150 passengers. In both scenarios, the best available composition in the Netherlands to appoint to the trip is a train with capacity for 1200 passengers. Therefore, we only take adjusted passenger demand on an aggregated level into account.

To model the adjusted passenger demand in a mathematical formulation, we denote $P$ as the set of possible compositions, where a composition is a combination of train units that can be used on a trip. Subsequently, $cap_p$ is the capacity for passengers in composition $p \in P$. This capacity is measured as the maximum number of passengers that can possibly fit in the rolling stock units present in the composition. Denote $V$ as the unique set of trips, meaning that $V$ contains exactly every trip with the same origin and destination once.

Then, the decision variable $X_{t,p}$ is equal to 1 if composition $p \in P$ is appointed to trip $t \in T$. With the introduced parameters and decision variables, the following constraints are able to keep track of passenger flows in the railway network:

\[
F_t + SS_t = \hat{ID}_t + F_{OD_t} - OD_{t} + SS_{k(t)} \quad \forall t \in T \tag{3.6}
\]

\[
F_t \leq \sum_{p \in P} X_{t,p} \cdot cap_p \quad \forall t \in T \tag{3.7}
\]

\[
\sum_{s'_{t} = s_{t}, s''_{t} = s''_{t}} \hat{OD}_t = \sum_{s'_{t} = s_{t}, s''_{t} = s''_{t}} OD_{t} \quad \forall v \in V \tag{3.8}
\]

\[
\sum_{t' \in T : \tau_{t'} \leq \tau_{t}} \sum_{s'_{t'} = s_{t'}, s''_{t'} = s''_{t'}} (\hat{OD}_{t'} - OD_{t'}) \geq 0 \quad \forall t \in T \tag{3.9}
\]

Constraints (3.6) state that the passenger flow on trip $t \in T$ is equal to the incoming demand, plus the passenger flow on the previous trip, minus the amount of passengers that get off the train at the end of the previous trip, plus the capacity shortage of the previous trip, and finally minus the number of passengers that do not fit in the composition appointed to the trip. Constraints (3.7) make sure that the passenger flow can not exceed the appointed capacity for the trip. Constraints (3.8) denote that all passengers have to arrive at their planned destination before the end of the planning horizon. Constraints (3.9) state that none of the passengers can arrive at their destination before they originally planned to arrive, because the number of outgoing passengers cannot exceed the number of planned outgoing passengers.

Both simplifying assumptions (1. and 2.) are satisfied with these constraints. First of all assumption 1 is satisfied, because $\hat{ID}_t$ is a parameter. For the second assumption, let us assume that passengers are able to leave the railway system at stations before reaching their destination. That means that the total number of passengers getting off at some station is larger than the scheduled number for at least a single station, because not only passengers with this station as destination leave the railway system, but also passengers who prematurely leave the system there. As a result it holds that:

\[
\sum_{t' \in T : \tau_{t'} \leq \tau_{t}} (OD_{t'} - \hat{OD}_{t'}) > 0 \quad \forall v \in V \tag{3.10}
\]

However, this causes a conflict with Constraints (3.8). So, passengers can not leave the railway system before reaching their destination. The same reasoning holds for passengers taking a detour; more passengers would get off a train after a unique trip ($v \in V$), and so Eq. (3.10) would hold in this case as well, which causes a conflict with Constraints (3.8).

Next to minimizing the total number of seat-shortages, we also want to minimize the total passenger delay on an aggregated level. The variable $Q_t$ denotes the passenger delay after trip $t \in T$ and is defined as in Eq. (3.11).

\[
Q_t = \sum_{t' \in T : \tau_{t'} \leq \tau_{t}} (\hat{OD}_{t'} - OD_{t'}) \cdot (\tau_{t'} - \tau_{t}) \quad \forall t, t_1 \in T : \lambda(t_1) = t \tag{3.11}
\]

The first part of the equation for the passenger delay denotes the number of passengers that wanted to leave the railway system after a trip at station $s_{t'}$, but were not yet present at $s_{t}$ because the capacity of the train was too little. The second part of the equation represents the minimum amount of time these passengers have to wait before they can arrive at their destination by using the next train that arrives at $s_{t}$.
Finally we have to note that with constraints (3.8), (3.9), and with the objective of minimizing the amount of seat-shortages and the total passenger delay, it holds that $OD_t = \overline{OD}_t$ as long as there is enough capacity to satisfy all passenger demand. However, when there is not enough capacity available for all passenger demand on a trip, then the model decides which passengers with which destinations will board the train and which do not. This is done in such a way that the total number of seat-shortages and the total passenger delay is minimized, but this boarding strategy is not necessarily the strategy that actually takes place in real-life. Therefore, we present two extreme boarding strategies in the next section, to compare the “optimal” boarding strategy with.

3.3. Boarding strategies

In real-time it is unknown which passengers will board the train and which passengers will wait for the next train to depart to their destination, when the appointed capacity to a trip is too little for the corresponding passenger demand. By using only constraints (3.6)-(3.9) the model is allowed to decide which passengers will board the train. This will be done such that the total number of seat-shortages and the total passenger delay are minimized. In reality, however, it is not likely that precisely those passengers which minimize the global objective will board the train. Unfortunately, it is not possible to precisely predict what passengers will do. For instance, if there are two passengers waiting at a platform and both of them have a different destination: station $a$ and $b$. The arriving train has only room for one of the two passengers, then there is no way to tell which of the two passengers will board the train. That is why we introduce two different boarding strategies to test their influence on the rescheduled rolling stock circulation.

The results of using only constraints (3.6)-(3.9) will be compared with models that force passengers with a certain destination to board a train. We will compare the model with two extreme cases and an average case. The first extreme case assumes that, if the appointed capacity is too little for the passenger demand, that passengers with the nearest destination to come will board the train first. The second extreme case assumes that passengers with the furthest destination on the line to come will board the train first in case the appointed capacity for a trip is too small. In reality most likely passengers with different destinations will board the train. The third boarding strategy is in between. It forces that half of the passengers with the nearest destination will board the train and the other half that boards the train will have a different destination. In reality, however, most likely passengers with different destinations will board the train than in any of the cases.

