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Stochastic dominance statistics for risk averters and risk seekers: an analysis of stock preferences for USA and China

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We derive the limiting process of stochastic dominance statistics for risk averters as well as for risk seekers when the underlying processes are dependent or independent. We take account of the dependency of the partitions and propose a bootstrap method to decide the critical point. In addition, we illustrate the applicability of the stochastic dominance statistics for both risk averters and risk seekers to analyse the dominance relationship between the Chinese and US stock markets in the entire period as well as the sub-periods before and after the crises, including the internet bubble and the recent sub-prime crisis. The findings could be used to draw inferences on the preferences of risk averters and risk seekers in investing in the Chinese and US stock markets. The results also enable us to examine whether there are arbitrage opportunities in these markets, and whether these markets are efficient and investors are rational.

Keywords: Stochastic dominance; Risk aversion; Risk-seeking; Test statistic; Hypothesis testing

JEL Classifications: C12, G0

1. Introduction

There are two basic approaches to the problem of portfolio selection under uncertainty. One approach is the mean-risk (MR) analysis. In this approach, the portfolio choice is made with respect to two measures—the expected portfolio mean return and portfolio risk. A portfolio is preferred if it has higher expected return and smaller risk. Among the MR analyses, the most popular measure is the Sharpe Ratio (SR) introduced by Sharpe (1966, 2009). As the SR requires strong assumptions that the returns of assets being analysed have to be iid, various measures for MR analysis have been developed to improve the SR, including the Sortino ratio, the conditional SR, the modified SR, Value-at-Risk (VaR), conditional VaR, expected shortfall, mixed Sharpe ratio and others. Readers may refer to Leung and Wong (2008), Bai *et al.* (2009, 2012) and the references therein for more information. A disadvantage of this approach is that it is derived by assuming the Von Neumann-Morgenstern quadratic utility function and that returns are normally distributed.

The other approach is to apply the concept of stochastic dominance (Hanoch and Levy 1969). The statistics developed in this paper use the second approach as it offers a mathematically rigorous treatment for portfolio selection. Moreover, there are convenient computational recipes and geometric interpretations of the trade-off between the two measures. A disadvantage of the former approach is that it is derived by assuming the Von Neumann-Morgenstern quadratic utility function and that returns are normally distributed (Feldstein 1969). In order to circumvent these limitations, a stochastic dominance (SD) test is introduced because an SD comparison is equivalent to an expected utility comparison.

It is well known that investors could be risk-averse and risk-seeking. Quirk and Saposnik (1962), Hanoch and Levy (1969), and others develop stochastic dominance (SD) theory for risk averters while Hammond (1974) and others develop the SD theory for risk seekers. On the other hand, Davidson and Duclos

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(hereafter DD, 2000) and others have developed an SD test for risk averters. In this paper, we extend their work to develop SD statistics, $T_j(x)$ $(j = 1; 2; 3)$, to determine the levels of significance for the j-order SD for two distributions F and G, drawing inference for the preferences for risk averters and risk seekers. We derive the limiting process of SD statistics when the underlying processes are dependent or independent. We take account of the dependency of the partitions and propose a bootstrap method to decide the critical point.

Thereafter, we illustrate the applicability of the SD statistics for both risk averters and risk seekers and analyse the dominance relationship between the Chinese and US stock markets in the entire period as well as the sub-periods before and after the financial crises, including the internet bubble and the recent sub-prime crisis. The findings could be used to draw inferences on the preferences of risk averters and risk seekers in investing in the Chinese and US stock markets. In addition, the results enable us to examine whether there is any arbitrage opportunity in these markets and whether these markets are efficient and investors are rational. We also note that a hierarchical relationship exists in SD mathematically (Sriboonchitta *et al.* 2009) but, in our analysis, we observe that the SD hierarchical relationship may not exist statistically. We will discuss this issue in our illustration section and propose an inference of this new finding for the literature.

We note that Bai *et al.* (2011) have extended the DD test to SD test for investors with S-shaped and reverse S-shaped utility functions, our paper extends their work in the sense that we (1) compliment their work to develop SD tests for risk averters and risk seekers, and (2) take care of the dependent and independent situations. In this paper, we also propose a bootstrap method to decide the critical point of the modified DD tests so that their critical values will be closer to the true critical values.

2. Definitions, notations and basic properties

We let Y and Z be random variables defined on $[a, b]$ with cumulative distribution functions (CDFs) F and G, and probability density functions (PDFs), f and g, respectively. We define $h(x) = H_0^A(x) = H_0^D(x)$ and

$$H_j^A(x) = \int_a^x H_{j-1}^A(y)\,dy, \quad H_j^D(x) = \int_a^x H_{j-1}^D(y)\,dy, \quad j = 2, 3; \tag{2.1}$$

where $H = F$ or G and $h = f$ or g.† $\mu_F = \mu_Y = \int_a^b x\, d\,F(x)$ is the mean of Y and $\mu_G = \mu_Z = \int_a^b x\, d\,G(x)$ is the mean of Z.

We note that H_j^A in (2.1) can be used to develop the SD theory for risk averters and thus we call this type of SD the ascending SD (ASD) and call H_j^A the jth order ASD integral or the jth order cumulative probability, since H_j^A is integrated from H_{j-1}^A in ascending order from the leftmost point of downside risk. On the other hand, H_j^D can be used to develop the SD theory for risk seekers and thus we call this type of SD the descending SD (DSD) and call H_j^D the jth order DSD integral or the jth order reversed cumulative probability, since H_j^D is integrated H_{j-1}^D in descending order from the rightmost point of upside profit. Y is said to dominate Z or F dominates G in the sense of j-order ASD [DSD], denoted by $Y \succeq_j^A Z$ or $F \succeq_j^A G$ [$Y \succeq_j^D Z$ or $F \succeq_j^D G$] if and only if

$$F_j^A(x) \le G_j^A(x) \ \left[F_j^D(x) \ge G_j^D(x)\right] \quad \forall x \in [a, b]\,. \tag{2.2}$$

If there exists strict inequality for any x in $[a, b]$, we say that Y straightly dominates Z and F straightly dominates G in the sense of strictly j-order ASD [DSD], denoted by $Y \succ_j^A Z$ or $F \succ_j^A G$ [$Y \succ_j^D Z$ or $F \succ_j^D G$] for $j = 1, 2, 3$. We define of sets of utility functions‡, $U_j^A(U_j^{SA})$ $[U_j^D(U_j^{SD})]$, for risk averters [risk seekers]:

$$U_j^A(U_j^{SA})\ [U_j^D(U_j^{SD})] = \{u : (-1)^i u^{(i)} \le (<) 0 \ \ [u^{(i)} \ge (>) 0],\ i = 1, \ldots, j\} \tag{2.3}$$

where $u^{(i)}$ is the ith derivative of u.

Choosing between F and G in accordance with a consistent set of preferences will satisfy the von Neumann-Morgenstern consistency properties. Accordingly, F is (strictly) preferred to G, or equivalently, Y is (strictly) preferred to Z if $\Delta Eu \equiv u(F) - u(G) \equiv u(Y) - u(Z) \ge 0 (> 0)$, where $u(F) \equiv u(Y) \equiv \int_a^b u(x) dF(x)$ and $u(G) \equiv u(Z) \equiv \int_a^b u(x) dG(x)$. The ASD and DSD approaches are regarded as two of the most useful tools for ranking uncertain investment prospects, since ranking assets have been proven (Li and Wong 1999) to be equivalent to utility maximization for the preferences of risk-averse and risk-seeking investors such that for $j = 1, 2$ and 3, $F \succeq_j^A (\succ_j^A) G$ if and only if $u(F) \ge (>) u(G)$ for any u in U_j^A (U_j^{SA}), and $F \succeq_j^D (\succ_j^D) G$ if and only if $u(F) \ge (>) u(G)$ for any u in U_j^D (U_j^{SD}).

