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▶ **To cite this version:**

Meyer Nicolas, Olivier Wintenberger. Discussion on "Graphical models for extremes" by Sebastian Engelke and Adrien Hitz. 2020. hal-02537123

HAL Id: hal-02537123

https://hal.archives-ouvertes.fr/hal-02537123

Preprint submitted on 8 Apr 2020

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Discussion on
Graphical models for extremes
by Sebastian Engelke and Adrien Hitz

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Independence and conditional independence require distributions supported on product spaces. Engelke and Hitz define a notion of conditional independence for a multivariate Pareto distribution $\mathbf{Y} = \lim_{u\to\infty} \mathbf{X}/u \mid \|\mathbf{X}\|_\infty > u$ when

$$\mathbb{P}\big(\mathbf{Y} \in \tilde{\mathcal{E}}\big) = 1\,, \tag{1}$$

where $\tilde{\mathcal{E}} = (0,\infty)^d = \{\mathbf{x} \in [0,\infty)^d,\, \min_{1\le k\le d} x_k > 0\}$. By conditioning the limit vector $\mathbf{Y}$ on the event that $\{Y_k > 1\}$, the authors actually work on the product space $\mathcal{L}^k = \{\mathbf{x} \in \mathcal{E},\, x_k > 1\}$ with $\mathcal{E} = [0,\infty)^d \setminus \{\mathbf{0}\}$.

The definition of $\tilde{\mathcal{E}}$ encourages to use a characterization of regularly varying random vectors in terms of the minimum of their marginals. A random vector $\mathbf{X} \in \mathcal{E}$ is regularly varying on the space $\tilde{\mathcal{E}}$ if and only if

$$\min_{1\le k\le d} X_k \text{ is regularly varying} \quad \text{and} \quad \big(u^{-1}\mathbf{X} \mid \min_{1\le k\le d} X_k > u\big) \xrightarrow{d} \mathbf{Y}', \quad u \to \infty\,, \tag{2}$$

where $\mathbf{Y}'$ takes values in the product space

$$\big\{\mathbf{x} \in \mathcal{E},\, \min_{1\le k\le d} x_k > 1\big\} = (1,\infty)^d = \bigcap_{1\le k\le d} \mathcal{L}^k\,,$$

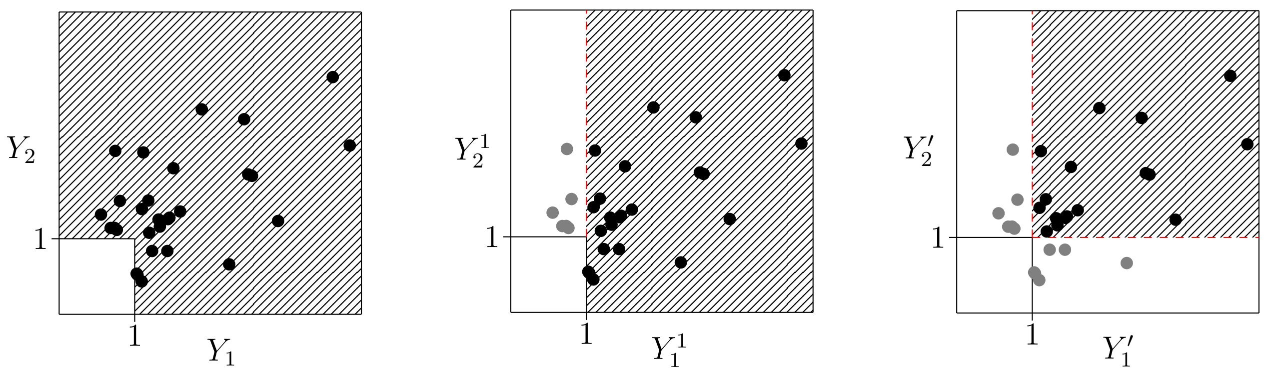
see Segers et al. (2017), Proposition 3.1. Restricting the regular variation condition of $\mathbf{X}$ to the space $\tilde{\mathcal{E}}$ is necessary to capture its asymptotic behavior through the events $\{\min_{1\le k\le d} X_k > u\}$ for $u > 0$.

In the framework of Engelke and Hitz (i.e. regular variation on $\mathcal{E}$ and assumption (1)), both conditions in (2) hold and the limit vector $\mathbf{Y}'$ corresponds to the vector $\mathbf{Y}$ conditioned on the event that $\{\min_{1\le k\le d} Y_k > 1\}$, see Figure 1. Conversely, the two assumptions in (2) are more general since they include for instance the case where the marginals of $\mathbf{X}$ are independent (and then $\mathbf{Y}$ exists and its distribution concentrates on the axes).

An alternative notion of conditional independence for a multivariate Pareto distribution can thus be defined through conditional independence of $\mathbf{Y}'_A$ and $\mathbf{Y}'_C$ given $\mathbf{Y}'_B$. Such property holds if the density $f_{\mathbf{Y}'}$ of $\mathbf{Y}'$ factorizes as

$$f_{\mathbf{Y}'}(\mathbf{y}') f_{\mathbf{Y}',B}(\mathbf{y}'_B) = f_{\mathbf{Y}',A\cup B}(\mathbf{y}'_{A\cup B}) f_{\mathbf{Y}',B\cup C}(\mathbf{y}'_{B\cup C})\,, \quad \mathbf{y}' \in (1,\infty)^d\,.$$

Regarding the original vector $\mathbf{X}$, the study of $\mathbf{Y}'$ instead of $\mathbf{Y}$ provides two advantages. First, the vector $\mathbf{Y}'$ models the extremal behavior of $\mathbf{X}$ when all its marginals are simultaneously large. Therefore, it provides accurate models for extremal dependent data. Note that if the strong



(a) (b) (c)

Figure 1: The shaded areas correspond to (a) the support of $\mathbf{Y}$, (b) the support of $\mathbf{Y}^1$ and (c) the support of $\mathbf{Y}'$.

condition that all marginals are extreme together is not satisfied, then (2), and thus (1), is very unlikely. Second, conditional independence of $\mathbf{Y}'$ can be interpreted in terms of $\mathbf{X}$. Indeed, if $\mathbf{X}_A$ is conditionally independent of $\mathbf{X}_C$ given $\mathbf{X}_B$, then $\mathbf{Y}'_A$ is conditionally independent of $\mathbf{Y}'_C$ given $\mathbf{Y}'_B$. In the case $B = \emptyset$, we obtain independence of $\mathbf{X}_A$ and $\mathbf{X}_C$ and thus of $\mathbf{Y}'_A$ and $\mathbf{Y}'_C$, whereas $\mathbf{Y} \in \mathcal{E} \setminus \tilde{\mathcal{E}}$ (as soon as $\mathbf{Y}$ exists). There, (2) is satisfied but not (1).

References

Segers, J., Zhao, Y., and Meinguet, T. (2017). Polar decomposition of regularly varying time series in star-shaped metric spaces. *Extremes*, 20(3):539–566.