3.3.1. Nearest destination first

In the first extreme case it is assumed that passengers with the nearest destination on the line to come board the train first in case the appointed capacity is too little. This can be done by setting constraints on the number of passengers leaving the railway system after the arrival of an incoming train. At every arrival of a train at a station, we determine the amount of passengers that have that station as their destination. This can either be the same as in the original situation (\(\overline{OD}_t\)) or larger, because the demand on the previous trip exceeded the capacity of the train. We define $X_O_t$ as this number of passengers that have station $s^i_t$ as their destination at time $t^i_T$, see Eq. (3.12). Here, the first sum of the right hand side of the equation denotes all passengers that want to leave the railway system at station $s^i_t$ up to and including the arrival of trip $t \in T$. The second term defines the number of passengers that have actually arrived at station $s^i_t$ up to the arrival of trip $t$. Thus, $X_O_t$ denotes the number of passengers that have station $s^i_t$ as their destination at time $t^i_T$.

$$X_O_t = \sum_{t^i_T; s^i_t = s^i} \overline{OD}_t - \sum_{t^i_T; s^i_t = s^i; t^i_s < t^i_T} OD_t, \quad \forall t \in T$$  \hspace{1cm} (3.12)

Note that $X_O_t \geq 0$ due to Constraint (3.9). In order to force passengers with the earliest destination to board the train, we want to archive that $OD_t = \min \{R_i, X_O_t\}$. Thus at every arrival of a train at a station, either $X_O_t$ or $F_t$ passengers have to leave the train. To model this, we furthermore introduce the binary variable $PO_t$, which equals 1 if the passenger flow on trip $t \in T$ is larger than the number of passengers that want to get out at the end of trip $t$, and 0 otherwise, see Eq. (3.13):

$$F_t + M(1 - PO_t) \geq X_O_t \geq F_t - M \cdot PO_t, \quad \forall t \in T$$ \hspace{1cm} (3.13)

Note that $M$ represents a large number in this inequality.

Now, adding constraints (3.12) and (3.13), together with constraints (3.14) – (3.15), ensures that either all passengers that want to get out at the end of trip $t \in T$ will leave the train, or the complete passenger flow leaves the train. This holds true due to Constraints (3.8) and (3.9).

$$OD_t \geq F_t - M \cdot PO_t, \quad \forall t \in T$$ \hspace{1cm} (3.14)

$$OD_t \geq X_O_t - M \cdot (1 - PO_t), \quad \forall t \in T$$ \hspace{1cm} (3.15)

At every station we force the maximum number of passengers to leave the train. As a consequence, we have forced that passengers with the earliest destination have boarded the train on the previous stations.
3.3.2. Furthest destination first

In the second extreme case it is assumed that passengers with the furthest destination on the line to come board the train first in case the appointed capacity is too little. This is achieved by setting constraints on the number of passengers in an incoming train continuing their journey on the line after the arrival of an incoming train. At every arrival of a train at a station we determine the number of passengers that have a further station on the line as destination. This can either be the same as in the original situation ($\bar{F}_i - \bar{OD}_t$) or larger, due to the fact that the demand exceeded the appointed capacity of the previous train. We define this number of passengers as $XC_t$, as described in Eq. (3.16). Here, the first term of the right hand side of the equation denotes the total number of passengers that want to continue to a station further on the line than station $s^T_t$ up to and including trip $t \in T$. The second term denotes the number of passengers that have actually continued to a station further on the line up to trip $t \in T$. Thus, $XC_t$ denotes the number of passengers that have a station further on the line than station $s^T_t$ as their destination at time $t$. 

$$XC_t = \sum_{t' \in T,\ t' \geq t} (\bar{F}_{t'} - \bar{OD}_{t'} - \sum_{t' \in T,\ t' < t} (\bar{F}_{t'} - \bar{OD}_{t'})) \quad \forall t \in T \tag{3.16}$$

Note that $XC_t \geq 0$ due to Constraint (3.9). In order to force passengers with the latest destination to board the train, we want to achieve that: $\bar{F}_i - \bar{OD}_t = \min\{\bar{F}_i, XC_t\}$. Here, $\bar{F}_i - \bar{OD}_t$ represents the number of passengers continuing their journey after the arrival of train $t \in T$ at station $s^T_t$. This should either be equal to $XC_t$ or equal to the total passenger flow $\bar{F}_i$. To model this, we introduce a binary variable $PC_t$ that is equal to 1 if the passenger flow on trip $t \in T$ is larger than the number of passengers that want to continue their journey after trip $t$, and 0 otherwise, see Eq. (3.17).

$$F_i + M(1 - PC_t) \geq XC_t \geq F_i - M \cdot PC_t \quad \forall t \in T \tag{3.17}$$

In case $PC_t = 0$, it means that the number of passengers that want to continue their journey after trip $t$ is larger than the total passenger flow on the trip. As a result, no passenger will leave the train after trip $t$ ($\bar{OD}_t = 0$), and so $\bar{F}_i - \bar{OD}_t = \bar{F}_i$. see Constraint (3.18). On the other hand, if $PC_t$ is equal to 1, it means that the passenger flow is larger than the number of passengers that want to continue after trip $t$. So, the actual number of passengers continuing after trip $t$ ($\bar{F}_i - \bar{OD}_t$), equals the total number of passengers that want to continue their journey after trip $t$: $XC_t$. See Constraint (3.19).

$$\bar{F}_i - \bar{OD}_t \geq F_i - M \cdot PC_t \quad \forall t \in T \tag{3.18}$$

$$\bar{F}_i - \bar{OD}_t \geq XC_t - M \cdot (1 - PC_t) \quad \forall t \in T \tag{3.19}$$

In this way we force passengers with the latest destination to board the train at the previous stations.

3.3.3. Average boarding strategy

The above two models simulate extreme instances. It is unlikely that all passengers will leave the train at a station that is not a final station. It is also unlikely that none of the passengers will leave the train at an intermediate station. Therefore, we have included a third boarding strategy. This strategy is in between the furthest and nearest destination first strategies. We assume that at most half of the passengers with the nearest destination will board the train, and, at least, the other half will have a destination further away. Consequently, we want to achieve at the end of every trip that $\bar{OD}_t = \min\{\frac{1}{2}\bar{F}_i, XO_t\}$. In other words, at the end of every trip $t \in T$, the number of passengers that leave the train equals all passengers that have station $s^T_t$ as their destination if that number is smaller than half of the passenger flow in the train, or half of the passenger flow leave the train.