The existence of ASD implies that the expected utility of the risk-averse investor is always higher when holding the dominant asset than when holding the dominated asset, and consequently, the dominated asset would not be chosen. We note that a hierarchical relationship exists in ASD: FASD implies SASD, which, in turn, implies TASD. However, the converse is not true. Similarly, the hierarchical relationships also exist in DSD. Thus, it is a common practice to report only the lowest dominance order of ASD and DSD.

3. SD tests for risk averters and risk seekers

The tests developed by Davidson and Duclos (DD, 2000), Barrett and Donald (2003) and Linton *et al.* (2005) are the most commonly used statistics to investigate the preference for risk averters. Since the test developed by DD is found§ to be one of the most powerful SD statistics and yet one of the least conservative in size, in this paper we discuss only the DD test and extend the theory by modifying only the DD test for risk averters and risk seekers.

We assume the data $\{f_i\}$ $(i = 1, 2, \ldots, N_f)$ and $\{g_i\}$ $(i = 1, 2, \ldots, N_g)$ are observations drawn from the independent random variables Y and Z with distribution functions F and G,

†see, for example, Li and Wong (1999), Wong and Li (1999), and Wong (2007) for more information.

‡We note that the theory can be easily extended to satisfy utilities defined to be non-differentiable and/or non-expected utility functions (Wong and Ma 2008).

§See Lean *et al.* (2008) and the references therein for more information.

respectively. We first propose to test the following hypothesis, for $j = 1, 2, 3$, H_0^A : $F_j^A \equiv G_j^A$, against three alternatives

$$H_1^A : F \not\equiv_j^A G, \ H_{1l}^A \ F \succ_j^A G_j, \text{ and } H_{1r}^A \ F \prec_j^A G. \quad (3.1)$$

The three hypotheses are equivalent to $H_1^A : F_j^A(x) \neq G_j^A(x)$ for some x, $H_{1l}^A : F_j^A(x) \leq G_j^A(x), \forall x$ and the inequality is strict for at least one x, and $H_{1r}^A : F_j^A(x) \geq G_j^A(x), \forall x$ and the inequality is strict for at least one x. The integrals F_j^A and G_j^A for F and G are defined in (2.1) for $j = 1, 2$ and 3. For a grid of pre-selected points $\{x_k, k = 1, \ldots, K\}$, one could obtain the following jth order ascending DD test statistic, $T_j^A(x)$ ($j = 1$, 2 and 3) to test for H_1^A, H_{1l}^A, and H_{1r}^A:

$$T_j^A(x) = \frac{\hat{F}_j^A(x) - \hat{G}_j^A(x)}{\sqrt{\hat{V}_j^A(x)}}, \quad (3.2)$$

where

$$\hat{V}_j^A(x) = \hat{V}_{F_j}^A(x) + \hat{V}_{G_j}^A(x), \ \hat{H}_j^A(x) = \frac{1}{N_h(j-1)!}\sum_{i=1}^{N_h}(x - h_i)_+^{j-1}, \quad (3.3)$$

$$\hat{V}_{H_j}^A(x) = \frac{1}{N_h}\left[\frac{1}{N_h((j-1)!)^2}\sum_{i=1}^{N_h}(x - h_i)_+^{2(j-1)} - \hat{H}_j^A(x)^2\right], \quad H = F, G; \ h = f, g.$$

It is not difficult to show that under the null hypothesis H_0^A, for each fixed $x \in (a, b)$, $T_j^A(x) \to N(0, 1)$, the standard normal distribution. However, the limiting joint distribution of $(T_j^A(x_1), T_j^A(x_2))$ does not have independent components for any $x_1 \neq x_2$. More importantly, the limiting correlation depends on the null in which the distribution functions $F = G$. Therefore, it is empirically difficult to test the hypotheses described in (3.1). One may incorporate the idea proposed by Bishop *et al.* (1992) to test the null hypothesis for a pre-designated finite number of values $\{x_k, k = 1, \ldots, K\}$ and then test the following weaker hypotheses: $H_0^{AK} : F_j^A(x_k) = G_j^A(x_k)$ for all x_k; $H_1^{AK} : F_j^A(x_k) \neq G_j^A(x_k)$ for some x_k; $H_{1l}^{AK} : F_j^A(x_k) \leq G_j^A(x_k)$ for all x_k and $F_j^A(x_k) < G_j^A(x_k)$ for some x_k; $H_{1r}^{AK} : F_j^A(x_k) \geq G_j^A(x_k)$ for all x_k and $F_j^A(x_k) > G_j^A(x_k)$ for some x_k.

We modify the DD test for risk averters to be DD test statistic for risk seekers and we call it the descending DD test statistic. For a grid of pre-selected points $\{x_k, k = 1, \ldots, K\}$, the jth order descending DD test statistic, $T_j^D(x)$ ($j = 1, 2$ and 3), is:

$$T_j^D(x) = \frac{\hat{F}_j^D(x) - \hat{G}_j^D(x)}{\sqrt{\hat{V}_j^D(x)}}, \quad (3.4)$$

where

$$\hat{V}_j^D(x) = \hat{V}_{F_j}^D(x) + \hat{V}_{G_j}^D(x), \quad \hat{H}_j^D(x) = \frac{1}{N_h(j-1)!}\sum_{i=1}^{N_h}(h_i - x)_+^{j-1},$$

$$\hat{V}_{H_j}^D(x) = \frac{1}{N_h}\left[\frac{1}{N_h((j-1)!)^2}\sum_{i=1}^{N_h}(h_i - x)_+^{2(j-1)} - \hat{H}_j^D(x)^2\right], \quad H = F, G; \ h = f, g;$$

in which the integrals ${F_j}^D$ and ${G_j}^D$ are defined in (2.1) for $j = 1, 2$ and 3. For $k = 1, \ldots, K$, the following hypotheses are tested for risk seekers: $H_0^D : F_j^D(x_k) = G_j^D(x_k)$ for all x_k; $H_1^D : F_j^D(x_k) \neq G_j^D(x_k)$ for some x_k; $H_{1l}^D : F_j^D(x_k) \geq G_j^D(x_k)$ for all x_k, and $F_j^D(x_k) > G_j^D(x_k)$ for some x_k; and $H_{2r}^D : F_j^D(x_k) \leq G_j^D(x_k)$ for all x_k and $F_j^D(x_k) < G_j^D(x_k)$ for some x_k.

To implement the DD test, T_j^D, for risk seekers for $j = 1, 2, 3$, one should test the following hypothesis at each grid point being computed: H_0^D : $F_j^D \equiv G_j^D$, against three alternatives

$$H_1^D \ F \not\equiv_j^D G, \ H_{1l}^D : F \succ_j^D G_j, \text{ and } H_{1r}^D \ F \prec_j^D G. \quad (3.5)$$

The three hypotheses are equivalent to $H_1^D : F_j^D(x) \neq G_j^D(x)$, for some x, $H_{1l}^D : F_j^D(x) \geq G_j^D(x)$, $\forall x$ and the inequality is strict for at least one x, and $H_{1r}^D : F_j^D(x) \leq G_j^D(x)$, $\forall x$ and the inequality is strict for at least one x.

To investigate the limiting distribution of $T_j^A(x)$ and $T_j^D(x)$ regarded as stochastic processes, we establish the following theorem.