To this end, the binary variable $PM_t$ equals 1 if $\frac{1}{2}\bar{F}_i > XO_t$ and 0 otherwise. The constraint (3.20) makes sure that this holds true.

$$\frac{1}{2}\bar{F}_i + M \cdot (1 - PM_t) > XO_t \geq \frac{1}{2}\bar{F}_i - M \cdot PM_t \tag{3.20}$$

Due to this constraint it must be that $XO_t \geq \frac{1}{2}\bar{F}_i$ if $PM_t = 0$. Furthermore, in case $PM_t = 1$ it must hold that $\frac{1}{2}\bar{F}_i > XO_t$. Then, Constraints (3.21)-(3.24) sets the passenger flow as in our goal. Constraints (3.21) and (3.22) forces the number of passengers to be equal to $\frac{1}{2}\bar{F}_i$ if PM = 0. Note that the -1 in Constraints (3.21) and the +1 in Constraints (3.22) are necessary due to rounding of $\frac{1}{2}\bar{F}_i$. Furthermore, Constraints (3.23) and (3.24) set the number of outgoing passengers equal to XO is $PM_t = 1$.

$$\bar{OD}_t \geq \frac{1}{2}\bar{F}_i - PM_t \cdot M - 1 \quad \forall t \in T \tag{3.21}$$

$$\bar{OD}_t \leq \frac{1}{2}\bar{F}_i + PM_t \cdot M + 1 \quad \forall t \in T \tag{3.22}$$

$$\bar{OD}_t \geq XO_t - (1 - PM_t) \cdot M - 1 \quad \forall t \in T \tag{3.23}$$
\[ OD_t \leq XO_t + (1 - PM_t) \cdot M + 1 \quad \forall t \in T \quad (3.24) \]

All three boarding strategies will be compared with the model where the passengers are guided to board the train in an optimal way with respect to the global objective.

### 3.4. Complete model

In this subsection the complete MIP model used to reschedule the rolling stock while including dead-heading trips and adjusted passenger demand is discussed. The notation of the previous subsections still holds and is extended with the following parameters and variables. Let \( \mathcal{M} \) be the set of rolling stock types, then \( \nu_m(p) \) denotes the number of train units of type \( m \in \mathcal{M} \) in composition \( p \in P \). Different stations have different platform lengths, as a result not all stations can cope with all possible composition lengths. To that end denote \( \eta(t) \) as the set of allowed compositions on trip \( t \in T \), with respect to the platform lengths at station \( s^d \), \( s^i \), and all the stations in between.

At the end of a trip the composition of a train can possibly be changed, depending on the shunting rules at the station, before departing on its successive trip. A composition change consists of the composition of the incoming and outgoing trip and which units are coupled and uncoupled during the composition change. At the end of trip \( t \in T \), \( \rho(t) \) denotes the set of allowed composition changes. Furthermore, \( p_q \) denotes the composition of the incoming trip of composition change \( q \in \rho(t) \) and \( p_d \) denotes the composition of the outgoing trip in the composition change. For a given composition change \( q \in \rho(t) \) at the end of trip \( t \in T \), \( \alpha_{q,m} \) denotes the number of uncoupled units of type \( m \in \mathcal{M} \) and \( \beta_{q,m} \) denotes the number of coupled units of type \( m \in \mathcal{M} \) in this composition change. The time at which coupling takes place at the end of trip \( t \in T \) is denoted by \( \tau^+_{q} \) and the time at which an uncoupled unit is available after uncoupling is denoted by \( \tau^-_{q} \).

The available number of units \( m \in \mathcal{M} \) at station \( s \in S \) at the beginning of the planning period is denoted by \( I_{0,m}^s \) and the desired number of available units of type \( m \in \mathcal{M} \) at station \( s \in S \) at the end of the planning period is given by the parameter \( I_{\infty,m}^s \).

Finally, the following additional decision variables are used in the model:

- \( X_{t,p} \in \{0, 1\} \) denotes whether composition \( p \in \eta(t) \) is used on trip \( t \in T \).
- \( Z_{t,q} \in \{0, 1\} \) denotes whether composition change \( q \in \rho(t) \) is used at the end of trip \( t \in T \).
- \( k_{m} \in \mathbb{Z}_+^Q \) and \( \theta_{s,m} \in \mathbb{Z}_+^Q \) denote the number of units \( m \in \mathcal{M} \) that are coupled and uncoupled at the start and end of trip \( t \in T \), respectively.
- \( k_{m} \in \mathbb{Z}_+^Q \) denotes the number of units of type \( m \in \mathcal{M} \) in the inventory at station \( s^{dep} \) immediately after time \( \tau^+_q \).
- \( l_{m} \in \mathbb{Z}_+^Q \) denotes the number of units of type \( m \in \mathcal{M} \) at station \( s \in S \) at the end of the planning period.
- \( W_{s,m} \) denotes the deviation from the desired end-of-day balance of rolling stock type \( m \in \mathcal{M} \) in station \( s \in S \).
- \( Y_{d,m} \in \{0, 1\} \) denotes whether dead-heading trip \( d \in D \) is covered by a rolling stock unit of type \( m \in \mathcal{M} \).

With the above variables and parameters, and the ones introduced in the previous sections, we can form the mathematical model that is used to reschedule the rolling stock during a disruption while including dead-heading trips and adjusted passenger demand. In this model, the subsets \( A_t, B_t, E_t \) and \( G_t \) are defined as:

1. \( A_t = \{ t' \in T : s^d_{t'} = s^d_t, t' \leq \tau^+_{q} \} \)
2. \( B_t = \{ t' \in T : s^d_{t'} = s^d_t, t' \leq \tau^+_{q} \} \)
3. \( E_t = \{ d \in D : s^u_d = s^d_t, t' \leq \tau^+_{q} \} \)
4. \( G_t = \{ d \in D : s^u_d = s^d_t, t' \leq \tau^+_{q} \} \)

\[
\min f(X, Z, W, Y, SS, Q) \quad (3.25)
\]

Subject to:

\[
\sum_{p \in \eta(t)} X_{t,p} = 1 \quad \forall t \in T \quad (3.26)
\]

\[
X_{t,p} = \sum_{q \in \rho(t)} Z_{t,q} \quad \forall t \in T, p \in \eta(t) \quad (3.27)
\]

\[
X_{t,p} = \sum_{q \in \rho(\sigma(t))} Z_{\sigma(t),q} \quad \forall t \in T, p \in \eta(t) \quad (3.28)
\]
\[
C_{t,m} = \sum_{q \in \rho(\sigma(t))} \beta_{q,m} Z_{\sigma(t),q} \quad \forall t \in T, m \in \mathcal{M}
\]

\[
U_{t,m} = \sum_{q \in \rho(t)} \alpha_{q,m} Z_{q} \quad \forall t \in T, m \in \mathcal{M}
\]

\[
I_{t,m} = i^0_{s^0, m} - \sum_{t' \in A_t} C_{t', m} + \sum_{t' \in B_t} U_{t', m} - \sum_{d \in D} Y_{d,m} + \sum_{d \in E} Y_{d,m} \quad \forall t \in T, m \in \mathcal{M}
\]

\[
F^\infty_{t,m} = i_{s^0, m} + W_{s,m} - \sum_{t \in T, s \in S} \sum_{m \in \mathcal{M}} C_{t,m} + \sum_{t \in T, s \in S} U_{t,m} - \sum_{d \in D, s^0 = s} Y_{d,m} + \sum_{d \in D, s^0 = s} Y_{d,m} \quad \forall s \in S, m \in \mathcal{M}
\]

\[
F_t + SS_t = \tilde{ID}_t + F_{\sigma(t)} - OD_{\sigma(t)} + SS_{\tilde{\sigma}(t)} \quad \forall t \in T
\]

\[
R_t \leq \sum_{p \in P} X_{t,p} \hat{cap}_p \quad \forall t \in T
\]

\[
\sum_{t \in T} \sum_{s^0 = s^1 = s^2} \tilde{OD}_t = \sum_{t \in T} \sum_{s^0 = s^1 = s^2} OD_t \quad \forall v \in V
\]

\[
\sum_{t \in T \cap t^0 \leq t^1} \sum_{s^0 = s^1 = s^2} (\tilde{OD}_{t^2} - OD_{t^2}) \geq 0 \quad \forall t \in T, v \in V
\]

\[
Q_t = \sum_{t \in T \cap t^0 \leq t^1} (\tilde{OD}_{t^1} - OD_{t^1}) \cdot (\tau_{t^1}^d - \tau_{t^1}^d) \quad \forall t \in T : \lambda_{t^1} = t
\]

In the above model Constraints (3.26) and Constraints (3.33) are the same as in Nielsen (2011). All other constraints are either altered for the use of dead-heading trips (Constraints (3.31) and (3.32)) or new in order to include adjusted passenger demand (Constraints (3.34) – (3.38)).

Constraints (3.26) specify that to each trip exactly one composition is assigned. All compositions before and at the start of the disruption are fixed, because these trips are already underway. In that case, the set of allowed compositions, \(\eta(t)\), only consists of one composition. Note that a trip is cancelled if the empty composition is appointed to it. Constraints (3.27) state that if composition \(p \in \eta(t)\) is assigned to trip \(t \in T\), then only a composition that can originate from \(p\) can be assigned to the succeeding trip. Constraints (3.28) state that if composition \(p\) is assigned to trip \(t\), then only a composition which can be changed into \(p\) can be assigned to the predecessor trip \(\sigma(t)\).

Constraints (3.29) specify the number of coupled train units at the beginning of a trip and Constraints (3.30) specify the number of uncoupled train units at the end of a trip.

Constraints (3.31) stipulate the inventory of rolling stock type \(m \in \mathcal{M}\) at station \(s^0_t\) immediately after time \(\tau_{t^0}^d\). This is equal to the start inventory at the associated station, minus all train units that are coupled up to time \(\tau_{t^0}^d\), plus all uncoupled units that are available before time \(\tau_{t^0}^d\), plus all empty units that entered the station before time \(\tau_{t^0}^d\) due to dead-heading trips, minus all empty units that departed from the station up to time \(\tau_{t^0}^d\) due to dead-heading trips.

Constraints (3.32) define that the end inventory at a station \(s \in S\) is equal to the start inventory at \(s\) minus all train units that are coupled at \(s\), plus all train units that are uncoupled at \(s\), plus all empty units that are transferred to \(s\), and minus all empty train units that are transferred from \(s\) to another station. Constraints (3.33) states that the end-of-day balance is equal to the planned end-of-day balance plus a deviation.

Constraints (3.34)–(3.38) set the passenger flows as explained in the previous Section 3.2.
The objective function (3.25) is a linear function that depends on which composition is assigned to a trip (X), what composition changes are made (Z), the deviation from the end-of-day balance at stations (W), the number of dead-heading trips that are scheduled (Y), the total number of seat shortages (SS), and the total passenger delay (∑). In order to define the objective function in detail, we first introduce the parameters ψ0 and C∈ as the empty composition p0 ∈ P and the number of carriage kilometers driven on trip t ∈ T when using composition p ∈ P respectively. With these additional parameters the first part of the objective function is shown in Eq. (3.39). The first term defines the total costs of cancelling trips. Every cancelled trip has a cost equal to α. The second term defines the total costs for driving carriages, where β is the penalty for driving a single carriage kilometer.

\[ \sum_{t \in T} (\alpha \sum_{p \in \eta(t): p \neq p_0} X_{t,p} + \beta \sum_{p \in \eta(t)} X_{t,p} C_{t,p}) \] (3.39)