THEOREM 3.1 *Let $\{f_i\}$ ($i = 1, 2, \ldots, N_f$) and $\{g_i\}$ ($i = 1, 2, \ldots, N_g$) be random observations drawn from the independent random variables Y and Z, with CDFs, F and G, respectively. Under the null hypothesis $F \equiv G$, the j-order ASD [DSD] test statistics, $T_j^A(x)$ $[T_j^D(x)]$ ($j = 1$, 2 and 3), weakly tends to a limiting Gaussian process with mean 0, variance 1, and correlation function $r^A(x, y)$ $[r^D(x, y)]$ in which for the case $j = 1$, we get*

$$r_1^A(x, y) = \frac{F(x \wedge y) - F(x)F(y)}{\sqrt{F(x)F(y)(1 - F(x))(1 - F(y))}},$$

$$r_1^D(x, y) = \frac{F(x \wedge y) - F(x)F(y)}{\sqrt{F(x)F(y)(1 - F(x))(1 - F(y))}},$$

and for the case $j > 1$, we have

$$r^A(x, y) = \frac{\int_a^x \int_a^y (x - t)^{j-2}(y - s)^{j-2}(F(t \wedge s) - F(t)F(s))dtds}{\sqrt{V_j^A(x)V_j^A(y)}},$$

$$r^D(x, y) = \frac{\int_x^b \int_y^b (t - x)^{j-2}(s - y)^{j-2}(F(t \wedge s) - F(t)F(s))dtds}{\sqrt{V_j^D(x)V_j^D(y)}},$$

where $\hat{H}_j^A(x)$ and $\hat{V}_j^A(x)$ are defined in (3.2) and $\hat{H}_j^D(x)$ and $\hat{V}_j^D(x)$ are defined in (3.4) for $H = F$ and G, $V_1^A(x) = F(x)(1 - F(x))$, $V_j^A(x) = \int_a^x \int_a^x (x - t)^{j-2}(x - s)^{j-2}(F(t \wedge s) - F(t)F(s))dtds$, $V_1^D(x) = F(x)(1 - F(x))$, and $V_j^D(x) = \int_x^b \int_x^b (t - x)^{j-2}(s - x)^{j-2}(F(t \wedge s) - F(t)F(s))dtds$.

The proof of this theorem is given in Appendix. Based on this theorem, to test the hypotheses in (3.1), we propose to reject the null hypothesis H_0^A if $\max_{a<x<b} |T_j^A(x)| > M_{\alpha/2}^A$, for the alternative H_1^A; $\min_{a<x<b} T_j^A(x) < -M_\alpha^A$, for the alternative H_{1l}^A; and $\max_{a<x<b} T_j^A(x) > M_\alpha^A$, for the alternative H_{1r}^A. Similarly, to test the hypotheses (3.5), we propose to reject the null hypothesis H_0^D if $\max_{a<x<b} |T_j^D(x)| > M_{\alpha/2}^D$, for the alternative H_1^D; $\max_{a<x<b} T_j^D(x) > M_\alpha^D$, for the alternative H_{1l}^D; and $\min_{a<x<b} T_j^D(x) < -M_\alpha^D$, for the alternative H_{1r}^D. We suggest to obtain the critical values M_α^A and M_α^D by a bootstrap approach that will be described later in the paper.

In many cases, the asset random variables Y and Z are not independent of each other. Assume that they have a joint distribution $Q(y, z)$ with marginal distributions F and G. Suppose that the sample is drawn in a way that $\{(f_i, g_i), i = 1, \ldots, m,\ f_k, g_l, k = m+1, \ldots, N_f; l = m+1, \ldots, N_g\}$ are mutually independent. In such a case, the covariance of $\hat{F}_j^A(x)$ and $\hat{G}_j^A(x)$ and the covariance of $\hat{F}_j^D(x)$ and $\hat{G}_j^D(x)$ can be estimated by

$$\hat{V}^A_{FG_j}(x) = \frac{1}{N_f N_g((j-1)!)^2} \sum_{i=1}^{m} (x - f_i)_+^{j-1}(x - g_i)_+^{j-1} - \frac{m}{N_f N_g} \hat{F}_j^A(x)\hat{G}_j^A(x)$$

$$\hat{V}^D_{FG_j}(x) = \frac{1}{N_f N_g((j-1)!)^2} \sum_{i=1}^{m} (f_i - x)_+^{j-1}(g_i - x)_+^{j-1} - \frac{m}{N_f N_g} \hat{F}_j^D(x)\hat{G}_j^D(x),$$

respectively. Hence, the variance of $\hat{F}_j^A(x) - \hat{G}_j^A(x)$ and the variance of $\hat{F}_j^D(x) - \hat{G}_j^D(x)$ can be estimated by

$$\hat{V}_j^A(x) = \hat{V}^A_{F_j}(x) + \hat{V}^A_{G_j}(x) - 2\hat{V}^A_{FG_j}(x), \tag{3.6}$$

$$\hat{V}_j^D(x) = \hat{V}^D_{F_j}(x) + \hat{V}^D_{G_j}(x) - 2\hat{V}^D_{FG_j}(x), \tag{3.7}$$

Thereafter, one may define the test statistics $T_j^A(x)$ and $T_j^D(x)$ by (3.2) and by (3.4) with the new $\hat{V}_j^A(x)$ and $\hat{V}_j^D(x)$ defined in (3.6) and (3.7), respectively. We summarize the result in the following theorem:

THEOREM 3.2 *Let $\{f_i\}$ $(i = 1, 2, \ldots, N_f)$ and $\{g_i\}$ $(i = 1, 2, \ldots, N_g)$ be random observations drawn from the dependent random variables Y and Z, with continuous joint distribution function $Q(x, y)$ and marginal distribution functions F and G, respectively. Suppose the samples are drawn as described earlier; that is, we assume that (f_i, g_i); $i = 1, \ldots, m$ are pairwisely drawn and the rest are independent. Suppose that $m/(N_f + N_g) \to \lambda$. Then, under the null hypothesis $F \equiv G$, the j-order ASD [DSD] test statistic, $T_j^A(x)$ $[T_j^D(x)]$ $(j = 1, 2$ and $3)$, weakly tends to a limiting Gaussian process with mean 0, variance 1, and correlation function $r^A_{j,\,dep}(x, y)$ $[r^D_{j,\,dep}(x, y)]$ in which for the case $j = 1$, we get*

$$r^A_{1,\,dep}(x, y) = \frac{F(x \wedge y) - \lambda Q(x, y) - (1 - 2\lambda)F(x)F(y)}{\sqrt{F(x)F(y)(1 - F(x))(1 - F(y))}},$$

$$r^D_{1,\,dep}(x, y) = \frac{F(x \wedge y) - \lambda Q(x, y) - (1 - 2\lambda)F(x)F(y)}{\sqrt{F(x)F(y)(1 - F(x))(1 - F(y))}},$$

and for the case $j > 1$, we have $r^A_{j,\,dep}(x, y) = V_j^A(x, y)/\sqrt{V_j^A(x)V_j^A(y)}$ and $r^D_{j,\,dep}(x,y) = V_j^D(x,y)/\sqrt{V_j^D(x)V_j^D(y)}$, where $\hat{H}_j^A(x)$ and $\hat{V}_j^A(x)$ for $H = F$ and G are defined in (3.3) and (3.6), $\hat{H}_j^D(x)$ and $\hat{V}_j^D(x)$ for $H = F$ and G are defined in (3.3) and (3.7), respectively, $V_j^A(x) = \int_a^x \int_a^x (x-t)^{j-2} (x-s)^{j-2}(F(t \wedge s) - F(t)F(s))dtds$, $V_j^A(x, y) = \int_a^x \int_a^y (x-t)^{j-2}(y-s)^{j-2}[(F(t \wedge s) - \lambda(Q(t, s) + Q(s, t)) - (1-2\lambda)F(t)F(s))]dtds$, $V_j^D(x) = \int_x^b \int_x^b (t-x)^{j-2}(s-x)^{j-2} (F(t \wedge s) - F(t)F(s))dtds$, and $V_j^D(x, y) = \int_x^b \int_y^b (t-x)^{j-2} (s-y)^{j-2}[(F(t \wedge s) - \lambda(Q(t, s) + Q(s, t)) - (1-2\lambda)F(t)F(s))] dtds$.