The second part of the objective function penalizes the number of additional shunting movements made in the rescheduled rolling stock circulation. Every cancelled shunting movement gets a penalty equal to κr and every unplanned new shunting movement gets penalized by κa. In order to measure the number of additional shunting movements, we first define the binary parameter s_t^q as whether a new shunting movement takes place when using composition change q ∈ Q on trip t ∈ T compared with the original rolling stock circulation. Furthermore, s_t^q defines whether a shunting movement is cancelled or not when composition change q ∈ Q is used on trip t ∈ T. Then, we can penalize the number of additional shunting movements as in Eq. (3.40).

\[ \sum_{t \in T} \sum_{q \in Q} (s_t^q \kappa^a + s_t^q \kappa^r) Z_{t,q} \] (3.40)

The third part penalizes deviations from the scheduled end-of-day balance, as shown in Eq. (3.41). Each deviation from the scheduled end-of-day balance is penalized with ψ.

\[ \psi \sum_{s \in S} \sum_{m \in M} W_{s,m} \] (3.41)

The fourth part of the objective function penalizes the use of dead-heading trips. Every dead-heading trip has a cost equal to Ψ, see Eq. (3.42).

\[ \Psi \sum_{d \in D} \sum_{m \in M} Y_{d,m} \] (3.42)

The final part of the objective function penalizes seat-shortages for passengers and the total passenger delay. Every seat-shortage is penalized with θ per kilometer and the total passenger delay is penalized with φ, see Eq. (3.43).

\[ \sum_{t \in T} (SS_t \theta + Q_t \phi) \] (3.43)

This leads to the complete objective function as expressed in Eq. (3.44).

\[ \sum_{t \in T} (SS_t \theta + Q_t \phi + \alpha \sum_{p \in \eta(t): p \neq p_0} X_{t,p} + \beta \sum_{p \in \eta(t)} X_{t,p} C_{t,p} + \sum_{q \in Q} (s_t^q \kappa^a + s_t^q \kappa^r) Z_{t,q}) + \sum_{m \in M} (\psi \sum_{s \in S} W_{s,m} + \Psi \sum_{d \in D} Y_{d,m}) \] (3.44)

The decision maker should state, in advance, the costs of having deviations from the end-of-day balance at a station, of having not enough capacity for all passengers on a trip, of having different composition changes than in the original schedule, of sending dead-heading trips, and of having too little capacity. The objective is most of the time a trade-off between passenger service and costs.

The following variable domains are used in the model:

\[ X_{t,p} \in \{0, 1\} \quad \forall t \in T, p \in \eta(t) \] (3.45)

\[ C_{t,m}, U_{t,m}, \ell_{t,m} \in \mathbb{R}_+ \quad \forall t \in T, m \in \mathcal{M} \] (3.46)

\[ L_{s,m} \in \mathbb{R}_+ \quad \forall s \in S, m \in \mathcal{M} \] (3.47)

\[ Z_{t,q} \in \mathbb{R}_+ \quad \forall t \in T, q \in \rho(t) \] (3.48)

\[ Y_{d,m} \in \mathbb{R}_+ \quad \forall d \in D, m \in \mathcal{M} \] (3.49)

\[ F_t, SS_t, OD_t, Q_t \in \mathbb{R}_+ \quad \forall t \in T \] (3.50)
4. Computational tests

In this section we give an overview of the computational results of the various instances on which we have tested our models. All computational tests are performed on an Intel Core i5-3210M processor with 2.50 GHz and 8GB RAM by using CPLEX 12.6.1.

4.1. Variants of the model

The complete model, as presented in Section 3.4, was tested in six versions:

1. Original Model (OM): no dead-heading trips and no adjusted passenger demand have been included.
2. Dead-Heading Model (DHM): dead-heading trips have been included, but no adjusted passenger demand has been included.
3. Adjusted Demand Model (ADM): no dead-heading trips have been included, but adjusted passenger demand has been included.
4. Earliest Destination Board Model (ADM(E)): no dead-heading trips have been included, but adjusted passenger demand has been included, with the addition that passengers with the nearest destination are assumed to board the train (see Section 3.3.1).
5. Latest Destination Board Model (ADM(L)): no dead-heading trips have been included, but adjusted passenger demand has been included, with the addition that passengers with the furthest destination are assumed to board the train (see Section 3.3.2).
6. Dead-Heading Adjusted Demand Model (DHADM): both dead-heading trips and adjusted passenger demand are taken into account.

We have not included the two versions where an extreme boarding strategy and dead-heading trips are combined. This is because those two versions did not lead to additional conclusions: dead-heading trips are used to decrease the number of cancelled trips, and the extreme boarding strategies lead to many seat-shortages.

All six versions are tested on real life instances from NS. In Section 4.2 all computational experiments used to test the models are explained in detail. Thereafter, in Section 4.3, the results of applying the models to the instances are discussed. The results of the models that make use of adjusted passenger demand are difficult to compare with the results of the models that do not include adjusted passenger demand. Consequently, we split the comparison of the results. In Section 4.3.2 we start with investigating the added value of using dead-heading trips by comparing OM with DHM. Following, in Section 4.3.3, we compare all models where adjusted passenger demand is taken into account: ADM, ADM(L), ADM(E), and DHADM. Finally, in Section 4.3.4, all models are compared with each other with respect to their computation times.

4.2. Case description

Fig. 6 gives an overview of the railway lines that are included in our case study. All important and busy lines of the western part of the Netherlands in 2012 are taken into account. In this way a large part of the complete Dutch railway network is covered by our instances. The dataset consists of 2276 different trips. Two different rolling stock types are used on these trips. They differ in their number of carriages (either 4 or 6). By using these rolling stock types, in total 11 different compositions (including the empty one) are available. This case has been used in multiple different papers as well, see Haahr et al. (2016) for instance.
Besides the timetable of a large part of the Dutch railway network, NS provided us with information regarding the passenger demand. For every scheduled trip we are given the expected passenger demand and the expected percentage of passengers getting off at the end of that trip. As a consequence, we can determine the incoming demand on every trip.

The models have been developed to be used in real-time. In order to test them adequately, we simulate different disruption scenarios. These scenarios are summarized in Table 1. As can be seen, there are 12 different disruptions simulated at different locations and different time slots on the railway network.

Besides the simulated disruptions, also the parameter settings are of importance for testing the models. There is a trade-off between the different objective components. For instance, reducing the number of seat-shortages for passengers leads to an increase in the number of carriage kilometers. Consequently, the penalty settings will influence the results of the RSRP. We will test different penalty settings for every disruption case and every model. First of all, cancelling additional trips must be prevented, so the penalty for cancelling a trip is always the largest. We use a penalty value of 1 000 000 in half of the settings. We like to investigate the trade off between cancelling trips and the other objective components, so we use a penalty of 100 000 for the other half of the settings.

With respect to the other objective coefficients, the following penalties are used. First of all, a penalty of 1000 is used for a single negative deviation from the scheduled end-of-day balance. Negative deviations from the scheduled end-of-day balance are solved during the day by using dead-heading trips. This is expensive for the railway operator and these operations need therefore to be minimized. However, it is more important to have a good passenger service, so the penalty is not that large. Furthermore, penalties are set upon having seat shortage kilometers, passenger delay, and on driving a single carriage for one kilometer. For the first and second, a general penalty of 1 is used, and for the latter a general penalty value of 0.1 is imposed. The penalties for seat shortages and carriages kilometers seem to be small, however, they are measured per kilometer. The shortest trip has a length of 16km and the longest trip has a length of 67km. As a result, having, for example, 100 seat-shortages on a trip with length 50km already leads to a penalty of 100 · 50 · 1 = 5000. The same holds for the carriage kilometers, this is measured per carriage per kilometer. Note that the smallest composition already contains 4 carriages. Furthermore, the penalty for passenger delay equals 1. This seems to be small as well, but note that it is measured per minute per passenger. Thirdly, the penalty for sending a single dead-heading trip during the day equals 2000 and the penalty for a single minute of passenger delay equals 1. The final two penalty values are for performing a shunting operation at a location and time where no shunting operation was scheduled, and for not doing a shunting operation at a location and time where a shunting operation was scheduled. An unplanned shunting movement means that a crew member must be appointed to the corresponding station to perform the shunting movement, this takes time and costs manpower. On the other hand, cancelling a shunting movement means that a crew member is unnecessarily appointed, and this has certain costs as well. We use a penalty value of 1000 for adding a new shunting operation and of 100 for cancelling a shunting operation.

In order to test the influence of the penalty settings, the models are tested on different settings as well. In every of those settings, the penalty value of one of the objective coefficients is changed, while all other penalty values are the same as in the general setting. First of all, the penalty value for a single negative deviation from the scheduled end-of-day balance has been increased to 2000 and 5000. Secondly, the penalty for having a seat shortage kilometer is changed to either 0.1, 0.5, 2, or 5. The carriage kilometer penalty is always kept the same, because only the ratio \( \frac{\text{km}}{\text{passenger}} \) is of importance. Thirdly, the penalties for scheduling a single dead-heading trip during the day have changed to 200, 1000, 5000, and 10000. Finally, the penalty settings for (unplanned, cancelled) shunting activities are changed to (2000, 200) and (5000, 500).

This leads to a total of 26 different penalty settings, 13 instances per base case. A short summary of the penalty values is given in Table 2.

### 4.3. Results

As mentioned before, it is not correct to compare the results of the models that include adjusted passenger demand with the models that do not include adjusted passenger demand. Therefore, we start with giving an example of the difference between the models using adjusted passenger demand and the models that do not in Section 4.3.1. Thereafter, we compare the two versions OM and DHM with each other in Section 4.3.2. In Section 4.3.3 we compare the four versions ADM, ADM(L), ADM(E), and DHADM.

---

**Table 1**

Disruption cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Disrupted area</th>
<th>Disruption Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 3</td>
<td>Gd – Ut</td>
<td>07:00–10:00</td>
</tr>
<tr>
<td>4, 5</td>
<td>Rd – Gv</td>
<td>16:00–19:00</td>
</tr>
<tr>
<td>6</td>
<td>Ledn – Ut</td>
<td>16:00–19:00</td>
</tr>
<tr>
<td>7, 8</td>
<td>Amf – Ut</td>
<td>16:00–19:00</td>
</tr>
<tr>
<td>9, 10</td>
<td>Gv – Ledn</td>
<td>16:00–19:00</td>
</tr>
<tr>
<td>11, 12</td>
<td>Asd – Ut</td>
<td>16:00–19:00</td>
</tr>
</tbody>
</table>
Table 2
Summary of the different penalty settings. The first column denotes the penalty values for cancelling an additional trip. The second column for having a negative deviation from the scheduled end of day balance (End Dev). The third and fourth column for the seat-shortages per kilometer (SS km) and the total passenger delay (Pass Delay). The fifth column denotes the carriage kilometers (Carr km). The sixth column shows the penalty values for using a dead-heading trip (DH). Finally, the seventh column denotes the penalty values for having an unplanned (U) or cancelled (C) shunting movement.

<table>
<thead>
<tr>
<th>Cancel trip (α)</th>
<th>End Dev (ψ)</th>
<th>SS km (θ)</th>
<th>Pass Delay (ϕ)</th>
<th>Carr km (β)</th>
<th>DH (Ψ)</th>
<th>Shunting (U, C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000000</td>
<td>1000</td>
<td>1</td>
<td>1</td>
<td>0.1</td>
<td>10000</td>
<td>(1000, 100)</td>
</tr>
<tr>
<td>10000</td>
<td>2000</td>
<td>0.1</td>
<td></td>
<td>5000</td>
<td>2000</td>
<td>(2000, 200)</td>
</tr>
<tr>
<td>5000</td>
<td>0.5</td>
<td></td>
<td></td>
<td>2000</td>
<td>1000</td>
<td>(5000, 500)</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>200</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 7. Passenger demand.

4.3.1. Difference example
We use disruption case 1 (see Table 1) as the example to explain the difference between the models that use adjusted passenger demand and the models that do not. A disruption between Gouda (Gd) and Utrecht (Ut) takes place from 07:00-11:00 and as a consequence the rolling stock circulation needs to be rescheduled for the remainder of the day.