4. Determination of critical values

In this section, we discuss only the situation in which the variables are independent. The situation in which the variables are dependent could be obtained similarly. Suppose the variables being examined are independent and the sample series $\{f_i, i = 1, 2, \ldots, N_f\}$ and $\{g_i, i = 1, 2, \ldots, N_g\}$ are iid. We draw two resamples $\{f_i^*, i = 1, 2, \ldots, N_f\}$ and $\{g_i^*, i = 1, 2, \ldots, N_g\}$ from the pooled sample $\{f_{(i)}, g_{(j)}, i = 1, 2, \ldots, N_f, j = 1, 2, \ldots, N_g\}$, the DD test statistics ($\hat{T}_j^A$ and $\hat{T}_j^D$) $(j = 1, 2, 3)$ defined in (3.2) and (3.4) can then be bootstrapped. Using this method, one can approximate the null distribution of the test statistics. The details to obtain the **critical values** of statistic $\hat{T}_j^A$ and $\hat{T}_j^D$, $j = 1, 2, 3$, are as follows:

Step 1 Draw a sample set $\{f_i^*, i = 1, 2, \ldots, N_f\}$ from $\{f_i, g_j, i = 1, 2, \ldots, N_f, j = 1, 2, \ldots, N_g\}$ with replacement and draw another sample set $\{g_i^*, i = 1, 2, \ldots, N_g\}$ in the same way.

Step 2 Compute: $A_j = \max_{a<x<b} \left|\hat{T}_j^{*A}(x)\right| = \max_{a<x<b} \left|\left[\hat{F}_j^{*A}(x) - \hat{G}_j^{*A}(x)\right]/\sqrt{\hat{V}_j^{*A}(x)}\right|$, and $D_j = \max_{a<x<b} \left|\hat{T}_j^{*D}(x)\right| = \max_{a<x<b} \left|\left[\hat{F}_j^{*D}(x) - \hat{G}_j^{*D}(x)\right]/\sqrt{\hat{V}_j^{*D}(x)}\right|$, where $\hat{H}_j{}^*(x)$ and $\hat{V}_j^*(x)$ for $H = F$ and G, respectively, are similarly defined in (3.2) with resample sets in Step 1. Repeat this process N times.

Step 3 Find $A_j(\alpha)$ such that $\sharp\{|A_{jk}| \geq A_j(\alpha), k \leq N\} = [N_\alpha]$; that is, the α percentile of the distribution of $\hat{T}_j^{*A}$ and find $D_j(\alpha)$ such that $\sharp\{|D_{jk}| \geq D_j(\alpha), k \leq N\} = [N_\alpha]$; that is, the α percentile of the distribution of $\hat{T}_j^{*D}$.

We note that the approach we used to get the critical values is similar to that used in Bai *et al.* (2011). We explain in the following remark for the theory to determine the critical values outlined in section 4.

Remark 4.1 We use bootstrap method to get the approximative critical value under null hypothesis. First, since we draw resample sets $\{f_i^*, g_j^*, i = 1, 2, \ldots, N_f, j = 1, 2, \ldots, N_g\}$ by sampling with replacement, they are iid random samples with distribution function $F^*_{N_f,N_g} = \frac{N_f}{N_f+N_g}F_{N_f} + \frac{N_g}{N_f+N_g}G_{N_g}$. For large N_f and N_g, $F^*_{N_f,N_g}$ is asymptotic to F under the null hypothesis $F = G$ and further $A_j(\alpha)$ and $D_j(\alpha)$ are asymptotic to the α percentile of the distribution of $\max_{a<x<b}\left|\hat{T}_j^A\right|$ and that of $\max_{a<x<b}\left|\hat{T}_j^D\right|$ under the null hypothesis. When $F \neq G$, though $A(\alpha)$ and $D(\alpha)$ are different from the real critical point values, since resample sets $\{f_i^*, i = 1, 2, \ldots, N_f\}$ and $\{g_j^*, j = 1, 2, \ldots, N_g\}$ have the same distribution $F^*_{N_f,N_g}$, they still can be used to reject the null hypotheses, because $\max_{a<x<b}|\hat{T}_j^A|$ and $\max_{a<x<b}|\hat{T}_j^D|$ are large while $\max_{a<x<b}|\hat{T}_j^{*A}|$ and $\max_{a<x<b}|\hat{T}_j^{*D}|$ are not.

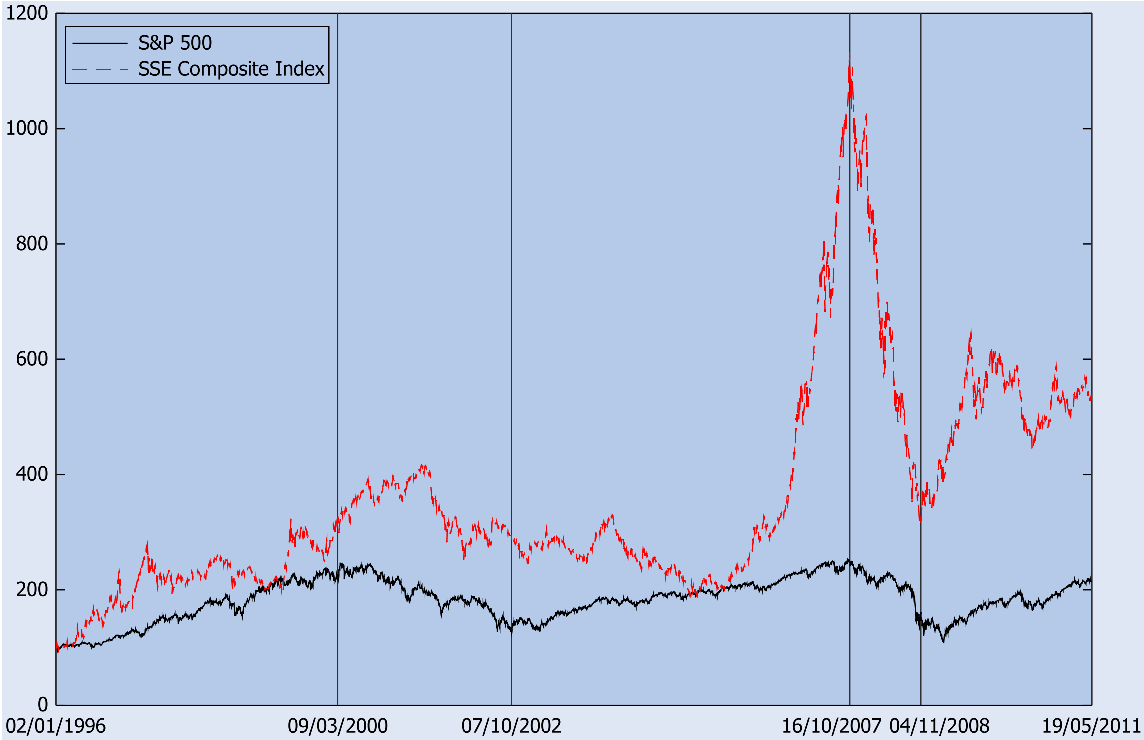

Figure 1. Time series plot of SSE and S&P500 from 02/01/1996 to 19/05/2011.

5. Empirical results

In this paper, we will illustrate the applicability of the SD statistics for both risk averters and risk seekers developed in this paper to analyze the dominance relationship between the Chinese and US stock markets in the entire period as well as the sub-periods before and after the financial crises (Barany *et al.* 2012), including the internet bubble and the recent subprime crisis (Lleo and Ziemba 2012). We are also interested in studying their preferences in these markets in the bull runs as well as in the bear markets. To do so, we apply the modified DD statistics developed above to examine the daily returns of both SSE and S&P 500 for the full sample period (02/01/1996 to 19/05/2011) and the five sub-periods: 01/1996 to 03/2000, 03/2000 to 10/2002, 10/2002 to 10/2007, 10/2007 to 11/2008, and 11/2008 to 05/2011 (figure 1). SSE denotes the Shanghai Stock Exchange Composite Index, while S&P 500 denotes the Standard & Poor 500 index.