We focus on the difference by rescheduling the rolling stock circulation with the original model (OM) and the adjusted demand model (ADM). Fig. 7 presents the passenger demand used by the OM (denoted by OM in the Figure), and the adjusted passenger demand used during rescheduling with ADM (denoted by ADM) for the trajectory Gouda-Utrecht of the 500 line. The time during the day is displayed on the horizontal axis. Note that we only display the passenger demand in a single direction, so no passenger demand is displayed for the direction Utrecht-Gouda. Furthermore, we only show the passenger demand for the trips on the 500 line. However, this is not the only line with a connection between Gouda and Utrecht. The other lines are taken into account when determining the adjusted demand between the stations.

As can be seen, the adjusted passenger demand and the original demand are equal as long as everything runs according to plan. However, after the disruption, there is a large peak for the adjusted demand, while the original passenger demand remains constant. In order to reduce this large peak in ADM, large compositions have to be appointed to the trips just after the disruption. This can be seen in Fig. 8, where the appointed capacity after rescheduling with ADM and OM are shown for the 500 line between Gouda and Utrecht. After rescheduling with OM there are no larger compositions appointed to the trips just after the disruption, while after rescheduling with ADM larger compositions are actually appointed to these trips. As a consequence, it takes approximately eight trips on the 500 line before the adjusted demand is equal to the original passenger demand again. Note that on other trips of other lines between Gouda-Utrecht also large compositions are appointed with ADM.

4.3.2. Dead-heading trips
By comparing the original model with the model that includes dead-heading trips we can emphasize the added value of using dead-heading trips. To this end, we will compare the two models based on their objective values and on the number of cancelled trips in this subsection. When the solution of DHM does not use any dead-heading trips, it means that its objective value is equal to the objective value of OM.
Table 3 gives an overview of the results of both models. As can be seen, in cases 1, 2, and 3 there is always one additional trip cancelled in OM, while there are no trips cancelled when rescheduling with DHM. This is due to the fact that dead-heading trips are used in these cases to overcome a shortage of rolling stock units at Ut just after the disruption. As a consequence, the average objective value of DHM is smaller than the average objective value of OM. In cases 4, 5, 6, 7, 8, and 10 there are no differences between OM and DHM and in the cases, 6, 7, and 10 there is only a small difference between the models. Here, dead-heading trips are used to either decrease the number of unplanned shunting movements, the negative end of day deviations, or the number of seat-shortages. These dead-heading trips are only scheduled if the benefits outweigh the costs. So, when dead-heading trips are cheaper, they are used more.

There is no difference between the instances with a penalty value of 1 000 000 or 100 000 for cancelling an additional trip. Both OM and DHM always prevent trips from getting cancelled, if this is possible. Changing the other objective coefficients does not influence the results much, it slightly alters the optimal solutions.

Summarizing the results of OM and DHM, we can conclude that dead-heading trips are important to prevent additional trips from getting cancelled. In this way the passenger service increases.

4.3.3. Adjusted passenger demand

In this section we present the results of the models that include adjusted passenger demand: ADM, ADM(L), ADM(E), ADM(A) and DHADM. We compare these models based on the number of cancelled trips and on the passenger service (seat-shortages and passenger delay).
Table 4
Results of the models taking adjusted demand into account: the first five columns denote the average number of cancelled trips for all models over the 26 parameter settings. The last column shows the average number of dead-heading trips used in DHADM.

<table>
<thead>
<tr>
<th>Case</th>
<th>Cancelled trips</th>
<th>Dead-heading</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADM</td>
<td>ADM(L)</td>
<td>ADM(E)</td>
</tr>
<tr>
<td>1</td>
<td>2.68</td>
<td>2.64</td>
</tr>
<tr>
<td>2</td>
<td>2.16</td>
<td>2.12</td>
</tr>
<tr>
<td>3</td>
<td>2.08</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0.44</td>
<td>0.08</td>
</tr>
<tr>
<td>12</td>
<td>0.48</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Table 5
Results of the models taking adjusted demand into account. The last five columns denote the average number of seat-shortages.

<table>
<thead>
<tr>
<th>Case</th>
<th>Seat shortages</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADM</td>
<td>ADM(L)</td>
</tr>
<tr>
<td>1</td>
<td>168,578</td>
</tr>
<tr>
<td>2</td>
<td>112,461</td>
</tr>
<tr>
<td>3</td>
<td>60,837</td>
</tr>
<tr>
<td>4</td>
<td>44,496</td>
</tr>
<tr>
<td>5</td>
<td>24,424</td>
</tr>
<tr>
<td>6</td>
<td>3854</td>
</tr>
<tr>
<td>7</td>
<td>25,108</td>
</tr>
<tr>
<td>8</td>
<td>24,823</td>
</tr>
<tr>
<td>9</td>
<td>52,657</td>
</tr>
<tr>
<td>10</td>
<td>23,471</td>
</tr>
<tr>
<td>11</td>
<td>69,537</td>
</tr>
<tr>
<td>12</td>
<td>49,559</td>
</tr>
</tbody>
</table>

Table 6
Results of the models taking adjusted demand into account. The last five columns show the average passenger delay for all models.

<table>
<thead>
<tr>
<th>Case</th>
<th>Passenger delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADM</td>
<td>ADM(L)</td>
</tr>
<tr>
<td>1</td>
<td>119,940</td>
</tr>
<tr>
<td>2</td>
<td>848,46</td>
</tr>
<tr>
<td>3</td>
<td>447,54</td>
</tr>
<tr>
<td>4</td>
<td>45196</td>
</tr>
<tr>
<td>5</td>
<td>2308</td>
</tr>
<tr>
<td>6</td>
<td>3824</td>
</tr>
<tr>
<td>7</td>
<td>21503</td>
</tr>
<tr>
<td>8</td>
<td>22776</td>
</tr>
<tr>
<td>9</td>
<td>53157</td>
</tr>
<tr>
<td>10</td>
<td>23808</td>
</tr>
<tr>
<td>11</td>
<td>64848</td>
</tr>
<tr>
<td>12</td>
<td>50211</td>
</tr>
</tbody>
</table>

Tables 4, 5, and 6 give an overview of the results for these models. The first conclusion we can draw from the tables is that DHADM performs best in terms of cancelling the least amount of trips, the number of seat shortages, and the total passenger delay. DHADM takes next to adjusted passenger demand also dead-heading trips into account. So, in the DHADM, dead-heading trips are used to either decrease the number of cancelled trips, the number of seat-shortages, or the passenger delay. The gains of scheduling a dead-heading trip are worth the costs in all instances.

Secondly, there are more seat-shortages when rescheduling with ADM(L) than with ADM(E). ADM(L) boards passengers with the furthest destination away, while ADM(E) boards passengers with the nearest destination. Consequently, seats are sooner available if passengers leave the train earlier. So, with respect to the number of seat shortages, it is better for the
railway operator if passengers who need to get off the train first also board first. The \(ADM(A)\) strategy is less extreme, and most likely more realistic. A strategy in between both extreme strategies leads to less seat-shortages. However, a guided boarded strategy is not optimal. The number of seat-shortages is much lower after rescheduling with \(ADM\). The \(ADM\) decides which passengers board the train depending on the global objective. To summarize, in the worst case (\(ADM(L)\)) there will be many more seat-shortages than necessary, because the “wrong” passengers board the train first.

Thirdly, it holds for all models that sometimes more trips are cancelled than absolutely necessary, which can be deduced from the fact that more trips get cancelled than after rescheduling the instances with OM. This means that some trips with small passenger demand are cancelled. As a consequence, the rolling stock units of the cancelled trips can be used on the trips with a large passenger demand. In this way, there is more capacity available on the very busy lines during a disruption. Thus, by cancelling trips with small passenger demand, the total number of seat-shortages is reduced. This might not be ideal in practice. Therefore, we introduced an additional case where the penalty for cancelling a train is increased, see Section 4.3.3.1.

The penalty for cancelling a trip is important in the models with adjusted passenger demand. With a penalty value of 1 000 000 on average only 0.27 trips get cancelled with ADM, while with a penalty value of 100 000 on average 0.92 trips get cancelled. However, as a consequence, with a penalty value of 1 000 000 there are on average 57,768 seat shortage kilometers, while there are only 52,406 seat shortage kilometers on average with a penalty of 100 000. The rolling stock units that were used on the cancelled trips can be used on other trips to increase the capacity there. So, a trade off must be made between cancelling trips and the number of seat shortages.

To summarize, adjusted passenger demand has a large influence on the results. The demand does not decrease, and, as a result, the trade-off between cancelling additional trips and using larger compositions on other trips is no longer evident. If passengers board in the “worst” case scenario, as in \(ADM(L)\), then the seat-shortages will be much larger than if the passenger follow the “optimal” boarding strategy as in \(ADM\). Finally, dead-heading trips can be used to reduce the number of seat-shortages and the number of cancelled additional trips.

### 4.3.3.1. Cancelling less trains.

The rescheduling solutions when using adjusted passenger demand cancel more trains than necessary. This is a solution that is not ideal in practice. Therefore, we have increased the penalty value for cancelling a trip to 10 000 000 in order to test the models with adjusted passenger demand while cancelling the least amount of trips as possible. The three models \(ADM\), \(ADM(L)\), and \(ADM(E)\) cancel now as many trips as OM, while the model \(DHADM\) now cancels just as many trips as \(DHM\). Tables 7, 8, and 9 give an overview of the average number of cancelled trains, the average number of seat shortages, and the average passenger delays for the models when using different penalty values.
for cancelling a trip. As can be seen, the larger the penalty for cancelling a trip, the less trips get cancelled, but the more seat-shortages and/or passenger delays there will be.

4.3.4. Computation times

The models need to be fast in order to be useful in real-time. Therefore, all six models are compared based on their computation times. Fig. 9 shows the average computation time per case per model. The computation times for both ADM(L) and ADM(E) are much larger than the acceptable norm of 300 seconds, so only the computation times of the other models are shown in the figure. The OM is fastest, however DHM is not much slower. On average OM takes 22.8 sec to solve an instance, DHM 28.3 sec, DHADM 59.2 sec, ADM 73.9 sec, ADM(L) 360.5 sec, ADM(E) 621.2 sec, and ADM(A) 490.6 sec.

We can conclude that OM, DHM, DHADM, and ADM all have computation times that are acceptable for usage in practice.

5. Conclusion

In current literature models are developed to tackle the Rolling Stock Rescheduling Problem. However, these models are not applied in practice yet, because not all practical aspects are taken into account. In this paper we have included two of these practical aspects in the rolling stock rescheduling model.

First of all, we introduced the possibility of scheduling dead-heading trips from a station with an excess of inventory to a station with a shortage of inventory. These trips are called dead-heading trips and can be used to reduce the number of cancelled trips.

Secondly, adjusted passenger demand is taken into account in the model. Passengers stay in the railway system until they arrive at their destination. As a consequence, trains with more capacity will be appointed to the trips where the actual passenger demand is large.

Six different rolling stock rescheduling model versions have been tested. Results show that by using dead-heading trips the number of additional cancelled trips and the number of seat-shortages are reduced in comparison with the model versions where no dead-heading trips are included. Furthermore, the model versions where adjusted passenger demand is taken into account have appointed trains with more capacity to the trips just after the end of the disruption. This is due to the fact that the adjusted passenger demand is there the largest and appointing trains with a large capacity thus reduces the number of seat-shortages. Finally, the computation times are applicable in practice for the models where dead-heading trips and adjusted passenger demand are included.

There are interesting research possibilities for further research. First of all, results showed that if passengers with a nearer destination board the train first when there is not enough capacity for all passengers to board the train, this leads to a better global objective than if passengers with a further destination board the train. As a consequence, it might be interesting to investigate the usage of trains skipping stations, such that passengers with a further destination board those trains instead of claiming the capacity of the other trains. In this way both passengers with a nearer destination and with a further destination might become satisfied. Secondly, the assumption that passengers do not leave the railway system prematurely and do not take a detour to their destination should be relaxed in further research. Finally, other practical aspects need to be included in the Rolling Stock Rescheduling Problem as well. For instance, rolling stock units that have a maintenance appointment somewhere during the day should be included in the model together with adjusted passenger demand and dead-heading trips.

References