To analyse the preferences for risk averters, we adopt the modified DD statistics T_j^A (we call it ASD test) and T_j^D (we call it DSD test) for risk averters and risk seekers, respectively, where T_j^A is defined in (3.2) and T_j^D is defined in (3.4) for $j = 1, 2$, and 3 with F and G be the CDFs of the returns of $S\&P$ and SSE, respectively. We also follow Fong *et al.* (2005), Gasbarro *et al.* (2007) and others to use the 10 major partitions with 10 minor partitions within any two consecutive major partitions in each comparison and draw statistical inference. We display the results in tables 1 and 2, plot the ASD tests together with the CDFs, F and G, of the daily returns of $S\&P$ and SSE in figure 2, and plot the DSD tests together with the DSD integrals F_1^D and G_1^D defined in (2.1) in figure 3. At last, we summarize all SD relationship between SSE and S&P 500 for the full sample period as well as all the five sub-periods in table 3.

5.1. First-order stochastic dominance analysis

We first analyse the performance of the daily returns of the Chinese and US stock markets for risk averters in the full sample period. To minimize the Type II error and to use the idea of almost SD (Leshno and Levy 2002, Guo *et al.* 2013), we use a 5% cut-off point to the proportion of the values of the test statistic in (3.2) for SSE and S&P to draw statistical inferences. Using the 5% cut-off point, if S&P dominates SSE, one should find at least 5% of T_j^A to be significantly negative and no portion of T_j^A to be significantly positive (table 1). The reverse holds if SSE dominates S&P. From table 1, we find that 34.92% (27.92%) of T_1^A is significantly positive (negative) for the entire period. Thus, the results lead us reject the hypothesis that S&P stochastically dominates SSE or vice versa in the sense of first-order ASD (FASD).

Together with the plot of the modified ASD test for risk averters exhibited in figure 2, the results from table 1 show that T_1^A is significantly negative in the downside risk and significantly positive in the upside profit, inferring that investors with increasing utility functions will prefer to invest in S&P when facing with the downside risk, whereas they will prefer to invest in SSE in anticipation of the upside profit. This could also infer that investors prefer to invest in US stock market in the bear markets and prefer to invest in Chinese stock market in the bull runs. The result of the first-order DSD (FDSD) exhibited in table 2 and figure 3 draws the same inference as the result of FASD that S&P is preferred to SSE on the downside risk and SSE is preferred on the upside profits. These results do not reject the market efficiency hypothesis. In order to explore whether the market is efficient and investors are rational, we need to examine the higher order SD.

Table 1. Results of the modified ASD test statistic for the risk averters.

02/01/1996–19/05/2011	FASD		SASD		TASD	
F = S&P500, G = SSE	$T_1^A > 0$	$T_1^A < 0$	$T_2^A > 0$	$T_2^A < 0$	$T_3^A > 0$	$T_3^A < 0$
Total(%)	34.92	27.92	0	89.57	0	98.98
Positive domain(%)	34.92	0	0	0	0	0
Negative domain(%)	0	27.92	0	89.57	0	98.98
max $(\|T_j^A\|)$	9.10	7.36	0.68	7.16	1.00	6.20
02/01/1996–09/03/2000	FASD		SASD		TASD	
F = S&P500, G = SSE	$T_1^A > 0$	$T_1^A < 0$	$T_2^A > 0$	$T_2^A < 0$	$T_3^A > 0$	$T_3^A < 0$
Total(%)	36.91	31.59	0	91.29	0	99.32
Positive domain(%)	36.91	0	0	0	0	0
Negative domain(%)	0	31.59	0	91.29	0	99.32
max $(\|T_j^A\|)$	8.35	5.94	0.54	6.35	1.00	5.46
09/03/2000–07/10/2002	FASD		SASD		TASD	
F = S&P500, G = SSE	$T_1^A > 0$	$T_1^A < 0$	$T_2^A > 0$	$T_2^A < 0$	$T_3^A > 0$	$T_3^A < 0$
Total(%)	38.82	4.02	63.98	0	20.21	0
Positive domain(%)	38.82	0	63.98	0	20.21	0
Negative domain(%)	0	4.02	0	0	0	0
max $(\|T_j^A\|)$	4.83	3.59	3.52	1.00	2.14	1.00
07/10/2002–16/10/2007	FASD		SASD		TASD	
F = S&P500, G = SSE	$T_1^A > 0$	$T_1^A < 0$	$T_2^A > 0$	$T_2^A < 0$	$T_3^A > 0$	$T_3^A < 0$
Total(%)	30.34	23.81	0	87.25	0	98.98
Positive domain(%)	30.34	0	0	0	0	0
Negative domain(%)	0	23.81	0	87.25	0	98.98
max $(\|T_j^A\|)$	8.86	7.48	1.04	6.84	0.74	5.42
16/10/2007–04/11/2008	FASD		SASD		TASD	
F = S&P500, G = SSE	$T_1^A > 0$	$T_1^A < 0$	$T_2^A > 0$	$T_2^A < 0$	$T_3^A > 0$	$T_3^A < 0$
Total(%)	0	25.28	0	78.86	0	61.13
Positive domain(%)	0	0	0	0	0	0
Negative omain(%)	0	25.28	0	78.86	0	61.13
max $(\|T_j^A\|)$	2.79	4.39	1.41	3.49	1.39	2.74
04/11/2008–19/05/2011	FASD		SASD		TASD	
F = S&P500, G = SSE	$T_1^A > 0$	$T_1^A < 0$	$T_2^A > 0$	$T_2^A < 0$	$T_3^A > 0$	$T_3^A < 0$
Total(%)	15.94	0	0	0	0	0
Positive domain(%)	15.94	0	0	0	0	0
Negative domain(%)	0	0	0	0	0	0
max $(\|T_j^A\|)$	3.73	2.41	1.00	1.14	1.00	0.56

Note: This table summarizes the modified ASD test results for risk averters. The table reports the percentages of modified ASD statistic that are significantly negative or positive at the 5% significance level, based on the critical value generated from a bootstrap method discussed in Section 4. The test statistic $T_j^A(x)$ is defined in (3.2) for $j = 1, 2$, and 3 with $F = S\&P$ and $G = SSE$. FASD, SASD and TASD stand for first-, second-, and third-order ASD, respectively.

5.2. Higher order stochastic dominance

We now examine whether there is any higher order SD which is commonly used, see, for example, Fábián *et al.* (2011). From table 1, we observe that 89.57% of the second-order modified ASD statistic T_2^A is significantly negative and none of it is significantly positive at the 5% bootstrap-simulated critical level. Similarly, from table 1, we find that 98.98% of the third-order modified ASD statistic T_3^A is significantly negative and none of it is significantly positive at the 5% bootstrap-simulated critical level. Hence, we conclude that there is a dominance of S&P over SSE in terms of both second-, and third-order ASD (SASD and TASD) at the 5% significant level, inferring that second- and third-order risk averters prefer to invest in the US stock market rather than the Chinese stock market. In addition, we also apply the testing procedure by using $\max_x |T_j^A(x)|$. The inference drawn from this approach leads to the same conclusion. Following the suggestion from Falk and Levy (1989) and others, one may conclude that the markets are not efficient and investors are not rational. In order to check whether the market is efficient and investors are rational, we suggest studying the preference of risk seekers.

5.3. Preference of risk seekers

In order to study the preferences of risk seekers between the Chinese and US stock markets, we adopt the DSD theory and

Table 2. Results of the modified DSD test statistic for the risk seekers.

02/01/1996–19/05/2011	FDSD		SDSD		TDSD	
F = S&P500, G = SSE	$T_1^D > 0$	$T_1^D < 0$	$T_2^D > 0$	$T_2^D < 0$	$T_3^D > 0$	$T_3^D < 0$
Total(%)	27.92	34.92	0	96.42	0	98.96
Positive domain (%)	27.92	0	0	0	0	0
Negative domain (%)	0	34.92	0	96.42	0	98.96
max ($\|T_j^A\|$)	7.36	9.10	1.35	8.72	1.24	6.92
02/01/1996–09/03/2000	FDSD		SDSD		TDSD	
F = S&P500, G = SSE	$T_1^D > 0$	$T_1^D < 0$	$T_2^D > 0$	$T_2^D < 0$	$T_3^D > 0$	$T_3^D < 0$
Total (%)	31.59	36.91	0	96.95	0	99.46
Positive domain (%)	31.59	0	0	0	0	0
Negative domain (%)	0	36.91	0	96.95	0	99.46
max ($\|T_j^A\|$)	5.94	8.35	0.54	8.06	1.00	6.96
09/03/2000–07/10/2002	FDSD		SDSD		TDSD	
F = S&P500, G = SSE	$T_1^D > 0$	$T_1^D < 0$	$T_2^D > 0$	$T_2^D < 0$	$T_3^D > 0$	$T_3^D < 0$
Total (%)	4.02	43.38	0	0	0	0
Positive domain (%)	4.02	0	0	0	0	0
Negative domain (%)	0	43.38	0	0	0	0
max ($\|T_j^A\|$)	3.59	4.83	2.23	1.63	0.73	1.51
07/10/2002–16/10/2007	FDSD		SDSD		TDSD	
F = S&P500, G = SSE	$T_1^D > 0$	$T_1^D < 0$	$T_2^D > 0$	$T_2^D < 0$	$T_3^D > 0$	$T_3^D < 0$
Total (%)	23.81	30.34	0	96.30	0	98.36
Positive domain (%)	23.81	0	0	0	0	0
Negative domain (%)	0	30.34	0	96.30	0	98.36
max ($\|T_j^A\|$)	7.48	8.86	1.00	8.64	1.00	7.32
16/10/2007–04/11/2008	FDSD		SDSD		TDSD	
F = S&P500, G = SSE	$T_1^D > 0$	$T_1^D < 0$	$T_2^D > 0$	$T_2^D < 0$	$T_3^D > 0$	$T_3^D < 0$
Total (%)	25.28	0	0	0	0	0
Positive domain (%)	25.28	0	0	0	0	0
Negative domain (%)	0	0	0	0	0	0
max ($\|T_j^A\|$)	4.39	2.79	1.53	1.75	1.31	1.09
04/11/2008–19/05/2011	FDSD		SDSD		TDSD	
F = S&P500, G = SSE	$T_1^D > 0$	$T_1^D < 0$	$T_2^D > 0$	$T_2^D < 0$	$T_3^D > 0$	$T_3^D < 0$
Total (%)	0	15.94	0	0	0	0
Positive domain (%)	0	0	0	0	0	0
Negative domain (%)	0	15.94	0	0	0	0
max ($\|T_j^A\|$)	2.41	3.73	0.81	2.01	0.74	1.00

Note: This table summarizes the modified DSD test results for risk seekers. The table reports the percentages of modified DSD statistic that are significantly negative or positive at the 5% significance level, based on the critical value generated from a bootstrap method discussed in Section 4. The test statistic $T_j^D(x)$ is defined in (3.4) for $j = 1, 2$, and 3 with $F = S\&P$ and $G = SSE$. FDSD, SDSD and TDSD stand for first-, second-, and third-order DSD, respectively.

employ the DSD statistic, T_j^D, for risk seekers as stated in (3.4) for $j = 2$ and 3 to conduct the analysis. From table 2, we find that 96.42% of T_2^D is significantly negative and no portion of T_2^D is significantly positive at the 5% significant level for the whole sample period. This implies that SSE stochastically dominates S&P in the sense of second-order SD (SDSD), and thus we conclude that second-order risk seekers prefer SSE to S&P for the whole sample period. Similarly, from table 2, we find that 98.96% of T_3^D is significantly negative and no portion of T_3^D is significantly positive at the 5% level for the whole sample period. This implies that SSE stochastically dominates S&P in the sense of third-order DSD (TDSD), and thus third-order risk seekers prefer SSE to S&P for the whole sample period.

Different from the conclusion drawn in the ASD test in which one could conclude that risk averters prefer to invest in the US stock market rather than the Chinese stock market, our DSD analysis reveals the reverse preference for risk seekers that they are attracted to the Chinese stock market to maximize their expected utilities. Is there any inference on market efficiency and rationality from our findings? We will discuss the issue in next section.

5.4. *Robustness analysis in the sub-periods*

We turn to investigate the preferences for risk averters and risk seekers in each of the sub-periods and we first discuss their

Table 3. Results of ASD and DSD tests for the risk averters and risk seekers.

01/1996–05/2011	G = S&P500	SSE
F = S&P500		SASD
SSE	SDSD	
01/1996–03/2000	G = S&P500	SSE
F = S&P500		SASD
SSE	SDSD	
03/2000–10/2002	G = S&P500	SSE
F = S&P500		FASD/FDSD#
SSE		
10/2002–10/2007	G = S&P500	SSE
F = S&P500		SASD
SSE	SDSD	
10/2007–11/2008	G = S&P500	SSE
F = S&P500		FASD/FDSD*
SSE		
11/2008–05/2011	G = S&P500	SSE
F = S&P500		
SSE	FASD*/FDSD*	

Note: Here all the 'SD' results are 'F' (in the first column) SD 'G' (in the first row). * means SD marginally and # means SD when taking the concept of almost SD, Readers may refer to the text for more details.

relationship in the sense of the FSD. Table 1 indicates that 36.9, 38.8, 30.3, 0 and 15.9% of T_1^A are significantly positive in the first, second, third, fourth, and fifth sub-periods, respectively, and all are occurred in the positive domain. On the other hand, 31.6, 4.0, 23.8, 25.3 and 0% of T_1^A are significantly negative for the first, second, third, fourth, and fifth sub-periods, respectively, and all are occurred in the negative domain. Since the analysis of the FDSD is the same as that of the FASD, we skip the discussion of the FDSD analysis. From our finding, we notice that only the first and the third sub-periods (both are in the bull runs) are similar to the ASD results for the entire period that the results lead us to reject the hypothesis that S&P stochastically dominates SSE or vice versa in the sense of FSD. Thus, for the first and third sub-periods, the conclusion drawn from our FSD analysis is the same as that drawn for the entire period that there is no FSD between S&P and SSE but S&P is preferred to SSE on the downside risk and SSE is preferred on the upside profits.

For the second sub-period from 9 March 2000 to 7 October 2002, the period after the burst of internet bubble, investors may expect that the stock market in the USA does not perform well and the Chinese stock market will be doing better than the US stock market. Our FSD analysis supports this argument. From table 2, we find that 38.8% of T_1^A are significantly positive in the positive domain whereas 4.0% of T_1^A is significantly negative in the negative domain. One may infer that, same as those obtained in the first and third sub-periods, there is no FSD between S&P and SSE but S&P is preferred to SSE on the downside risk and SSE is preferred on the upside profits. However, if one uses a 5% cut-off point to incorporate the idea of almost SD, one could claim that SSE stochastically dominates S&P in the sense of FSD.

From FSD analysis in the period after the burst of the sub-prime crisis from 16 October 2007 to 4 November 2008, one may recommend investors to invest in the Chinese stock market instead of the US market when the USA is in a financial crisis. However, this may not be true. From table 1, we notice that no T_1^A is significantly positive whereas it possesses 25.3% to be significantly negative, inferring that, surprisingly, the S&P stochastically dominates SSE in the sense of FSD and thus, investors should invest in the US stock market rather than the Chinese stock market during the sub-prime crisis.

Furthermore, one may believe investing in the US stock market is better than the Chinese stock market since the investment is better even during the sub-prime crisis. Our SD analysis finds that this may not be true either. Table 1 exhibits that 15.9% of T_1^A are significantly positive whereas none of it is significantly negative in the period during recovery after the burst of sub-prime crisis from 4 November 2008 to 19 May

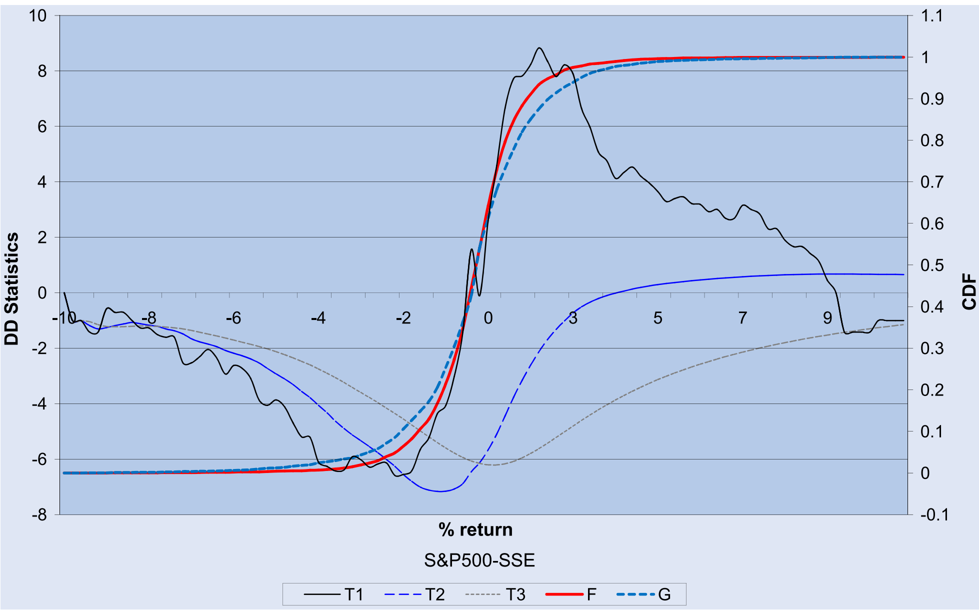

Figure 2. The modified ASD test statistics and the distribution functions F and G from 02/01/1996 to 19/05/2011.
Note: Tj is the test statistic $T_j^A(x)$ defined in (3.2) for $j = 1, 2,$ and 3 with $F = F_1^A = S\&P$ and $G = G_1^A = SSE$. F_1^A and G_1^A are defined in (2.1).

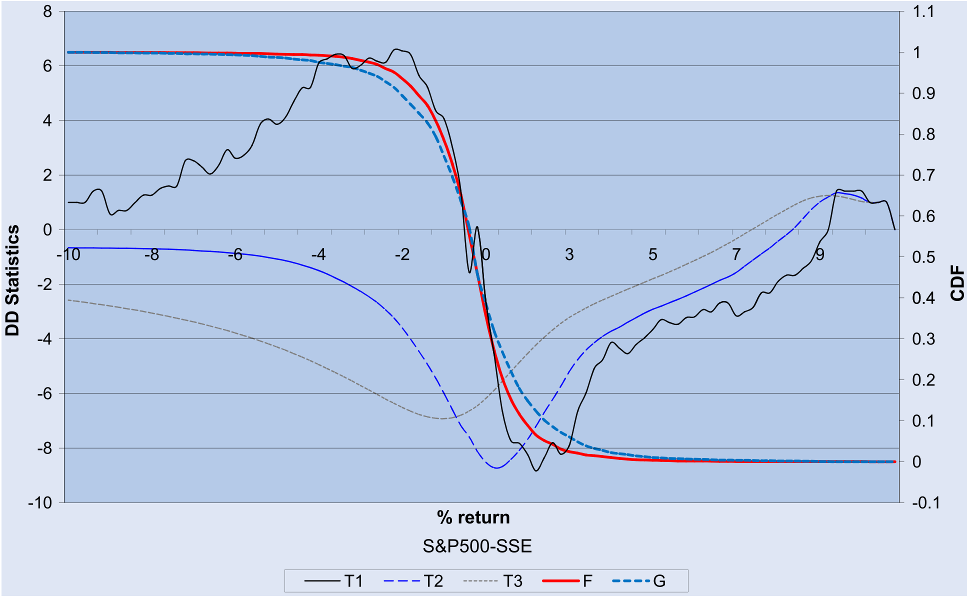

Figure 3. The modified DSD test statistics and the DSD integrals F_1^D and G_1^D from 02/01/1996 to 19/05/2011.
Note: Tj is the test statistic $T_j^D(x)$ defined in (3.4) for $j = 1, 2$, and 3 with $F = S\&P$ and $G = SSE$. F_1^D and G_1^D are defined in (2.1).

2011. One could conclude that SSE stochastically dominates the S&P in the sense of FSD and thus, investors should invest in the Chinese stock market instead of investing in the US stock market in the bull run during the recovery of the sub-prime crisis.

However, the situation is not that simple because though we find that SSE stochastically dominates the S&P in the sense of FASD, our findings indicate that SSE does not stochastically dominate the S&P in the sense of both SASD and TASD. One may believe that this result contradicts the hierarchical property for ASD that FASD implies SASD which, in turn, implies TASD and thus it is recommended that only the lowest dominance order of ASD is reported. However, our findings show that FASD does not imply SASD or TASD! We note that there is no contradiction. The hierarchical property does, in fact, hold for both ASD and DSD mathematically but this does not imply that the hierarchical property must hold statistically. Our finding in the fifth sub-period shows that it is possible that FASD statistically does not imply SASD or TASD statistically and thus in this paper we suggest that academics and practitioners report FSD, SSD and TSD, in case, say, X stochastically dominates Y FASD, but not SASD and TASD. We suggest to call it X stochastically dominates Y *marginally* in the sense of FASD.

6. Inference from our findings

The SD rules can be used to determine whether there is any opportunity for arbitrage and whether the markets are efficient and investors are rational. We discuss in this section.

6.1. Inference on arbitrage opportunity

Jarrow (1986) and Falk and Levy (1989) claim that if FSD exists, under certain conditions, arbitrage opportunities also exist, and investors will increase their wealth and expected utilities if they shift from holding the dominated asset to the dominant one. However, Wong *et al.* (2008) have shown that if FSD exists statistically, arbitrage opportunities may not exist, but investors can increase their expected wealth as well as their expected utilities if they shift from holding the dominated asset to the dominant one.

To check whether there is any arbitrage opportunity, one can apply the FSD analysis to the assets for comparison. In our analysis discussed in section 5.1, although we find that S&P FSD dominates SSE in the downside returns while SSE FSD dominates S&P in the upside returns, S&P does not FSD dominate SSE over the entire distribution and vice versa. This implies that there is no arbitrage opportunity in the US and Chinese stock markets over the entire period studied in this paper.

However, in section 5.1, we find that (1) in the second sub-period from 9 March 2000 to 7 October 2002, the period after the burst of internet bubble, if one adopts a 5% cut-off point rule, one could claim that SSE FSD dominates S&P; (2) in the period after the burst of the sub-prime crisis from 16 October 2007 to 4 November 2008, the S&P FSD dominates SSE; and (3) in the period during recovery from the sub-prime crisis from 4 November 2008 to 19 May 2011, SSE FSD dominates the S&P marginally.

Could these findings infer that there are arbitrage opportunities in the US and Chinese stock markets and these markets are not efficient? We will say, yes, our analysis supports the argument that there exists some arbitrage opportunities in the US and Chinese stock markets in some short periods and we will discuss the issue of market efficiency in the next subsection.

6.2. Inference on market efficiency and rationality

In section 5.5, in some short periods SSE FSD dominates S&P, whereas in some other short periods, the FSD relationship reverses. Indeed, these results do infer that there are some arbitrage opportunities in some short periods of time. However, since there is no FSD for a long period in these markets, we will say that our FSD findings do not reject that markets are efficient and investors are rational.

To further explore whether the market is efficient and investors are rational, we need to examine the higher order of SD as studied in section 5.2. If no SASD is found in the market containing X and Y, this suggests that risk-averse investors are indifferent between X and Y, so they will not switch X to Y, or vice versa, to increase their expected utility (Lean *et al.* 2010, Chan *et al.* 2012). In this situation, we claim that the market is rational and efficient. Similarly, if no TASD is found in the market containing X and Y, this implies that risk-averse investors with DARA are indifferent between X and Y. In this situation, we claim that the market is both rational and efficient.

Nonetheless, Falk and Levy (1989) claim that, given two assets, X and Y, if by switching from X to Y (or by selling X short and holding Y long), an investor can increase expected utility, the market is inefficient. SSD does not imply any arbitrage opportunities, but it does imply the preference of one asset over another by risk-averse investors. For example, as we found in section 5.2, S&P stochastically dominates SSE in the sense of both SASD and TASD. Thus, one may not make an expected profit by switching from SSE to S&P, but switching would allow risk-averse investors to increase their expected utility. In this situation, should we claim that the US and Chinese stock markets are inefficient and investors are irrational?

This claim could be made if one believes that the markets only contain risk-averse investors. However, it is well known that the market could have other types of investors (see e.g. Friedman and Savage (1948), Markowitz (1952), Thaler and Johnson (1990), Fong *et al.* (2008), Wong and Chan (2008), and Egozcue *et al.* (2011) for more discussion). If one believes that the markets could contain more than one type of investors, such as risk averters as well as risk seekers, one could find that one asset dominates another asset by ASD but is dominated by that asset by DSD. These are exactly the findings we obtained in this paper: S&P stochastically dominates SSE strictly in the sense of both SASD and TASD, while SSE stochastically dominate S&P strictly in the sense of SDSD and TDSD. Thus, risk averters could prefer to invest in the US stock market rather than the Chinese stock market, while risk seekers prefer to invest in the Chinese stock market rather than the US market. Then, in equilibrium, the number of trades that risk averters, who go long in the US market and/or short sell the Chinese stock market, would match the number of trades that risk seekers, who go long in the Chinese stock market and/or short sell the US market. In this situation, there is no pressure to push up or down the indices in the US and Chinese stock markets when both risk averters and risk seekers can attain what they seek. Under these conditions, we argue that the market remains efficient and investors are rational.

At last, we note that, conceptually, market rationality within the SD framework is not different from the conventional concepts captured by some of the rational asset pricing models, such as the CAPM. The only difference is that the latter approach defines an abnormal return as an excess return adjusted to some specific risk measure, while SD market rationality tests employ the whole distribution of returns. Given the imprecise knowledge of the best model, the SD approach with fewer restrictions on both investors' preferences and return distributions seems to help us understand the markets better.

7. Concluding remarks

We note that since Lean *et al.* (2008) find that the power and size of the test developed by DD is good, we expect the power and size of our proposed tests will be good because our proposed tests get better critical values than the DD test. In order to examine the power and size performance of our proposed tests, one has to conduct an extensive simulation to examine the power and size performance of our proposed tests and other SD tests to compare their performance. We leave this to future research.

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Appendix 1

Proof of Theorem 3.1 We only prove the DSD part of Theorem 3.1. The proof of the ASD part of the theorem could be obtained similarly. Without loss of generality, we assume $N_f = N_g = N$ in the proof. According to empirical process theorem†, for any continuous distribution function $F(x)$, we have

$$\sqrt{N}\big(F_N(x) - F(x)\big) \longrightarrow B\big(F(x)\big),$$

where F_N is the empirical distribution of N iid observations drawn from F and $B(\cdot)$ is the standardized Brownian bridge on the interval [0, 1].

Recall that

$$\hat{F}_j^D(x) = \frac{1}{N(j-1)!}\sum_{i=1}^{N}(f_i - x)_+^{j-1} = \frac{1}{(j-1)!}\int_x^b (t-x)^{j-1}dF_N(t) \quad \text{if } a \le x \le b\,.$$

Applying the empirical process theorem, for $j \ge 1$, we have

$$\sqrt{N}\big(\hat{F}_j^D(x) - F_j^D(x)\big) \to \frac{1}{(j-1)!}\int_x^b (t-x)^{j-1}dB\big(F(t)\big) \quad \text{if } a \le x \le b.$$

Because $F_j^D(x) = G_j^D(x)$ under null hypothesis, and $\hat{F}_j^D(x)$ and $\hat{G}_j^D(x)$ are independent, we get

$$\sqrt{N}\big(\hat{F}_j^D(x) - \hat{G}_j^D(x)\big) \to \frac{\sqrt{2}}{(j-1)!}\int_x^b (t-x)^{j-1}dB\big(F(t)\big) \quad \text{if } a \le x \le b.$$

By the law of large numbers, with probability 1, one could easily find that

$$\begin{cases} N\hat{V}_{FG_j}^D \to 0, \\ N\hat{V}_{F_j}^D \to \frac{1}{((j-1)!)^2}\int_a^x (t-x)^{2j-2}dF(t) - F_j^D(x)^2, \\ N\hat{V}_{G_j}^D \to \frac{1}{((j-1)!)^2}\int_a^x (t-x)^{2j-2}dG(t) - G_j^D(x)^2. \end{cases}$$

Thereafter, under the null hypothesis, we get

$$N(\hat{V}_{F_j}^D + \hat{V}_{G_j}^D) \to \frac{2}{((j-1)!)^2}\int_x^b (t-x)^{2j-2}dF(t) - F_j^D(x)^2 \overset{j\ge 1}{=} \frac{2}{((j-2)!)^2}\int_x^b\int_x^b (t-x)^{j-2}(s-x)^{j-2}(F(t\wedge s) - F(t)F(s))dtds\,.$$

†Readers may refer to Donsker (1952) and Wolfowitz (1954) for more information about empirical process theorem.

Using integration by parts, when $j > 1$, we have

$$\frac{\sqrt{2}}{(j-1)!}\int_x^b (t-x)^{j-1} dB(F(t)) = -\frac{\sqrt{2}}{(j-2)!}\int_x^b (t-x)^{j-2} B(F(t))dt\,.$$

Hence, when $j > 1$, we obtain

$$T_j^D(x) \to -\frac{\int_x^b (t-x)^{j-2}B(F(t))dt}{\sqrt{\int_x^b\int_x^b (t-x)^{j-2}(s-x)^{j-2}(F(t\wedge s)-F(t)F(s))dtds}} \quad \text{if } a \le x \le b\,.$$

From this expression, we know that the limiting process of T_j is Gaussian with mean 0, variance 1 and covariance function

$$r_j^D(x,y) = \frac{\int_x^b\int_a^y (t-x)^{j-2}(s-y)^{j-2}(F(t\wedge s)-F(t)F(s))dtds}{\sqrt{\mathbb{V}_j^D(x)\mathbb{V}_j^D(y)}} \quad \text{if } a \le x, y \le b\,,$$

where $\mathbb{V}_j^D(x) = \int_a^x\int_a^x (t-x)^{j-2}(s-x)^{j-2}(F(t\wedge s)-F(t)F(s))\,dtds$. For the case $j = 1$, we have $T_1^D(x,y) = \frac{B(F(x))}{\sqrt{F(x)-F^2(x)}}$. The limiting process is also Gaussian with mean 0, variance 1 and covariance function $r_1^D(x,y) = \frac{F(x\wedge y)-F(x)F(y)}{\sqrt{F(x)F(y)(1-F(x))(1-F(y))}}$ if $a < x, y < b$.

□